TPSEMath
Transforming Post-Secondary Education in Mathematics

Formative Assessment in Mathematics in Remote Settings

Panelists
Abbe Herzig, American Mathematical Society
Rena Levitt, Minerva Schools at KGI
Francis Su, Harvey Mudd College
Michelle Younker, Owens Community College
Kate Stevenson, CSU Northridge

Moderated by Nancy Sattler, Terra Community College & Walden University

AMS webinar recording policy

- The AMS is recording the webinar and reserves the right to show it again and distribute it.
- By participating, you are agreeing that your contributions become part of the recording.

Questions and comments

- To ask a question of the panelists, please submit it through the Q&A button. We will get to as many questions as we can.
- The Chat function is enabled for attendees to share ideas and resources with each other. The panelists and hosts may review the chat after the webinar, but may not be able to respond to questions posted there.
- To share ideas with other attendees, make sure to send messages to “all panelists and attendees.”

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Considerations

- Please be open to new ideas and be considerate of others in your comments.
- ***Please make sure that you are muted.***

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Formative Assessment

Abbe Herzig

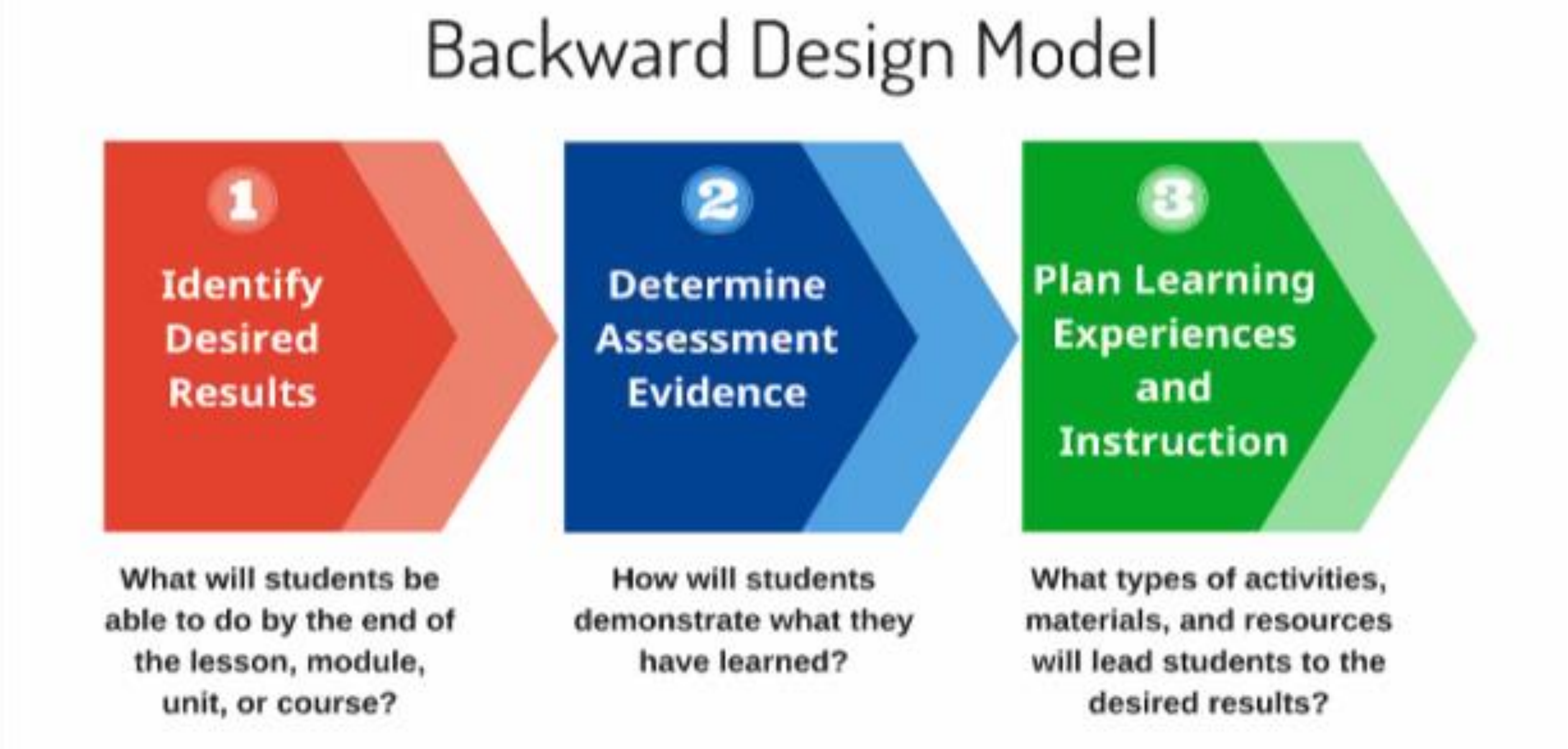
Director of Education, American Mathematical Society

A way to think about assessment

Assessment provides instructors and students a window into students’ progress toward course learning objectives.

In particular, assessment helps students understand

“Have I learned what I need to learn?”



Align assessment and learning experiences to your learning goals

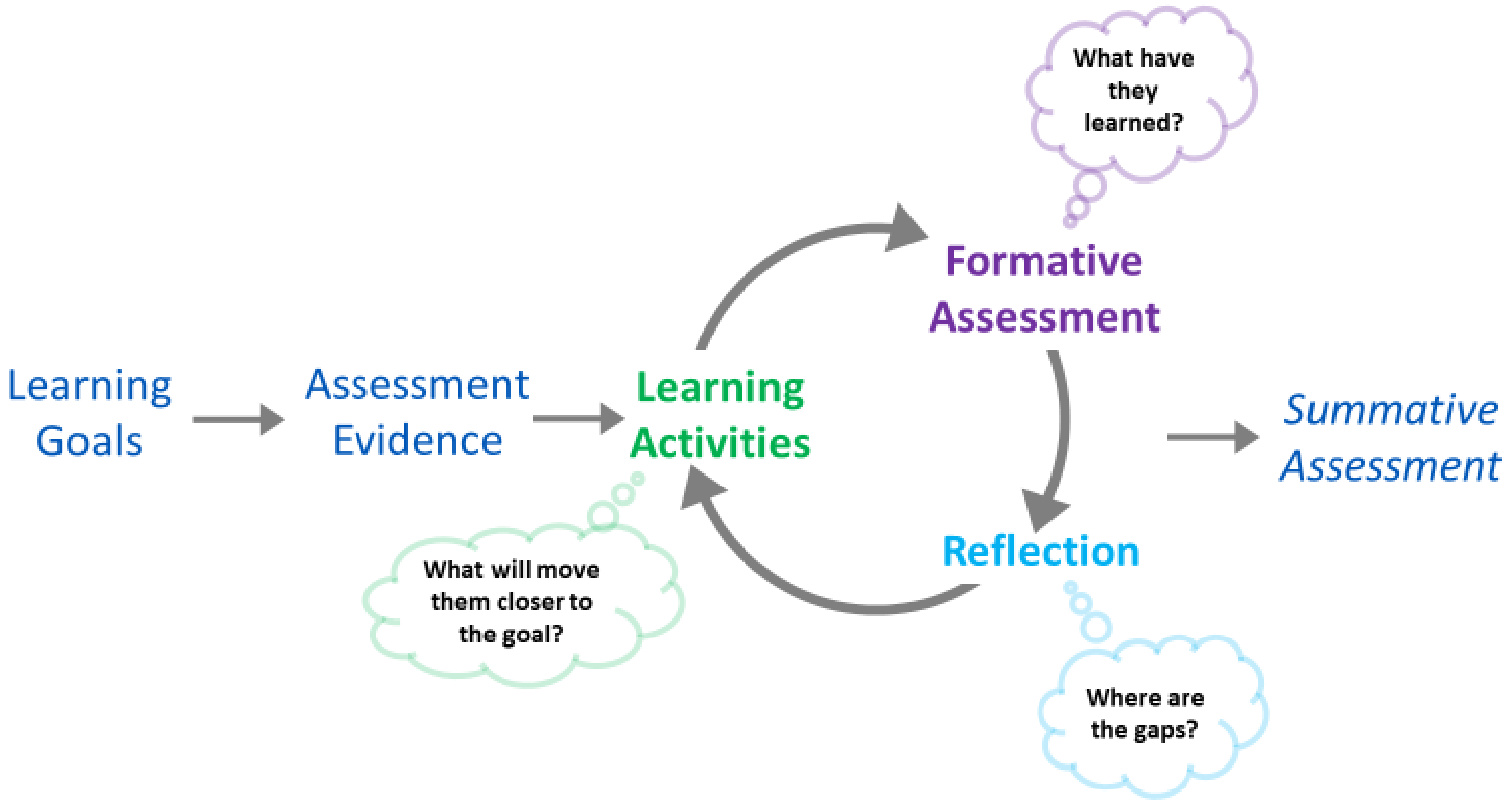
(How can you get them there?)

