## Proceedings of the

## Conference on

## Summer

Undergraduate
Mathematics
Research
Programs
Joseph A. Gallian Editor

Cover photographs courtesy of Penn State Erie, The Behrend College, Erie, PA, and AU Photos, Auburn University, AL.

This material is based upon work supported by the National Security Agency under Grant MDA 904-99-1-0063.

[^0]Copyright © 2000 American Mathematical Society
Printed in the United States of America.
All rights reserved.
10987654321050403020100

## Table of Contents

## Preface <br> Part I. Descriptions of Summer Research Programs

 ixAuburn University ..... 3
California State University Fullerton ..... 9
Carnegie Mellon University ..... 13
College of William and Mary ..... 19
College of Wooster ..... 31
Cornell University ..... 37
Cornell University ..... 41
Davidson College ..... 51
Drew University ..... 55
Hope College ..... 59
Indiana University ..... 69
Louisiana State University ..... 77
Miami University ..... 87
Michigan Technological University ..... 93
Mount Holyoke College ..... 105
National Security Agency ..... 117
National Security Agency ..... 121
Northern Arizona University ..... 123
Oregon State University ..... 127
Pennsylvania State University ..... 133
Rose-Hulman Institute of Technology ..... 139
SUNY Potsdam-Clarkson University ..... 147
Tulane University ..... 151
University of Maryland Eastern Shore ..... 155
University of Minnesota, Duluth ..... 163
University of Puerto Rico-Humacao ..... 169
University of Tennessee ..... 181
Univesity of Virgina ..... 189
University of Washington ..... 193
Williams College ..... 203
Worcester Polytechic Institute ..... 213
Part II. Descriptions of Summer Enrichment Programs
Carleton College and St. Olaf College ..... 223
College of Charleston ..... 231
George Washington Univerity ..... 237
Mills College and Berkeley ..... 245

## Part III. Student Evaluations of Summer Programs

"Cornell 1999 REU program evaluation summary" by Nora L.
Balfour............................................................................ . 259
"The SCAMP summer program" by Blair Bradley ........... . 265
"Mathematics research experiences" by Stephen G. Hartke.. 267
"Research Experience at the University of Maryland Eastern
Shore" by Andrea Height................................................ . 273
"REU and the nontraditional student" by Laura E. Johnson 275
"The experience of a lifetime" by Darren A. Narayan ....... . 279
"Reflections on the NSF/REU program" by Johnnell Parrish 281
"Experience in faculty mentored research" by Suzanne Sindi and
Gareguin Mikaelian ..................................................... . 285

## Part IV. Articles

"Helping students present their research" by Joseph A. Gallian
and Aparna Higgins ...................................................... 289
"Multi-faceted undergraduate research in mathematics at the University of Dayton" by Aparna Higgins 297
"MASS Program at Penn State" by Anatole Katok and Svetlana
Katok.......................................................................... 307
"How program design and implementation leads to achieving
goals" by Herbert A. Medina . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 315
"Evaluating summer math programs" by Deborah Nolan.... 323
"Scholarly experiences for all students" by Lynn A. Steen... 331
"e-REU" by Robby Robson............................................ . . 337
"Opportunities for Undergraduates at the University of Arizona" by Elias Toubassi .......................................................... 341

Part V. Summary of Conference Sessions

Summary Plenary Sessions ............................................. . . 347
Summaries of Small Group Discussions ............................. 363

Part VI. Surveys of Summer Programs

Summary of Summer Program Surveys ........................... . . 377
Summer Program Surveys ................................................ . 379

## Preface

During a chance meeting in 1998 between John Ewing of the AMS and Barbara Deuink of the National Security Agency (NSA), John made the comment that it was his belief that there were many professional mathematicians whose careers had been positively influenced by their participation in a summer undergraduate research program. Barbara asked John if he knew of any attempt to document the impact of such programs. Out of this exchange came the idea for the AMS to organize a three day conference with the sponsorship of the NSA that would bring together mathematicians from across the country who have been involved in summer mathematics programs for undergraduates. An organizing committee consisting of Barbara Deuink, John Ewing, Joe Gallian, Jim Maxwell, Herbert Medina and Deb Nolan planned the conference. Diane Mack of the AMS served as the coordinator.

Among those invited to participate in the conference were current and recent directors of the National Science Foundation Research Experience for Undergraduates (REU) programs, directors of summer programs for women and minorities, representatives from the NSF and NSA, and people active in promoting undergraduate research. The meeting took place on September 30 - October 2, 1999 at the Hilton Hotel in Crystal City, Virginia.

The purpose of the conference was to exchange ideas, discuss issues of common concern, establish contacts, and gather information that would be of use to those in the mathematics community who are interested in establishing summer mathematics programs for undergraduates. The proceedings of the conference provides a wealth of information about the structure and philosophy of successful summer programs. This volume includes detailed descriptions of the programs run by the conference participants, expanded versions of some of the presentations given at the conference, an article about helping students present their research, an article about establishing an online REU, surveys completed by summer program directors, a summary of the survey data from the program directors, articles and statements solicited from students who have participated in summer programs, and summaries
of the plenary sessions, panel discussions, and break-out sessions. The AMS has also committed to an effort to track participants in summer programs over a long period.