Formative assessments

- Occur throughout a class or course
- Seek to improve students growth toward learning objectives
- Can be graded or ungraded

Summative assessments

- Occur at the conclusion of a unit or course
- Measure student learning, knowledge, proficiency
- Usually graded and often heavily weighted (*but don't have to be*!)

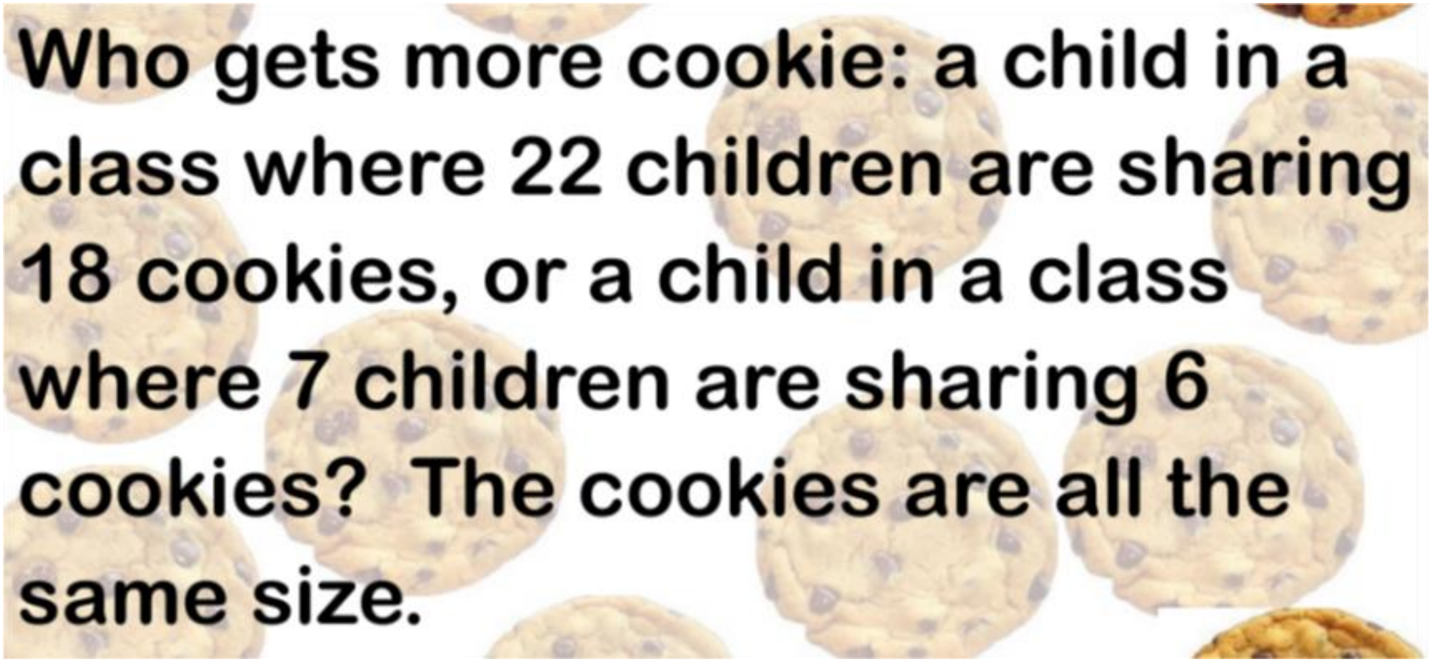

Learning Goals
Assessment Evidence
Learning Activities
Formative Assessment
What have they learned?
Reflection
Where are the gaps?
What will move them closer to the goal?
Summative Assessment

An example:

Learning goal: Students will be able to solve mathematics problems by applying reasoning and critical thinking skills.

Evidence: Students will develop a detailed solution to a simple problem and justify each step in their reasoning.

Learning activity: Students solved a non-standard problem, shared their solutions with each other in VoiceThread, and compared strategies.



Formative Assessment:

- Students posted an initial solution to VoiceThread.
- They challenged each other by asking for clarification, correcting errors, answering questions, and explaining concepts.
- They helped each other improve their solution strategies.

Summative Assessment

- Students submitted a written solution using any strategy they could justify.

Ground rules:

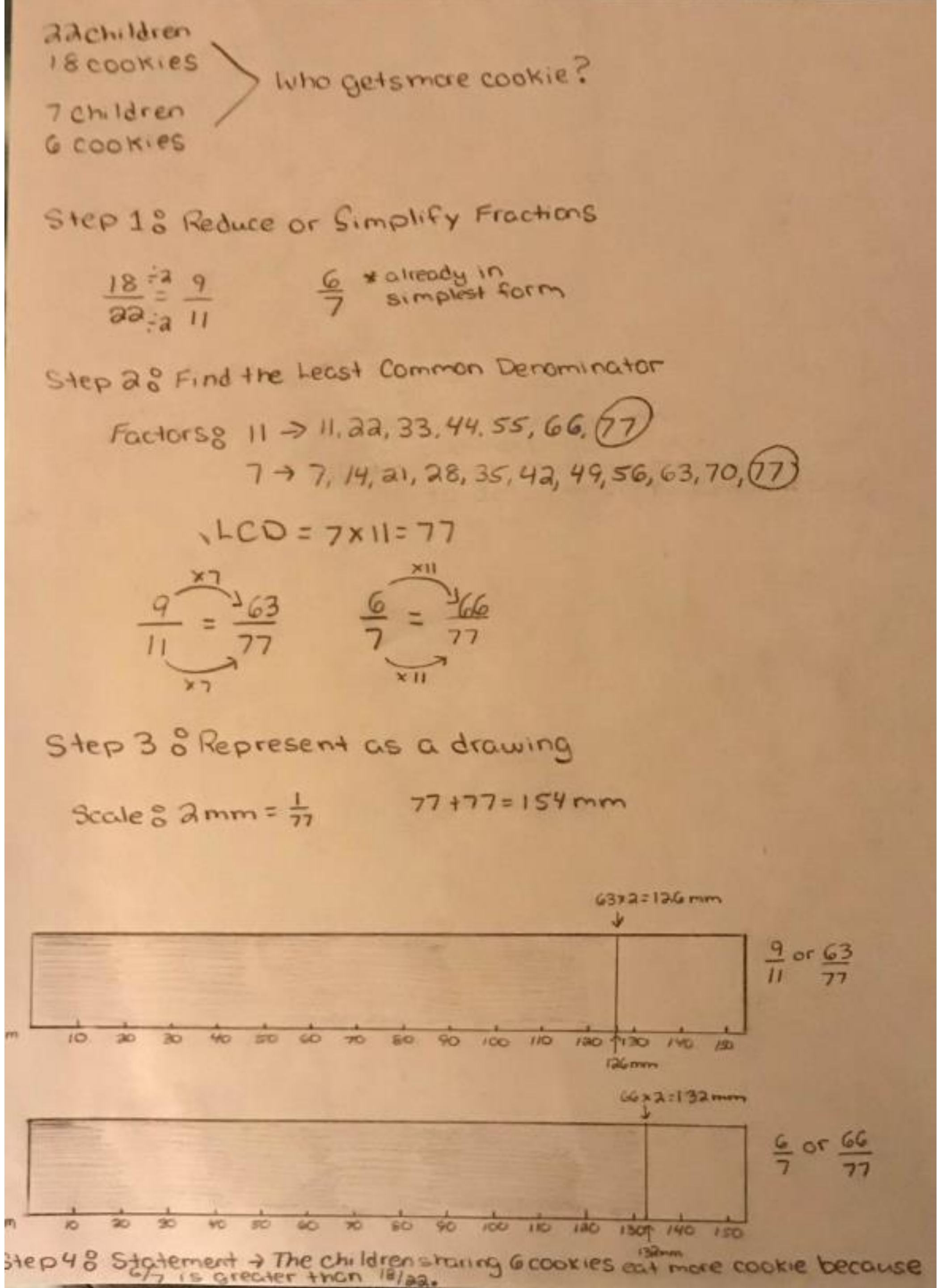
- Be creative! Step outside your comfort zone and try something new.
- Go beyond *how* to solve the problem, to *why* your solution makes sense.
- *Do not use standard algorithms or rote procedures*, unless you can clearly justify *why* they work and *why* they make sense to use for this problem.
- Your explanation should be detailed enough for others to understand your reasoning.
- Your strategy will not be assessed for being "right" or "wrong" but for its creativity and the clarity with which you explain your reasoning.

Learning Goal: To gain experience in solving a problem based on your own reasoning, rather than using rote procedures.

Ratios

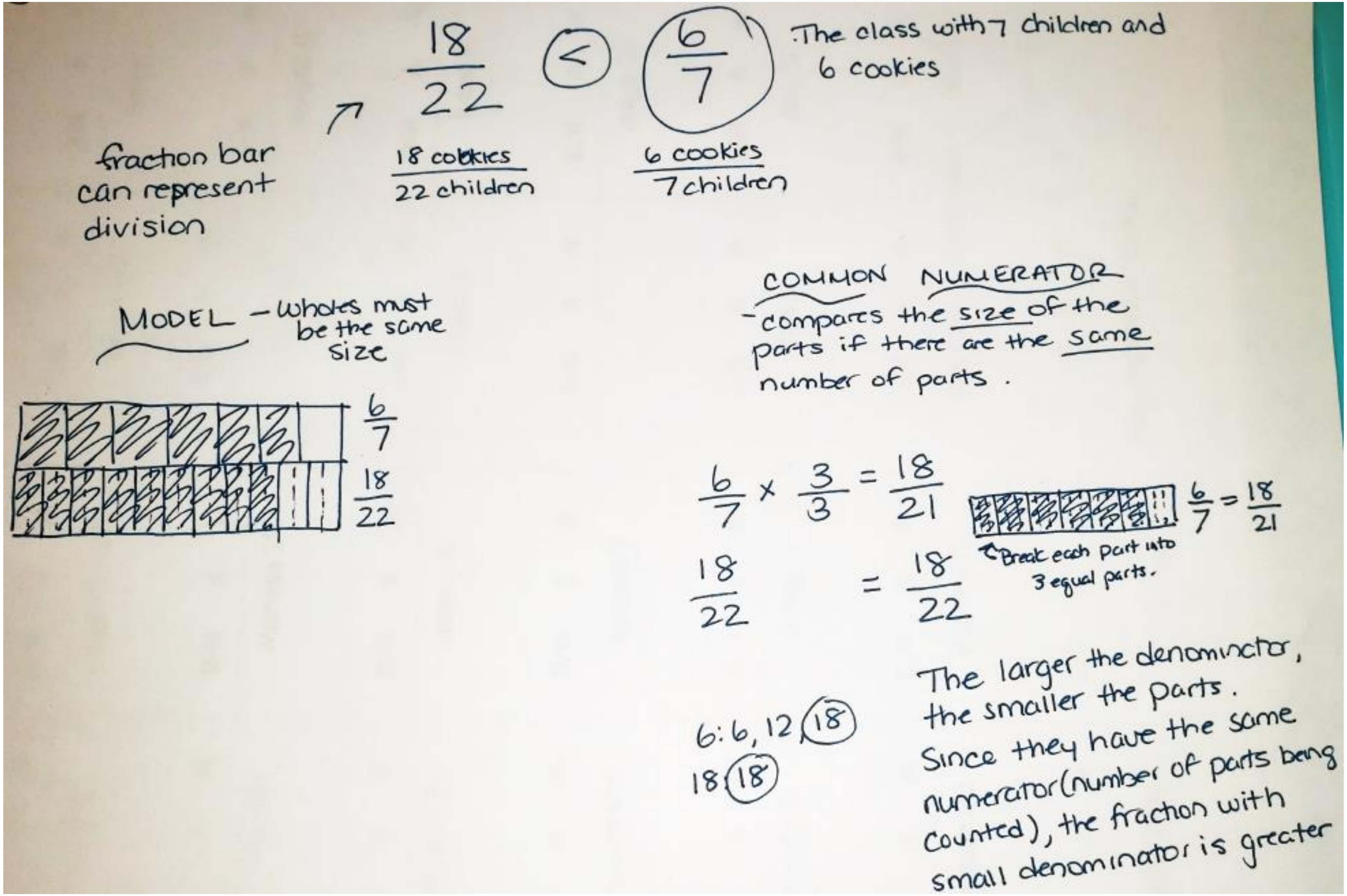
Common denominators

Scale model

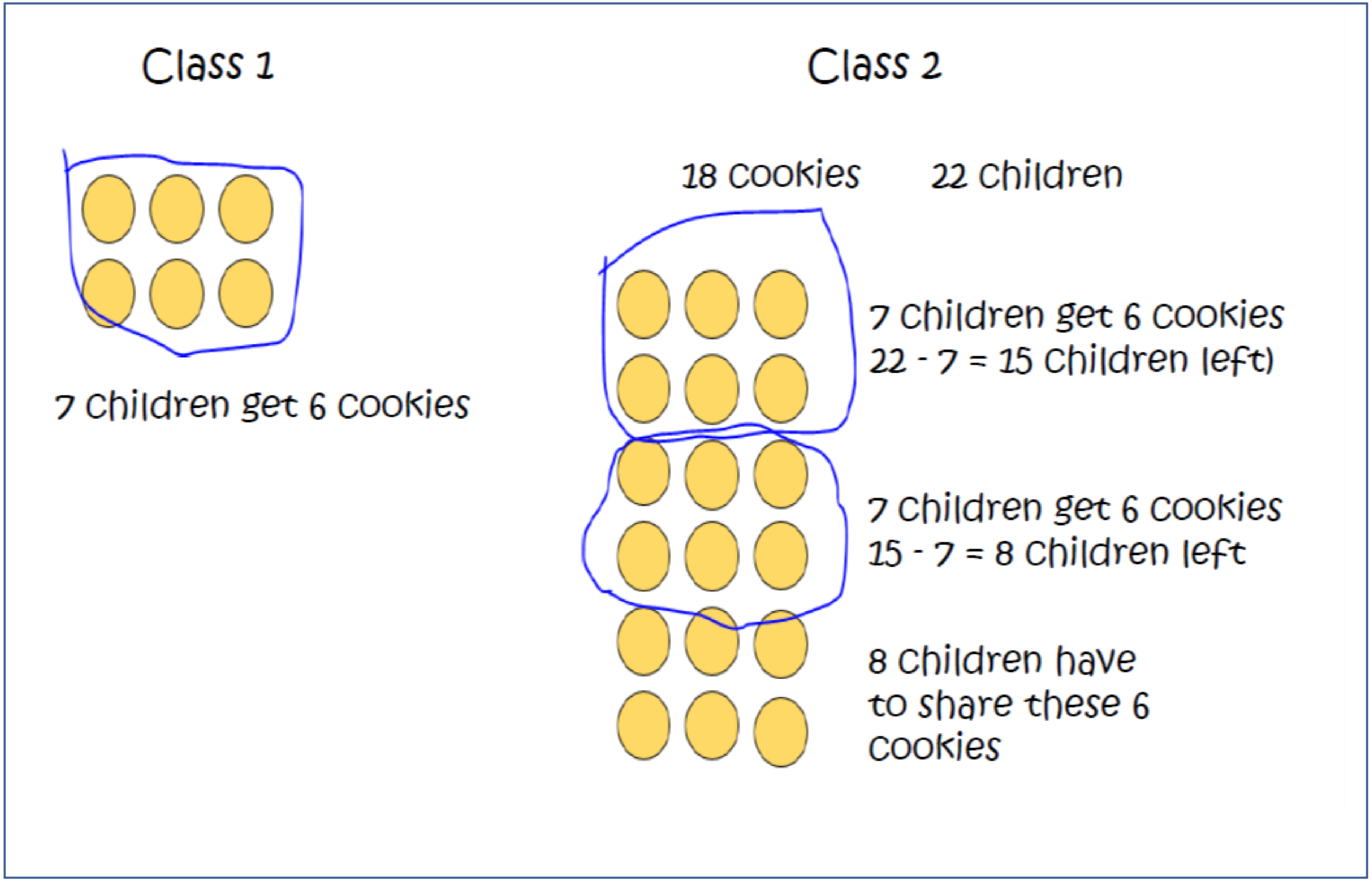


Ratios

Common numerators

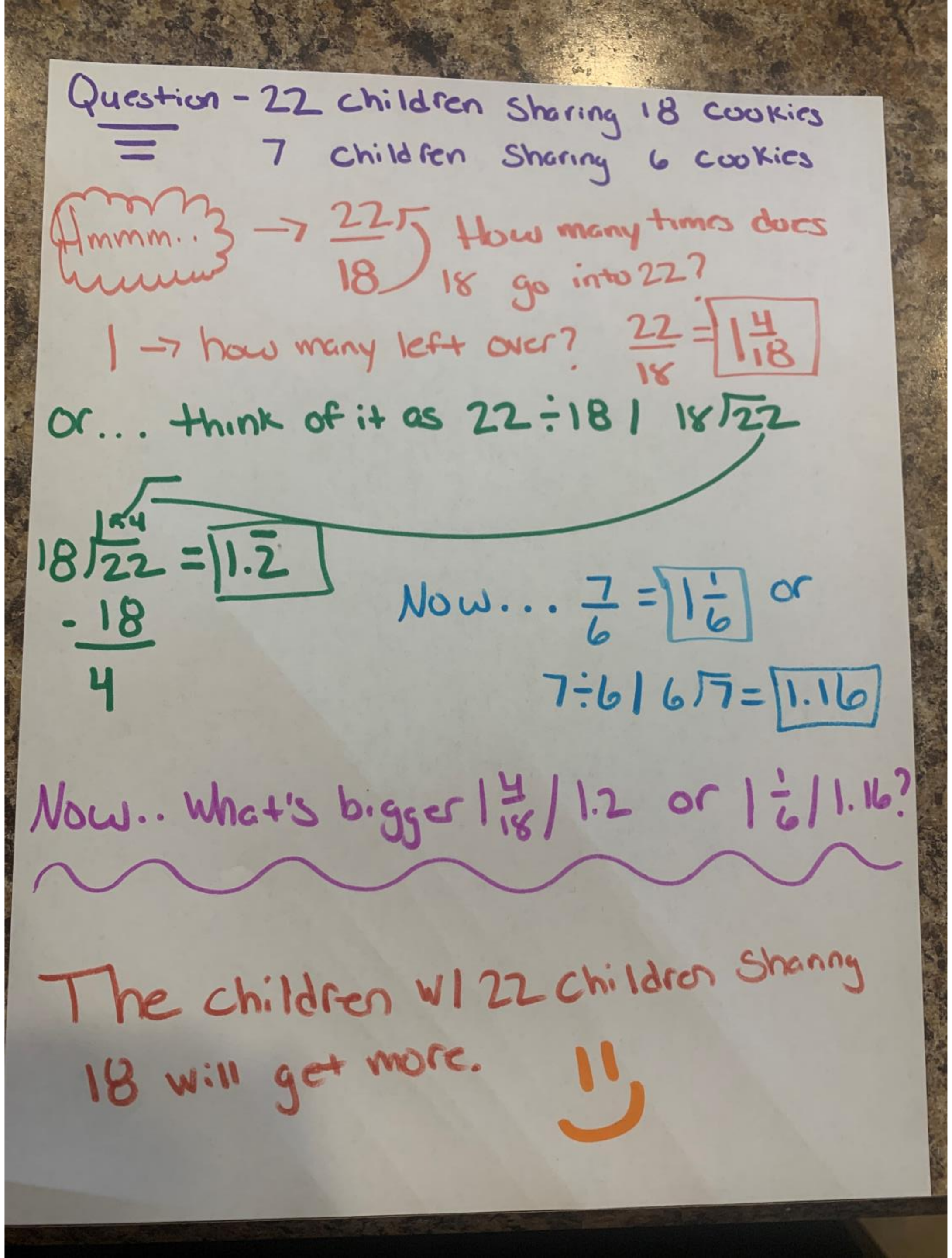


Proportional reasoning



Ratios

Confusion with units



Some formative assessment strategies

1. **Reflect** on what you’ve learned.
2. **Explain** three important concepts and provide an example of each.
3. **Connect** what you’ve learned to ideas from another course or discipline.
4. **Justify** the steps in a solved problem and **identify** errors.
5. **Correct** or **confirm** a counterexample to a statement.
6. **Summarize** ideas on an *Empty Outline* after a lecture or textbook chapter.
7. **Identify** the *Muddiest Point* in a lecture, discussion, or assignment

A few other strategies:

- Minute papers
- Polling
- 321 exit ticket: 3 things I learned, 2 things that were interesting, 1 question I have.
- Twitter model: tell me in 75 works or less . . . .

Classroom Assessment Techniques:

- https://vcsa.ucsd.edu/_files/assessment/resources/50_cats.pdf

Embedding formative assessments into your course

Rena Levitt, Minerva Schools at KGI

Embedding Formative Assessments into Course Structure

Before the semester:

- *Diagnostic interviews or surveys*

During the semester:

- *Frequent, low-stake quizzes*
- *Observations*

Pre-semester diagnostic assessment

- Determine students' backgrounds and interests
- Collect data on students' motivations and goals, plans for engaging with course material, and constraints while remote
- Gauge students' preparation for the course material
- Example for Linear Algebra course:
    - *Have you encountered matrices previously? If so, how have you used matrices in other contexts?*
    - *What methods to solve linear systems of equations are you comfortable applying? Give a brief description of each method you list.*
    - *Which of the methods you listed would you prefer to apply to the system $x + y + z = 0$, $x + 2y + 3z = 4$, $x - y = 2$? Why? Apply the method to solve the system. Make sure to provide a detailed solution.*

Daily, low-stakes quizzes

- Preparatory assessment ~ Gauges students comprehension of key concepts introduced before class.
  - You are trying to come up with a model for your laundry habits: on a given week, you can either do your laundry or not do your laundry. Write a Markov matrix that represents your own laundry habits and calculate the probability that you will do laundry two weeks from now.
- Closing assessment ~ Gauges student progress on learning goals during the class session.
  - Explain the relationship between eigenvalues, eigenvectors, and the long-term behavior of a Markov model.

Formative feedback on student contributions

- Review a portion of a recorded class session:
  - Specific activity ~ collaborative problem solving activity, debrief of a breakout group, student presentation of solutions
    - Example ~ Students rate a solution on a simple rubric: Incorrect / Correct but unclear / Correct and clear (Sundstrom, 2019). Instructor calls on students to justify their rating.
  - Instructor assesses student contributions during the activity and sends students targeted feedback tied to a specific learning outcome.

Other benefits of observing class recordings

- Identify potential adjustments for subsequent sessions
- Identify and track revisions for future semesters
- Use for peer feedback among faculty and professional development

Rewrites & Reflection in upper-division courses

Francis Su

Harvey Mudd College

Rewrites

- **Analysis I, course goals:**
  - Learn analysis
  - Learn to communicate math well
- **Writing:**
  - Uneven across backgrounds
  - Is a process, and can always improve
- **Allow Rewrites on the first 4 assignments**
  - Optional: Due week after HW is returned
  - Graders trained to look for writing improvements

Handout on Good Mathematical Writing

- https://math.hmc.edu/su/writing-math-well/

Or google

"Francis Su writing math well"

to find it

GUIDELINES FOR GOOD MATHEMATICAL WRITING

FRANCIS EDWARD SU

Communicating mathematics well is an important part of doing mathematics. Whether you are speaking or writing, learning to communicate effectively is not just a service to your audience; it is also an exercise in clarifying and structuring your own thinking. Moreover, beyond developing a basic competence, there is an art and elegance to good writing that every writer should strive for. And writing, as a work of art, can bring great personal satisfaction.