I wish to thank the following people for their contributions to making the conference and these proceedings possible: Barbara Deuink, John Ewing, Diane Mack, Jim Maxwell, Herbert Medina, Deb Nolan, and Jim Schatz. I am grateful to Vickie Ancona, Ed Dunne, Gil Poulin, and Janet Simoneau from the AMS for their excellent work in producing the volume. Robby Robson generously volunteered to take the notes of the plenary sessions that appear here. And, of course, the conference could have never taken place without funding from the National Security Agency and the cooperation of the summer program directors who participated in the conference.

December 17, 1999
Joseph A. Gallian, University of Minnesota, Duluth
E-mail address: jgallian@d.umn.edu

## Part I

## DESCRIPTIONS OF SUMMER RESEARCH PROGRAMS

# The Research Experience for Undergraduates in Discrete Mathematics at Auburn University, Summer, 1999 

Peter D. Johnson Jr.

Our program ran for eight weeks. We had 7 participants. There were only two of us (Overtoun Jenda and Peter Johnson), but we enjoyed the generous support of our colleagues, of our graduate students, and of several visitors during the summer. These supporters gave talks (three colleagues gave two-talk sequences), and were available for discussion and consultation. One of our graduate students, Michelle Foster, gave a talk that struck the fancy of one of the participants, and subsequently Michelle directed her inquiry for the duration, in Michelle's area (information theory). Two of our colleagues, Doug Leonard and Kevin Phelps, donated a significant amount of unpaid time in delivering concentrated introductions to computer algebra and coding theory, and to the software available in our computer laboratory.

The collegial, cooperative spirit of our department, and the fact that we are a noted research center in some areas of discrete mathematics, and can count on a steady stream of visitors, and also on the aid of graduate students of high quality and good cheer, were selling points in our proposal to the NSF (not to mention good reasons underlying our decision to attempt this program in the first place), and are reproducible, at least for the near future. However, when we apply for renewal, after next summer, we will give some thought to regularizing contributions like those of Doug, Kevin, and Michelle. How this will work, we are not sure.

Received by the editor March 13, 2000.

## 1. Recruiting and Selection

We found out that we were funded in late January and sent out fliers in February, to every four-year college and university in the Southeast that we could find the address of, plus a few places in other regions where we knew people.

We had announced in the proposal that we would recruit primarily in the Southeast, and this proved to be an advantage; evidently the NSF-REU people regard the Southeast as an academically underprivileged region, in need of enrichment activities. Of the seven participants finally chosen, six were from the Southeast. The seventh was from San Diego State University.

We asked for transcripts, two letters of recommendation, and an autobiographical sketch concentrating on what the applicant had liked about math so far, and/or what they hadn't liked, and where their current mathematical interests seemed to be (with a clear implication that it was okay not to have a specific current mathematical interest). We set March 31 as the closing date for applications, but subsequently pushed the date back to April 15.

We received 48 applications, for the seven places in the program. As you might expect, because no one would apply to such a program except one well qualified for participation by background and interest, every single applicant (but one, who was not an undergraduate) was perfect for what we had in mind. It is not true, as has been rumored, that we consulted the entrails of a freshly killed lizard in making our selection, but the lizard we keep for emergencies was getting nervous.

Given that only the roughest ranking of the applicants on merit and potential was possible, we felt quite justified in also serving "diversity" and preference goals in making our selection. As already mentioned, all but one of the participants was from the Southeast. Three of the seven chosen were female - a much higher fraction than the fraction of female applicants. When it came to race and ethnicity, we exercised a very light bias, if any, toward minorities. The only student to turn down our offer was a female African-American. Finally, our seven featured one Hispanic and no other significant minority representation.

We confess a reliance on recommendations from people we know; of the seven chosen, five came so recommended. (The one that got away did not.) However, we know a lot of people; probably more than half the applicants came from institutions where we know people, so we can debate the charge of cronyism. Further, our experience last summer suggests that it is not at all a bad thing to listen to the advice of "cronies".

We also took academic class and age into account. Finally, the chosen seven included two seniors, two juniors, two sophomores, and one freshman (as of April, 1999).

## 2. Remuneration and Housing

The participants were given a $\$ 3,000$ stipend for the eight weeks, plus a $\$ 670$ housing allowance - any of this not actually spent was theirs to keep. There was no transportation allowance.

The total offer was quite a bit above the NSF-recommended stipend of $\$ 2500$. Our thinking was that the paltriness of the recommended stipend would severely deter applications from students in special categories, such as those financially independent of their parents, or those who were parents themselves. As it happened, none of the participants belonged to any of those categories. Nonetheless, we feel that lowering the stipend amount is a false economy, and that raising it will have imponderable but significant effects on the attitudes of the participants toward the importance of what they are doing. [The stipend we gave is around one-fourth of the typical salary of a summer intern law student in law firms in New York or Chicago.]

We gave the participants the freedom to seek off-campus housing, or to have us arrange on-campus lodging. Again, the idea was to allow applications from students in special circumstances. Of the seven chosen, one chose off-campus housing, four chose on-campus housing in single rooms, and two chose on-campus housing in a "suite", as roommates (never having met).

A colleague who participated in a "summer research experience" as an undergraduate in Australia has suggested that it would be preferable to require participants to live in close proximity, in a sort of "summer camp" arrangement. Such an arrangement would build social ties among the participants, emphasize the social nature of mathematical research, and enrich the experience of the less-ready by putting them into greater contact with the more-ready.

This is a strong point. A propos, in our program, the bona fide star, Arthur Szlam, a senior from Emory University who proved a major theorem in the area of Euclidean Ramsey problems, his paper on which was accepted for publication in the Journal of Combinatorial Theory, Series A, around two-and-a-half months after the end of our program, was the one participant who chose off-campus housing. Arthur is far from being a shy recluse, but he worked alone, communicated mainly with faculty, and interaction with his fellow-participants seemed limited to three seminar talks in the course of the summer, plus his comments and questions in other seminars. Would the other participants have
benefited from closer proximity to Arthur? Undoubtedly, some would have.

On the other hand, collegiality and social cohesion among the participants, including Arthur, seemed quite high even with the arrangements as they were. Further, the idea of dealing with the possibility of special housing requirements by allowing freedom for all seems a good idea, even though no special housing requirements arose this time. But our main reason for eschewing the summer-camp housing model, for the foreseeable future, has to do with one of our primary goals overall, which is to provide an authentic mathematical research experience, warts and all. In a real research environment, besides the earnest worker ants, you get brilliant loners (like Arthur), you get slackers, hibernators, and people who go through stages. Bring it on! Maybe it was beginner's luck, but we felt that we brought it on last summer; it would not have been much different if all the participants had been living together, but it would have been more artificial.

## 3. What we meant to do, how we meant to do it, and how it went

As indicated above, our idea was to provide an authentic research experience for the participants, not to sell mathematical research as a career possibility. As we stated in our proposal, we would regard it as a possibly successful outcome if a participant decided not to go into mathematical research, as a result of their experience with us. This may have occurred with at least one of our participants, who now seems much more likely to go to medical school than to graduate school in mathematics. [To weigh things correctly, she clearly stated in her application her intention probably to go to medical school, ultimately. On the other side, Arthur Szlam is continuing at Emory University, as a graduate student in mathematics - the fact that he is doing so has little to do with us, really.]

Our plan was to have required meetings of $11 / 2-2$ hours every morning, and 1 hour every afternoon, for the full eight weeks. During the first two weeks, these would be occupied by introduction, to the computer lab, to necessary background, and to possible problems to work on. The announced plan was that, after two weeks, every participant would have to declare a problem area in which their final presentation would be made. Everybody would be able to work in any area, but the "final presentations" would have to be in the declared areas.

Our morning sessions initially were in computer algebra, computer usage, and coding theory. Our afternoon presentations initially were
in a variety of Euclidean Ramsey problems. The fact that we were, between us, well acquainted with problems in these areas, which we regarded as very suitable for an introduction to mathematical research, underlay our initial decision to apply to the NSF.

Aside from the requirement that each participant eventually concentrate on something, there was really no regimentation in our program beyond the required meetings. Our idea was that the "research experience" would be as it is in real life - with no immediate requirement of daily progress on set exercises, nor even of regular presentations.

As soon as the idea of what we were up to sank in, sometime in the second week, five of the seven participants set about various activities with great gusto. One of these five was Arthur Szlam, who is a special case; while we would love to point to Arthur's achievements as an indicator of the success of our program, it is undoubtedly of greater interest to look at what happened to the other six participants. Four of these arrived at the brink of publishable results - indeed, we have hopes that we will see some of their work last summer in print eventually, after some further work. In two cases, significant progress was made on problems posed to the participants during the first week of the program. In the other two cases of the four, the participants asked to be set on the road into problem areas they had heard about - whereupon they took off on their own (while consulting occasionally with available faculty and graduate students, and giving several talks on their inquiry).

The two participants that did not exhibit a great deal of activity attended all talks, asked intelligent questions from time to time, and gave quite competent presentations on their topics at the end of the summer, but it seems quite clear that they would have gotten more out of the experience with a more regimented program structure. Indeed, Robert Rubalcaba, our participant from San Diego, who was, next to Arthur, clearly the participant least in need of guidance and regimentation, gave specific recommendations, in the report/critique/evaluation that we asked for from all participants at the end of the summer, for increasing the degree of regimentation in the program, and we plan to implement some of his suggestions - such as requiring short presentations or progress reports from all participants, weekly after the first two weeks - next summer.

This seems to run counter to the "authenticity" goal, but we feel that it's worth trying anyway. (It should be noted, however, that Arthur, in his critique, specifically praised the freedom and lack of structure of the program.)