These guidelines may serve as a starting point for good mathematical writing.

1. BASICS

Know your audience. This is the most important consideration for writers. Put yourself in your reader's shoes. What background can we assume of the reader? What terminology should we define? What kind of "voice" do we want to project: casual or professional, serious or inviting, terse or loquacious?

If you are a student writing solutions for a homework set and your professor has not specified your audience, a good rule of thumb is to assume you are writing to another student in the course who has not yet done the assignment. Though you may assume that she has attended all the same lectures and has read the same textbook, it is standard courtesy to remind your reader of any relevant items that she has recently learned in class or from the textbook, or things she should know but might have forgotten.

For instance, if the concept of a rational number was only recently learned in class, you might insert "Recall that a rational number can be expressed as a fraction;" before saying: "since x is rational, we can set $x = m/n$ where m and n are integers."

Set an invitational tone. It is traditional to create an inviting atmosphere in one's mathematical writing. In effect, we invite readers to join us in our reasoning process by writing in the present tense, using the pronoun "we" instead of "I" (e.g., "we construct a tangent plane..."), and directing the reader with gentle commands (e.g., "let n be...", "recall that...", or "consider the set of...").

Use complete sentences. All mathematics should be written in complete sentences. Open any mathematics text and you'll see that this is true. Equations, even displayed ones, have punctuation that help you see where it fits in the context of a larger sentence. Consider this piece of writing:

Questions on HW#0

- To ensure students have read the handout:

Directly from the handout on good writing:

- 2. What is a good rule of thumb for what you should assume of your audience as you write your homework sets?
- 3. Do you see why the proof by contradiction on page 3 is not really a proof by contradiction?
- 4. Name 3 things a lazy writer would do that a good writer wouldn't.
- 5. What's the difference in meaning between these three phrases?
  Let $A = 12$.
  So $A = 12$.
  $A = 12$.

Rewrites

- **Graders:**
  - Grade for writing style, more strict on first assignments
  - Scores can only improve on rewrites (and they do!)
- **Outcomes:**
  - Writing drastically improves when students understand what is expected
- **Another variant in my IBL Topology class:**
  - Student create a joint solutions manual, and revise each others solutions
  - Write up 15 solutions over course of semester
  - Revise 15 other solutions over the course of the semester

Reflection Questions – follow up

- "Seven Exam Questions for a Pandemic" https://www.francissu.com/post/7-exam-questions-for-a-pandemic-or-any-other-time
- My Galois Theory course

2.(40 pts) In no more than 2 or 3 pages, explain Galois Theory to a friend who has taken a first course in algebra (Math 171) and has a basic understanding of polynomials. You are trying to give an overall picture of the subject, which means: you are not providing proofs, but you are providing conversational insight. This means that you are still trying to give motivation, and a good example can go a long way to providing insight, but you do not need to get into technical weeds.

Your description should devote at least a paragraph to these elements (and indicate where these may be found in your essay by marking them with these letters):

(a) what can I learn from a polynomial by studying its splitting field?

(b) what is the significance of the fundamental theorem of Galois theory say?

(d) give a couple examples of polynomials and their Galois groups, and what their groups tell us about the polynomial or anything else.

(e) how Galois theory helps us answer certain classical 'impossibility' results: give an insight you had when you learned this.

Points will be awarded based on the criteria above, and on strength of answer.

Reflection Questions – follow up

3.(10 pts)
Consider one mathematical idea from the course that you have found beautiful, and explain what the idea is (with an example), and why it is beautiful to you. Your answer should: (a) explain the idea in a way that could be understood by a classmate who has taken a first course in algebra (Math 171) and (b) address how this beauty is similar to or different from other kinds of beauty that human beings encounter.

a root in F. The uniqueness part of this result is beautiful because it helped me see $\mathbb{C}$ as a "natural" way to extend $\mathbb{R}$ even though it has quirky multiplication rules and is not an ordered field. This beauty is like the satisfaction at the end of a mystery novel when the strange characters or behaviors suddenly seem totally logical. The existence part is beautful in the way it streamlines so many arguments. Instead of showing ad-hoc that roots exist in some extension when working with arbitrary fields, we can just pull them from the algebraic closure. This is akin to the beauty we admire in complex machines like cars and computers where we don't have understand the particular components because we know how to operate a common abstract interface.

Similar to the Mona Lisa or other timeless classic art, these constructions can be appreciated by anyone, regardless of upbringing or ability. In the same way everyone finds a flower beautiful, these constructions can be appreciated as well. Additionally, the visual nature of these constructions makes their beauty similar to that of visual art. Both tell a story without the use of words. This concept in itself is beautiful, as it distills a concept down to its basics.

I find the depth of constructions similar to the relationship between traditional and modern art. The construction of the equilateral triangle is easily appreciated, just like a traditional painting. Both are accessible. On the other hand, the construction of a 17-gon is difficult to understand, and ever more difficult to attempt to replicate. When constructions are taken to their limits, they are obtuse. I feel like modern art is in a similar position. In my art history class last semester, my classmates and I were often left puzzled after Prof. Fandell discussed a banana taped to a wall or a toilet made of gold. We often had to ask, "what is the meaning of this piece?" Someone seeing the construction of a 17-gon for first time might ask the same question. Ultimately, I believe that the accessibility and universal nature of these constructions make them beautiful.

(b) This kind of beauty is reminiscent of the idea of a soul mate - someone who in some sense matches you perfectly and who you always want to be together with. Roots of the same minimal polynomial have the same "nature" in some way, and never occur without each other. The idea of two separate things being connected on some deep level also occurs in a lot ideas of dualism (you can't have shadow without light, you can't have cold without warmth, etc.) which have been present in religions and human beliefs for longer than recorded history, and must appeal to us on some deep level. I guess it is beautiful to see that this idea that is so deeply ingrained in us has an equivalent which can be proved even in the world of mathematics.

Formative Assessment
Self-assessment

Michelle L. Younker
Department of Mathematics/School of STEM
Owens Community College

Self-assessment allows students to:

- focus on their own work;
- monitor their own strengths and weaknesses;
- target areas that need attention; and
- identify how they learn

with the goal of improving understanding and performance.

Self-assessment tools

- Learning Logs
- Self-report note
- Pause
- Classroom Response Systems (Clickers)
- Quick nod/Exit slip/Thumbs up/Fingers up/Traffic Lights

Learning Log Prompts

- Explain how I organize my math notes. How does my organizational method help me?
- What do we do today? Why did we do it?
- What did I learn today? How can I apply it?
- What questions do I still have about it?

Classroom Response System Prompts

- From Calculus:

  True or False. If $f''(a)$ = 0, then f has an inflection point at a.

- From Differential Equations:

  Which of the following is not a differential equation?

  (a) $y' = 5y$

  (b) $tx\frac{dx}{xt} = 4$

  (c) $2x^2\mathrm{y} + y^2 = 8$

  (d) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

  (e) All are differential equations.

Formative Assessment Mastery via Many Micro-Tests with Multiple Attempts

Katherine (Kate) F. Stevenson
Department of Mathematics
California State University, Northridge

Context: Foundations of Mathematics
Reading, synthesizing, and communicating Math?

- Written weekly homework assignments

  30% (drop 1), please work in groups & write up your own work

- Bi-Weekly Journals & Presentations

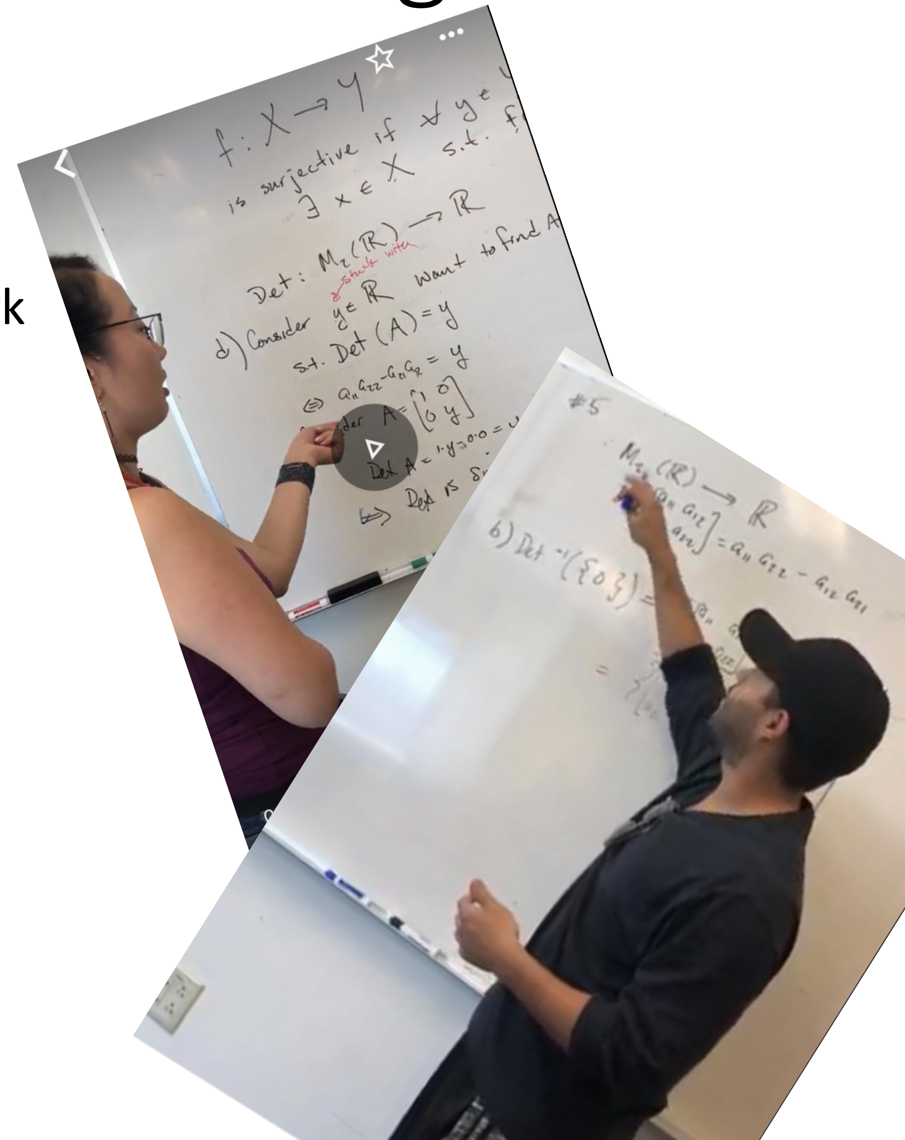
  10%, reflections & practice writing & speaking mathematics

- 2-3 Midterms

  30% (redo available for each in office hours)

- Cumulative final examination

  30%



How best to assess reading, synthesizing, and communicating?