## 4. Where are they now?

As mentioned above, Arthur Szlam is pursuing a Master's degree in mathematics at Emory University; the paper containing his astonishing Euclidean Ramsey theorem has just been accepted by the JCT(A), and he is about to become a co-author of two other papers, with faculty of our department, based on his work last summer.

Robert Rubalcaba is joining us as a graduate student in discrete mathematics this winter. One other participant indicated plans to come to graduate study in our department in the fall of 2000 . One of the participants will be working in hospitals for the next two summers, before entering medical school. Another just got married, and is taking a break from academia. The remaining two participants will be participants again, next summer, and seem quite enthusiastic about the prospect.

## 5. Changes next summer

We are applying for a REU supplement to allow us to add two high school teachers to the eight undergraduates we had planned for, for next summer. How this addition will affect things, we are not sure, but the NSF seems quite chuffed about the idea of extending these research experiences to high school teachers, and it does seem a worthy cause.

Besides implementing Robert R.'s suggestions for a more structured program (with, we hope, a minimum of damage to the free-wheeling atmosphere we enjoyed last summer), we will also try to act on another suggestion due to him and others, by arranging regular social hours for gathering of the participants, random faculty, and graduate students. We had some of this last summer, but were too swamped with first-time-around glitches with housing and payroll to do it really well.

Finally, we will aim to rethink our approach in computer algebra and coding theory. The difficulty is that getting to real research problems in these areas involves wading through a daunting amount of background and fundamentals, and it is just not feasible, for most, in eight weeks. Some of the participants enjoyed learning things in these areas, but none essayed a project in them. In view of our experience last summer, it seems a more fruitful approach would be to give less weighty, more gee-whiz introductions to these areas, offering them as possible subjects for rather expository projects, for those so inclined.

Department of Discrete and Statistical Sciences, 120 Math Annex, Auburn University, Alabama 36849

E-mail address: johnspd@mail.auburn.edu

# Undergraduate Research at Fullerton 

Mario Martelli

I started supervising undergraduate research in 1992. I have recruited the students exclusively from math or physics majors at California State University Fullerton and from high schools in the Orange County area, which have magnet programs in mathematics. The students from CSUF are usually recruited at the beginning of their junior year. Frequently, they are the best students in the course on Dynamical Systems and Chaos, which I have offered every year since 1991. However, I also consult with other faculty to get names of really promising individuals. I have never accepted more than three students per year, so that there are at most six undergraduates students working under my supervision at any given time. In general I introduce the group to one or two problems. My selection of the problems is guided by the following principles.

1. I need problems which are not too far above the student's level, but they are not exactly at their level. I want the students to learn something extra, not usually presented in the standard curriculum. Therefore I choose problems for which some preliminary background work is necessary.
2. I choose problems of which I normally know that a solution exists. In other words, even though I may not know exactly how to achieve a specific final results, I am pretty confident that this can be done. I give the students few details, and I try to guide their investigation and prevent them from taking wrong turns. I want them to obtain the results with their own methods and ideas, although I help them in developing both.
3. I prefer problems sophisticated enough that the solution can be published in a scientific journal, with my name appearing among the co-authors. This obviously limits considerably the choice

Received by the editor September 15, 1999.
of suitable problems. I have never tried different alternatives. However, now that several undergraduate journals are available, I am inclined to use also this strategy.
Here are some examples of problems I have assigned. Most of them have been completely successful, but one was successful only in part.

1. Find a discrete dynamical system in the plane which is C and has a unique fixed point which is an unstable attractor. Prove that such occurrence is impossible for discrete dynamical systems in the real line.
2. Collect many different definitions of chaotic behavior for discrete dynamical systems. Compare them and provide examples to show their advantages and their limitations. Prove that some of these definitions, although formulated differently, are equivalent when the underlying space is a compact metric space.
3. Obtain a formulation of the universal chord theorem in dimension higher than 2.
4. Give an elementary proof, without using any measure theory, that for a continuous non-constant function $f:[a, b] * R$, which is differentiable except possibly in countably many points, the set of points where the derivative is negative is either empty or non-negligible.
5. Provide a geometric approach to transversality (sufficient) conditions for the four different types of bifurcation for one-parameter families of scalar maps.
6. Obtain a global convergence theorem for discrete dynamical systems of triangular type, which can be applied to forward neural networks.