- Written weekly homework assignments

  30% (drop 1), please work in groups & write up your own work

- Bi-Weekly Journals & Presentations

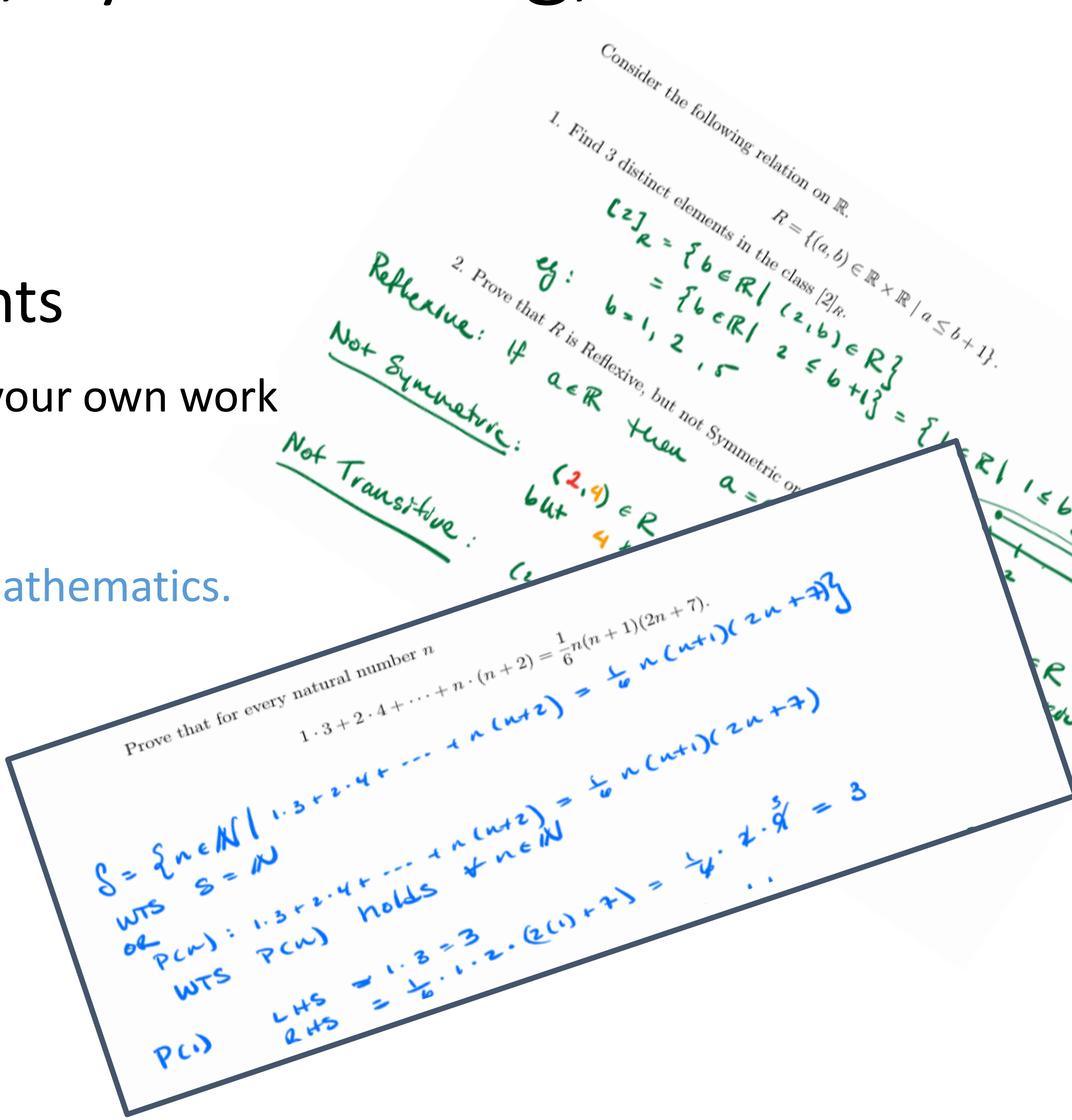
  10%, reflections & practice writing & speaking mathematics.

- **Bi-weekly micro-tests**

  30% (redo available for each in office hours)

- Cumulative final examination

  30%



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- Bi-Weekly Journals & Presentations

  10%, reflections & practice writing & speaking mathematics.

- **Bi-weekly micro-tests**

  30% (redo available for each in office hours)

- Cumulative final examination

  30%

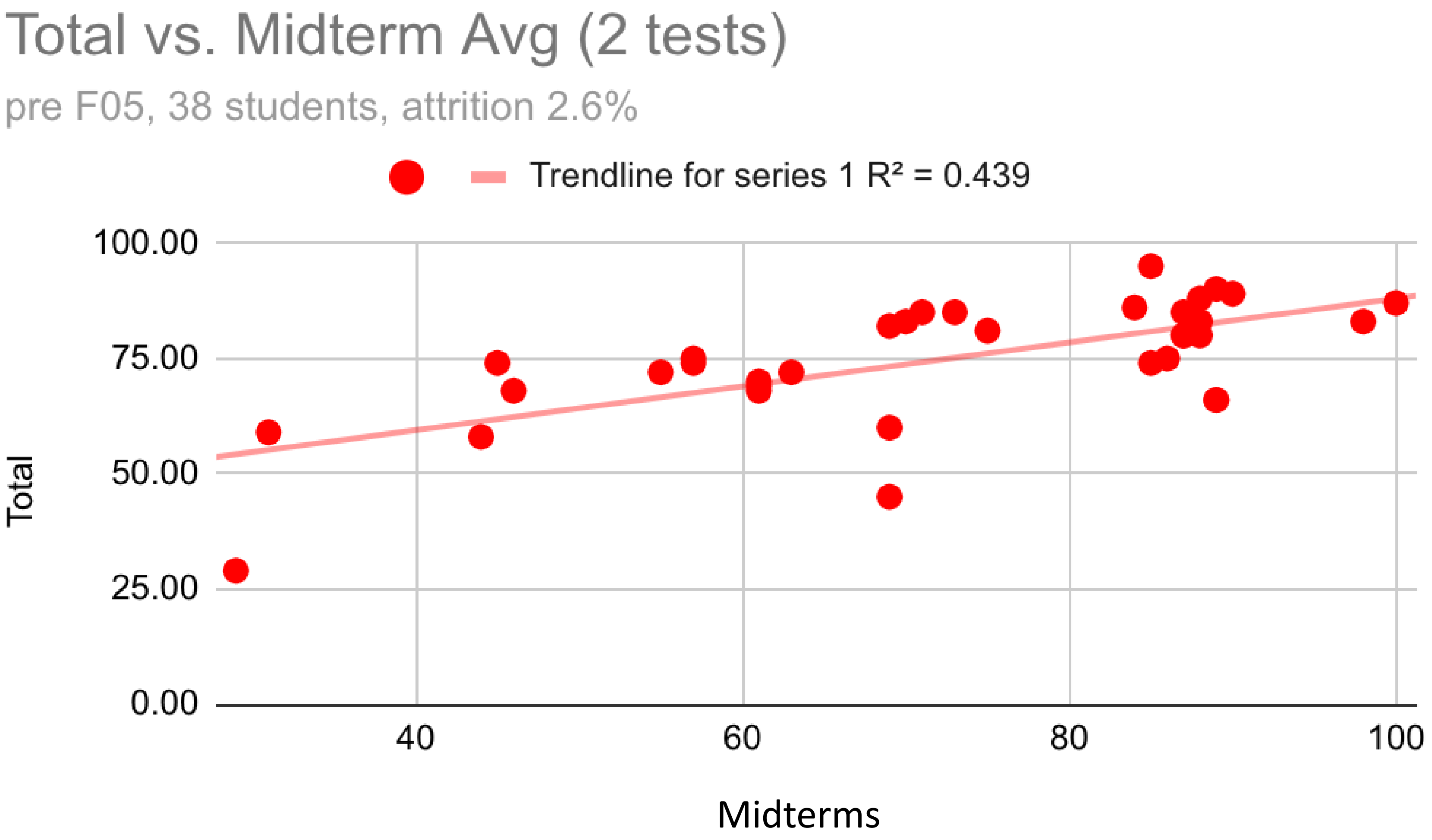
Differen
t weeks

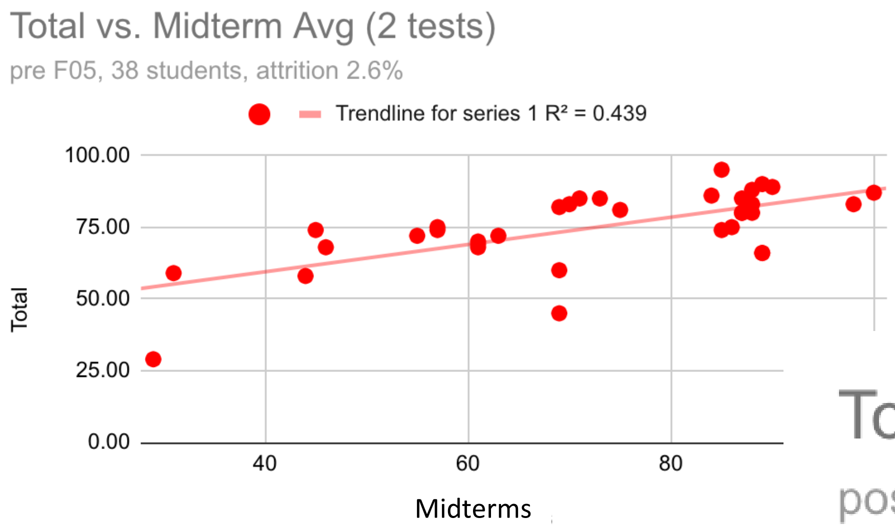
Homework turned in Wednesday, graded and returned Monday.

- Quiz based on returned homework, only.
- Given Wednesday, graded and returned Monday.

Transparent, predictable… equitable

What happened?