The presentation of the problems takes place preferably at the end of the academic year in which the students become juniors. We organize weekly or biweekly meetings over the summer, frequently at my house, to develop the background necessary to understand the problems. The work is continued in the Fall and no credit is given at this time for the extra time required by the research. We may start writing some preliminary results during this period. When the Spring semester comes, the students take my course on Dynamical Systems and Chaos. This provides me with the opportunity to see them on a more regular basis. If enough results have already been achieved, I prepare them for a poster presentation at the Spring Meeting of the Southern California Section of MAA and of SIAM. They also take part in the CSUF Research Competition. If selected, they participate in the CSU Research Competition. All expenses connected with this participation are paid
by a special CSUF fund. Usually they do quite well in both competitions. They have always been listed in the group representing CSUF at the CSU level. Moreover, one of them received first prize in the CSU Research Competition in 1994, and two were ranked third and second in 1995 and 1997 respectively. We continue our work over the summer of their junior year. When Falls come I usually enroll them in an independent study course. This strategy has the double purpose to give them some credit for what they have done in the previous year, and to provide the group with some fixed time in which start preparing for the various activities connected with the research. At this point we apply for the Research and Creativity Award offered by CSUF. The amount of the award is $\$ 1,000$. In the past all students have received this award. The money is used to pay their participation in the Undergraduate Student Poster Session which takes place every year in conjunction with the Annual Meeting of the AMS and MAA, and to pay for the CUR Poster Session in Washington in April. The Student Union provides extra funds for at least one trip, by paying the transportation expenses. The Dean of the School of Natural Science and Mathematics, the chair of my department and the Vice-President for Academic Affairs have also contributed sometimes when the money from other sources was insufficient. In the Spring of their senior year the students participate in the CSUF Research Competition for the second time, present their final results in the form of a poster at the Spring Meeting of the Southern California Section of MAA and frequently give talks to the Math Club on their research. We also apply for $\$ 500$ Publication and Presentation Prize offered by CSUF. The group has received the prize twice in the past five years. Of the various students who went through the program, four are in graduate school to obtain their Ph.D. in mathematics (Cornell, University of Arizona in Tucson, UC San Diego and UCLA); two have obtained their master in mathematics and are now teaching in community colleges in California; one plans to go to graduate school to get her Ph.D. in mathematics, but has not made up her mind yet about when and where. Two of the students I have at present time are planning to go to a Ph.D. program in Physics, and one to a Ph.D. program in Mathematics.

California State University Fullerton Fullerton, CA 92634
E-mail address: mmartelli@Exchange.FULLERTON.EDU

# Summer Undergraduate Applied Mathematics Institute at Carnegie Mellon University 

William O. Williams

## 1. History of the Institute

The Summer Institute grew out of a partnership, dating from 1991, between Hampton University, a historically black university, and Carnegie Mellon University. Working with James Turner, the Head of the Mathematics Department at Hampton University, Carnegie Mellon faculty organized a two-day Mathfest for undergraduate students from historically black colleges and universities. Faculty from both CMU and Hampton gave mathematical talks and met informally with students to talk about graduate school.

The following summer, in 1992, a four-week institute for eight undergraduate students, four of whom were black, was held at Carnegie Mellon, funded under a grant to the Center for Nonlinear Analysis from the Army Research Office. The Institute offered two classroom courses, one in elementary real analysis and one in numerical analysis, a Maple laboratory, and a series of weekly research seminars by faculty from the Department of Mathematics.

After this experimental Institute, in 1994 three-year funding was obtained from NSF to expand the Institute to include 12 students in a seven-week program. It was reorganized into essentially the current form, continuing and expanding the real analysis course, the Maple laboratory, and the research seminars, and adding a research project component. Renewed funding in 1996 from NSF and from NSA supported the Institute in the three succeeding years, with essentially the same format.

Received by the editor September 2, 1999.

Using a grant from a CMU alumnus and funds from a VIGRE grant to the Departments of Mathematical Sciences and Statistics, in 1998 one and in 1999 three CMU students were added to the Institute group, and an additional research component in statistics was included.

## 2. Philosophy of the Program

The Institute retains its emphasis on enrolling under-represented minorities and women: in the six years of the current format $46 \%$ of the students have been minorities, and $58 \%$ female. There also is a strong prejudice in admissions toward students from primarily undergraduate institutions.

The program might described as a virtual graduate-school experience. The rationale is as follows. Undergraduate students often are unclear on exactly what graduate study and research will require of them and what it can offer them. Unfortunately, many talented students decide against graduate studies in part because of this uncertainty; equally unfortunate is that many enter graduate school simply because they are 'good at math', and only discover after wasting a year or two that they do not really have the strong interest needed to pursue advanced work. Our program is designed to help students make a rational decision about their future by giving them a taste of the graduate experience, without excessive cost of time in their careers. Research: Undergraduate students in Mathematics can be unprepared by their undergraduate training to understand the nature of advanced work and research in the area. Indeed, in many areas of mathematics it is difficult to appreciate the nature of advanced work with only an undergraduate background. Applied mathematics has a lesser problem in this respect, and an introduction to research-style projects taken from applied areas can serve as an introduction to the nature of research, an introduction which is of use even to students who are interested in "pure" mathematics. Also, many students in traditional mathematical curricula are not aware of the possibility of doing research or working in applied areas, which offer the greatest number of non-teaching jobs for mathematicians. Our projects and our seminars are designed to give these students a chance to see the bigger picture. Graduate Studies: Strong students at research universities often are advised to take graduate-level courses in their upper-class years. In this way, they have the opportunity of seeing the nature of graduate mathematics and the effort that is required of graduate students, and to make an informed weighing of their own motivation and interest against the work involved in advanced study. Students at smaller schools do not
have this advantage, and have to rely only on the advice of their advisors as to how this balance falls in their case. The Institute, through its classes, attempts to fill this gap. Although the material covered is at undergraduate level, it is attempted to make the intensity level high enough that the students may develop a feeling for graduate-type classes. Equally importantly, the presence of a mixed audience - students from stronger and weaker schools, students with better or worse backgrounds - gives them a feeling of where they can place themselves with respect to their future peers. (In our experience this has cut both ways. There have been students who came away from the Institute with the recognition that previous worries about being Joe Blow from Small College USA were overblown, that they could, and wanted to, jump into graduate work, and there have been students with somewhat over-inflated self evaluation, who now realize that they will have to be very serious about their work if they are to be successful at the next level.)

Finally, the material which the students learn in these classes constitutes the tangible deliverable of the experience, since it should be of use to them in future studies at undergraduate and graduate level.
2.1. Recruiting and Selection. Recruitment of the target students can be difficult. In recent years the majority of applications have been downloaded from our web site. A reasonable assumption is that students who seek out REU sites on the web are likely already to be strongly interested in graduate study, so other means are needed to reach more of our target audience, that is, those who are unsure of their commitment to this path. We have attempted to reach out to such students through their faculty advisors. Through the years of the Institute, we have built up a relationship with faculty at many colleges, in particular HBCUs, and this has served as an excellent source of students. We have a mailing list, currently of 244 names, to whom we send a mailing each year. In addition, we make it a point to have a representing faculty member at the annual Undergraduate Mathfest, now run by National Association of Mathematicians, and at the Society for the Advancement of Chicanos and Native Americans in Science national meeting.

Selection of students is based on courses taken, grades, and, most importantly, faculty recommendations. By and large, we pass over the over-qualified students who have enough background that their best placement would be a purely-research REU position. Perforce we usually enroll a student or two who is more advanced than our ideal; however this has proved to work out well enough in practice, with the
team-spirit of the group high enough that the more advanced students help and work well with the less advanced. For those for whom the real analysis course would be too repetitive, we have offered advanced reading courses.

Applications nominally are closed on March first, although in practice we offer a week or two of leeway. Particularly attractive students may be given offers before the deadline, although we are careful to give them a longer period to respond, so that they may consider other opportunities.

## 3. Process

The Institute begins early in June and extends for seven weeks. The first day consists in orientation, with time devoted to outlining the goals and the schedule of the Institute, to formal registration as a CMU student and obtaining of identity cards, and to a tour of nearby areas where the students may shop or eat, the later conducted by the two graduate-student TA's associated to the Institute. The serious work begins on the second day, with both the analysis course and the Maple laboratory offering a first session and with presentation of the first of a series of lectures (two hours per day) on the possible project areas for the research projects. Each of the two courses continues to meet for five hours a week through the sixth week. Exams are given and a transferrable-credit grade assigned for the analysis course. Regular assignments are given in both classes.

In each year we select a different group of faculty to offer projects; the list of project areas from the last three years is:

The Marriage Algorithm<br>Variational Calculus<br>Problems in Continuum Mechanics<br>Mathematical Finance<br>Prof. Steve Rudich (CS)<br>Finite-difference Methods for PDE<br>Ramsey Numbers<br>Fractals and Wavelets<br>Prof. Bill Hrusa<br>Prof. Darren Mason<br>Prof. Steve Shreve<br>Prof. Jack Schaeffer<br>Prof. James Cummings<br>Computational Finance<br>Prof. Tim Flaherty<br>Prof. Reha Tutuncu<br>Computing Intermolecular Positions<br>from Distance Readings<br>Prof Tony Kearsley<br>Bayesian Analysis of Models Prof. Pantelis Vlachos (Stats)

At the end of the first week the students select a project, forming teams as appropriate to their and the project-director's preference (in the past one-person projects have been the exception, usually with only two or three students making such a choice in each year). A team size of two
to four is the norm. The students are expected to meet with their project director at least twice a week.

Research/graduate-experience seminars are presented weekly by faculty from CMU and from outside. In the last three years speakers from outside the mathematics faculty have included:
Matt Bishop, Graduate Student, CMU Automated Theorem Proving Rachel Rue, Graduate Student, CMU Independent Sets in Grids
Professor Bard Ermentrout, U. Pittsburgh, Synching in the Brain
Professor Bill Layton, U. Pittsburgh, Simulating Fluid Flow by Finite Element Methods
Professor Duane Cooper, U. Maryland, Binary Numbers for Preteens through Postdocs
Dr. Tony Kearsley, NIST Computing in Parallel for the Taxpayer: Some Computational Results from NIST
Prof. Jay Kadane, CMU, An Accusation of Examination Copying; a Study in Applied Statistics
Dr. Monica Brodzik, NSA, Mathematics at the National Security Agency
Prof. Ruth Williams, UCSD, Games, Queues and Brownian Motion -Highlights of my Personal Random Walk Through Mathematics
Robert Thrash, PhD candidate, CMU, Convergence of Binomial Models in Finance
Olivier Lessmann, Grad Student, CMU, From Complex Variables to Model Theory
Aris Winger, Grad Student, CMU, The Experiences of Graduate Life
The two courses terminate in the sixth week of the Institute, so that the last period is devoted to the wrapping up of the project and preparation for a one-hour presentation to the Institute students, Institute faculty and interested members of the faculty. No written version is required, due to the constraint of time.