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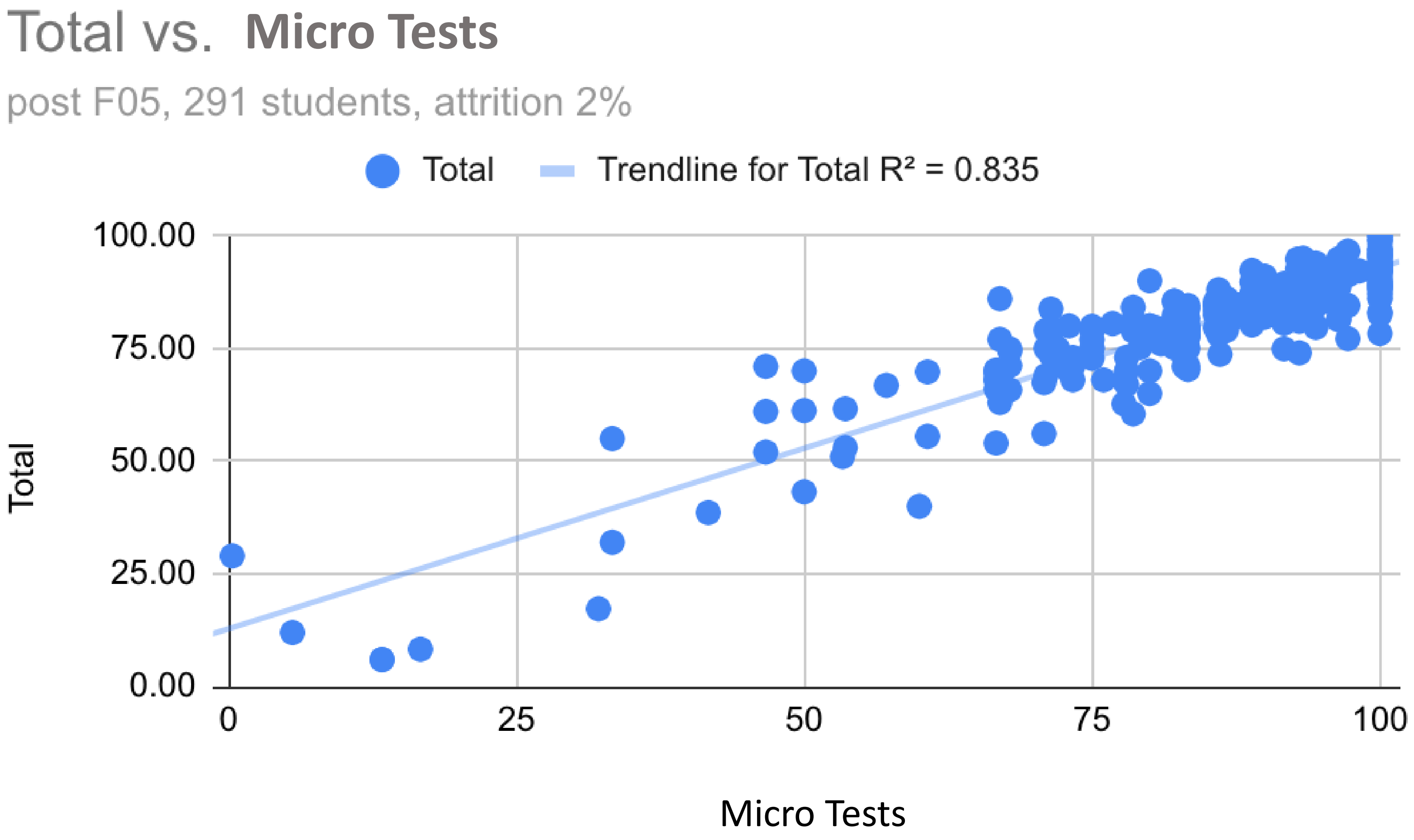

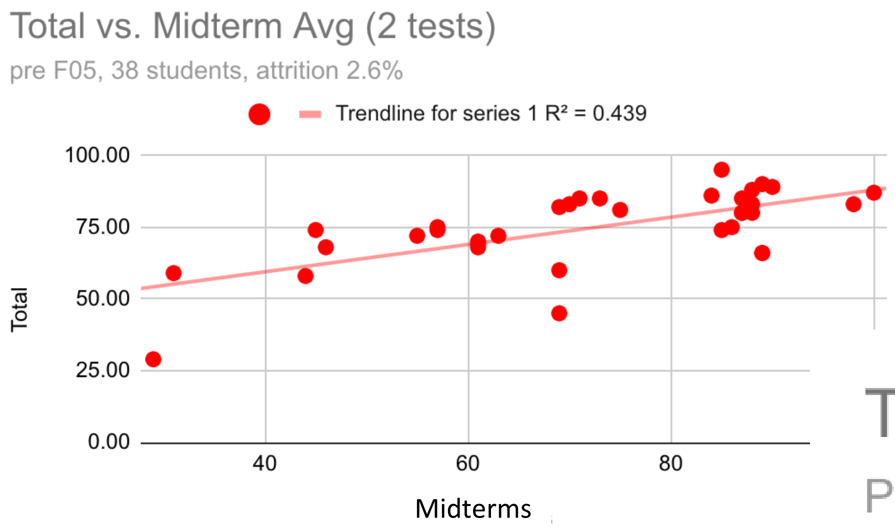

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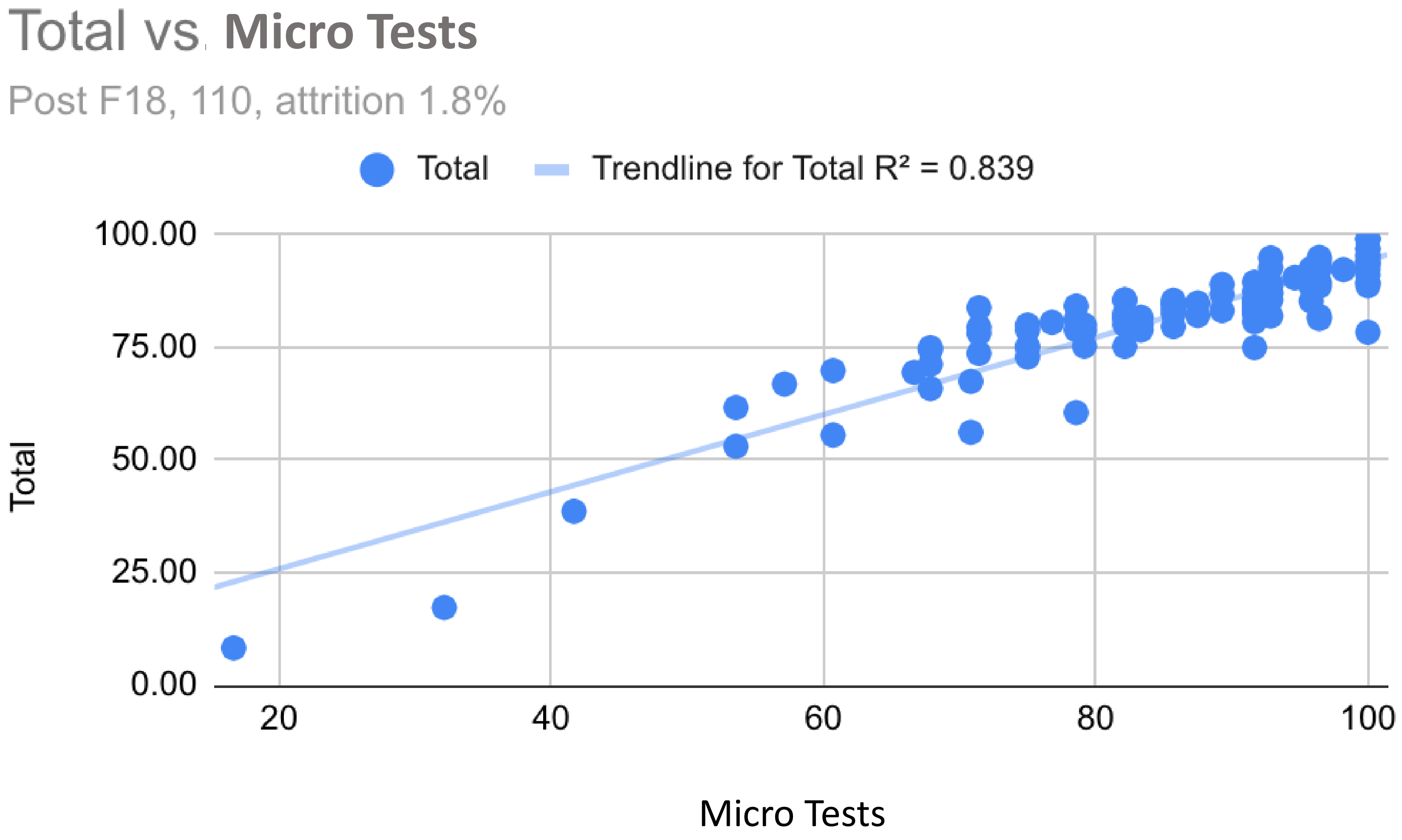