Social events planned for the group have included picnics and dinners at faculty's houses, river-boat rides, and trips to amusement parks. Because they are housed in a single corridor of a CMU dormitory and share classes and projects, it has been found that socialization amongst the students is automatic, rapid, and often continued after the Institute. In the six year period of the Institute there have only been two personal conflicts between students which required the intervention of the Director.

## 4. Outcomes

It is difficult to measure the success of a program such as ours quantitatively; there is no control group with which to compare. What we have done is to sample the students' opinions immediately after the institute ends and in the years following. Extensive student opinion surveys at the end of each Institute indicate almost universal appreciation for the experiences of the Institute, while the (spotty) returns from alumni are likewise uniformly positive.

It also has proven difficult to track the alumni through their careers. For the five years of the current form of the Institute (excluding this year, of course) we have had no response to our inquiries from from $30 \%$ of the total. Of those who responded , $8.5 \%$ ( $6 \%$ of the total number) are still undergraduates, $8.5 \%$ (6\%) have completed or are enrolled seeking a terminal master's degree, $67 \%$ (47\%) are seeking a PhD , and $16 \%(11 \%)$ are working; approximately half of the latter intend to return to graduate school.

Carnegie Mellon University, Pittsburgh, PA 15213
E-mail address: wow@nero.math.cmu.edu

# A Decade of REU at William and Mary 

Charles R. Johnson and David J. Lutzer

## 1. Institutional Overview and Program Goals.

The College of William and Mary is a state supported university that has always placed a major emphasis on providing high quality undergraduate education. Recent decades have seen the growth of graduate degree programs and today, in addition to its undergraduate program (with 5400 students), there are graduate programs available in 13 departments and Schools at William and Mary, involving some 2000 students.

The Department of Mathematics has 19 full-time faculty, each involved in teaching at the undergraduate level while maintaining an active research agenda. Building a bridge between faculty research and undergraduate teaching, the department has conducted NSF-funded REU programs each summer since 1990. Each of these REU programs was built around matrix analysis and its applications, one of the department's preponderant research strengths and an area in which the department is a leading reseach center.

The Mathematics department participates in an interdisciplinary doctoral program, cooperating with the College's Applied Science program to offer an apprentice-style program leading to a doctoral degree in applied mathematics. Currently there are four doctoral students enrolled in this graduate program, and the program has a linkage to REU, as described below.

A central objective of our REU programs is to provide talented students with experience in how mathematics is done, something that is quite different from students' typical classroom experiences. Our approach places great emphasis on the process and excitement of independent discovery by the student, working with challenging, unsolved

[^1]research problems, as well as on the interactions of students with faculty mentors, and of students with one another. Our hope is that this summer experience will lead students to pursue graduate school in one of the mathematical sciences.

Our REU program attempts to maximize the probability of success of the research experience for students by incorporating a number of short-term goals. For example, introductory lectures and weekly seminars are intended to increase the participants' knowledge of modern research areas as well as to whet their general mathematical curiosity. REU participants have access to, and receive special training in, mathematical software such as LaTex, MATLAB, and MAPLE, and are encouraged to use such tools. Furthermore, students are encouraged to keep a research diary to chronicle their daily progress, their conjectures and questions, as well as ideas that didn't appear to work. By the end of the program each participant will thus have a tangible record of accomplishments that will be the basis for the participant's final report.

Our program stresses the need for communicating results to peers and to the community at large. A number of early presentations introduce students to reading, writing, and explaining mathematics. Later on, students themselves present oral interim reports on their research to the entire group of faculty and students, and then present a final oral report in the eighth week. Students typically practice these reports with their REU mentors because one of our goals is to improve students' communication skills.

## 2. Mathematical Theme.

Throughout the decade of our REU efforts, the overall scientific theme of the program has been "Matrix Analysis and Applications." That theme is particularly appropriate because it encourages a combination of inductive and deductive approaches. For example, the individual research topics (samples appear below) readily lend themselves to exploration by hand or by computer and then subsequent generalization, with newly derived conjectures capable of further exploration. Significantly, only a modest undergraduate mathematics background is needed to appreciate and understand some interesting open problems formulated in terms of matrices (or graphs arising from matrices). As a result, students with varying degrees of formal mathematical training can be more easily accommodated. An added benefit is that knowledge of this area provides a solid background for further study in a number of mathematical specialties. Finally, having a common theme not only allows students to discuss problems with several faculty advisors but
also encourages students to exchange ideas and assist one another, in some cases working in small teams with one or two research mentors.