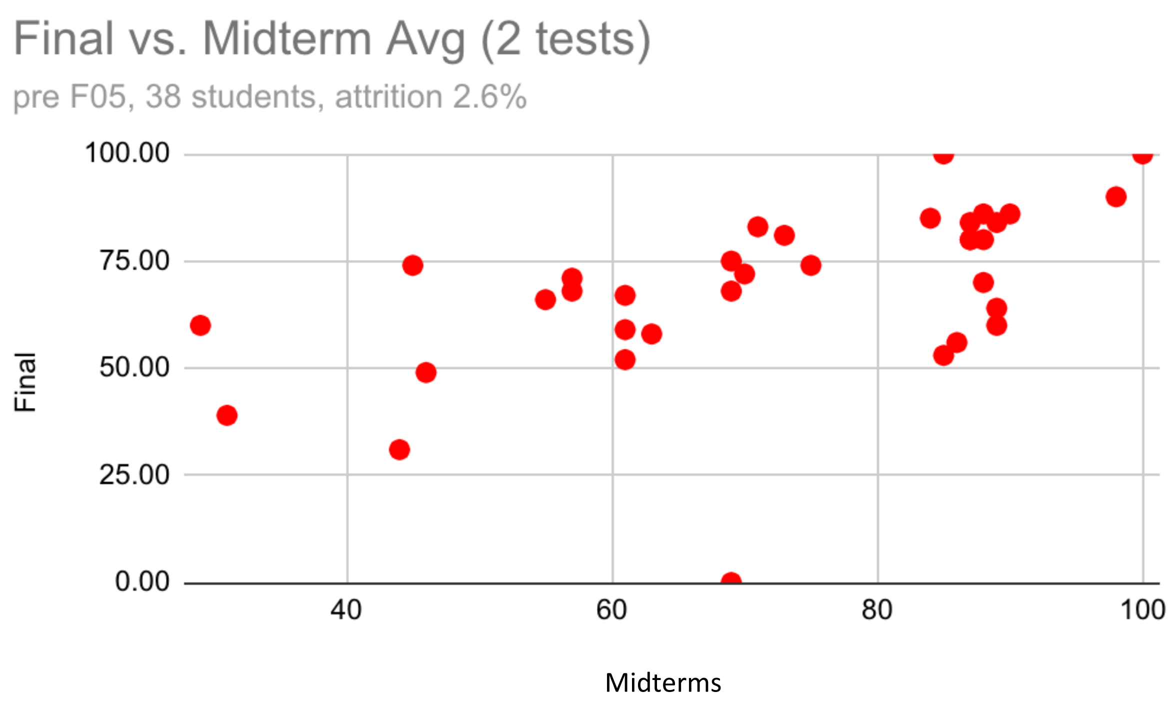

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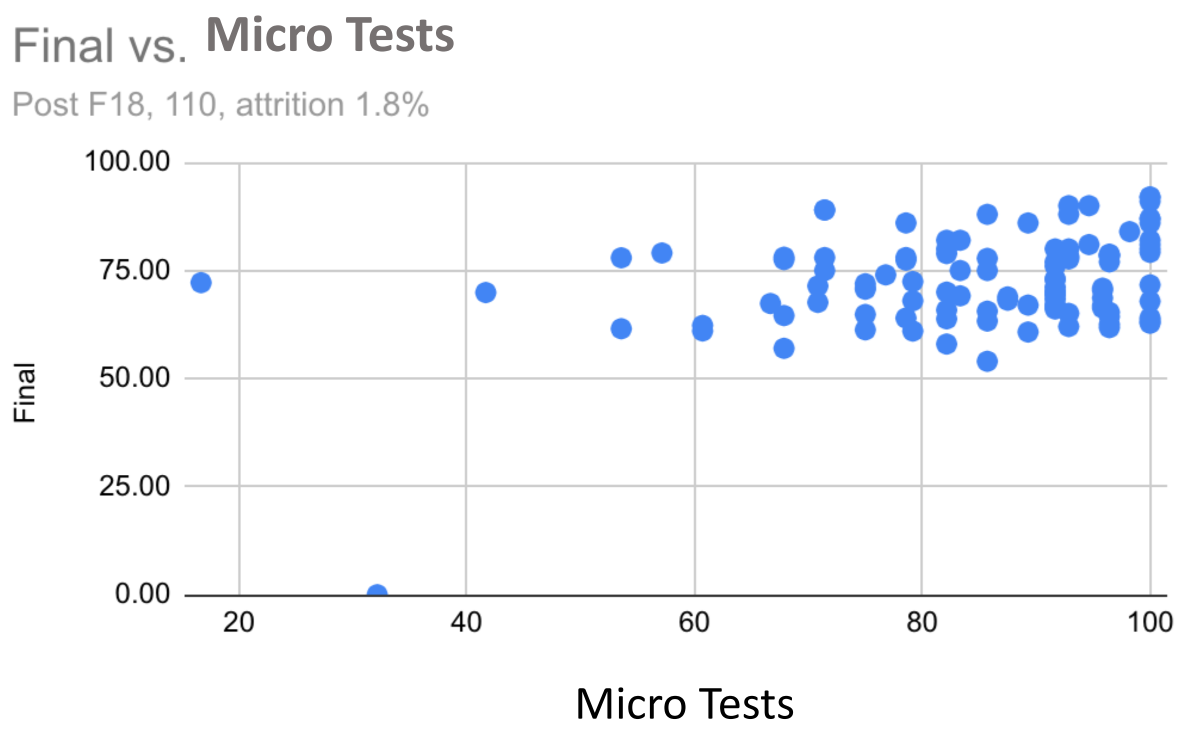


How do we measure this reading, synthesizing, and communicating **online**?

- Written weekly homework assignments

  30% (drop 1), please work in groups & write up your own work

- Bi-Weekly Journals

  10%, reflections & practice writing & speaking mathematics

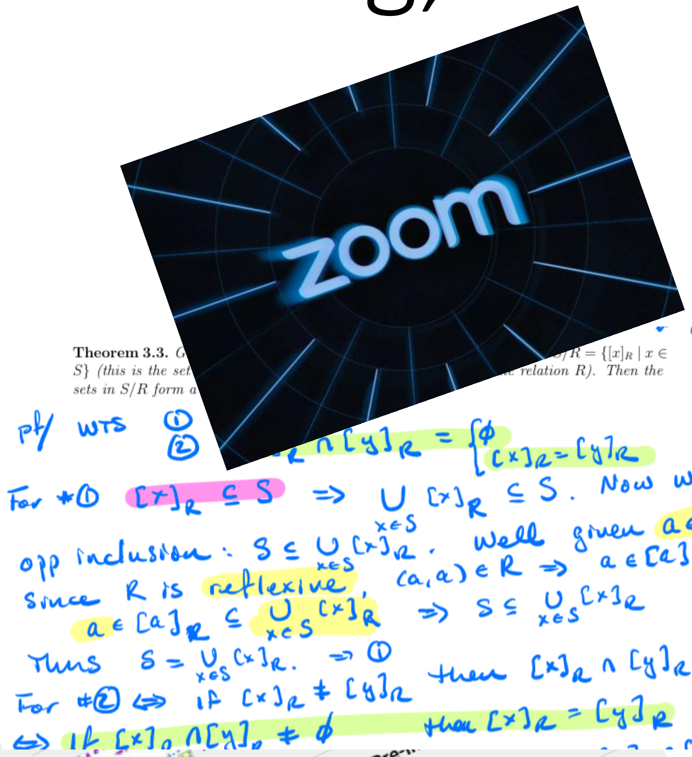
- Bi-weekly micro-tests

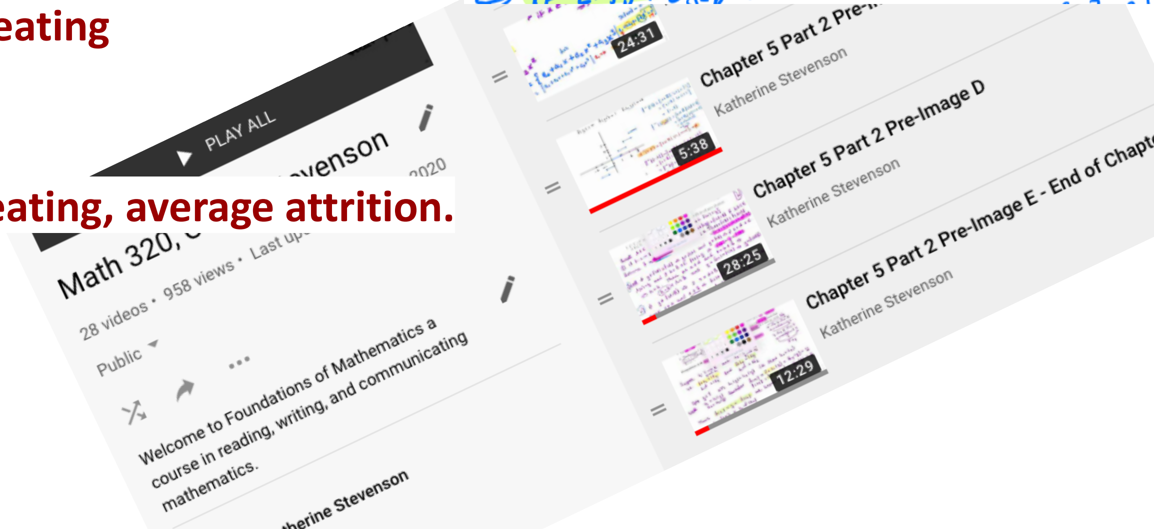
  30% (redo available for each in office hours)

  synchronous @ end of class period/office hour, little cheating

- Cumulative final exam

  30% **asynchronous, 4 day window, gradescope, little cheating, average attrition.**





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Questions/Sharing

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What topics are you interested in seeing in future TPSE webinars?