A wide variety of research topics have been studied by REU participants and mentors during the decade of this project, but certain broad themes unite them. Individual projects have been open ended, and a project begun in one summer by an REU student is often continued in later summers by another, using a report or publication by the first student as a starting point for further research. The key criteria for judging possible research topics include their accessibility to wellprepared and capable undergraduates, the modularity of the topic the extent to which it can be broken into steps of increasing depth and complexity - and the likelihood of obtaining interesting and satisfying results in an eight week summer session. By way of illustration, we list a few broad themes from which many REU problems have been drawn.
a) Linear preserver problems: The "linear preserver problem" asks for characterizations of linear operators on matrix spaces that leave invariant certain properties, relations, or subsets. Particular instances include the study of isometry problems and linear transformations that leave invariant controllable matrix pairs. In addition, many results from the matrix setting are being extended to infinite dimensional contexts and even to more general algebraic structures.
b) Determinants of matrices: Consider the set of all $n \times n$ matrices consisting of 0 s and 1 s , and having each row sum and each column sum equal to $k<n$. What are the possible determinant values for such matrices? Bounds are known, and complete solutions are known for special values of $n$ and $k$, but the general case is open. Next consider a real symmetric matrix $A$ with singular values $a_{1}, a_{2}, \ldots, a_{n}$ and a real skew symmetric matrix $B$ with singular values $b_{1}, b_{2}, \ldots, b_{n}$ What are the possible determinant values of $A+B$ ?
c) Qualitative matrix theory: A sign pattern $A=\left[a_{i j}\right]$ is a rectangular array of signs $(+,-)$ and zeros. Let $Q(A)$ be the set of all real matrices whose i,j entry has the same sign as $a_{i j}$. Qualitative matrix theory deals with the study of $Q(A)$ and, most often, with questions about whether there exists a $B \in Q(A)$ having a certain property $P$ (in which case we say that " $A$ allows $P$ " or whether all members of $Q(A)$ have property P (in which case " $A$ requires $P$ "). Classical questions of qualitative matrix theory deal with the properties $P=$ invertibility or stability, and more recently with questions involving row and column sums, semipositivity, structure of null vectors, and the probability of a positive determinant. Often there are corresponding questions for "patterns"
(arrays that specify the positions of zero and non-zero entries in a matrix) Historically, such questions arose from biology, economics, and chemistry, and more recently from computer science and mathematics.
d) Matrix convergence questions: A set $P=\left\{A_{i}: i \in I\right\}$ or real $n \times n$ matrices is "pointwise convergent" provided that for each $x \in R^{n}$ there is a sequence $\{p(x, j): j \geq 1\}$ of elements of $I$ such that $\lim _{k \rightarrow \infty}\left(\Pi_{j=k}^{1} A_{p(x, j)}\right) x=0$, and $P$ is "uniformly convergent" if a single sequence $p(j)$ can be chosen independently of $x$. Certain necessary and sufficient conditions for each type of convergence are known, and useful criteria for recognizing each type of convergence are available, given certain restrictions on $P$. Analogous criteria are needed for other sets of matrices. In particular, it is known that pointwise and uniform convergence are equivalent for classes of entrywise non-negative matrices, and a useful test for convergence in this case would be valuable.
e) Factorizations of almost periodic matrix functions. Almost periodic matrix functions are matrices with entries of the form $\Sigma_{j} c_{j} e^{i \lambda_{j} x}$, where the $c_{j}$ are complex numbers, the $\lambda_{j} \mathrm{~s}$ are real, and $x$ is a real variable. The sum may be finite or infinite. Factorizations of such functions as a product of at most three almost periodic matrix functions where the left factor has all $\lambda_{j} \mathrm{~s}$ non-positive, and the middle factor has basic exponetials on the main diagonal and zeros elsewhere, arise in applications to Weiner-Hopf and convolution type equations, and in mathematical physics (e.g., inverse wave scattering). Therefore it is of interest to study factorizations for special classes of almost periodic matrix functions and, if possible, to obtain explicit formulas for the factors. It turns out that this is a challenging mathematical problem and even for many seemingly simple classes such as $2 \times 2$ triangular matrices, there are no satisfactory answers so far.
f) Matrix completion problems: A partial matrix is one having some entries specified while others are free to be chosen from an agreed upon set (e.g., the real numbers). A completion of a partial matrix is a choice of values for the unspecified entries that results in a conventional matrix. A matrix completion problem asks which partial matrices have a completion of a designated type (e.g., positive definite, totally positive, etc.). Often, the arrangement of specified entries plays a key role in such problems and combinatorial issues become very important.

## 3. Student Recruitment and Selection.

Students in our program are recruited from a national pool. In recent years, the applicant pool has ranged from 110 to 160 students, and we have chosen eight students for our program. We also recruit
students from William and Mary and from nearby colleges and universities, but the overwhelming majority of our REU students come from other states. From time to time, foreign students have joined our program, with support from non-NSF sources.

Our recruitment efforts begin in the early fall and are carried out through a variety of mechanisms. Notices of the program are sent to almost 200 mathematics departments across the nation. Additional national exposure is obtained by posting notices of the program on various electronic networks (e.g., International Linear Algebra Society bulletin board, CSNET) and by providing information to relevant special interest newsletters (SIAM Activity Group on Linear Algebra Newsletter, ILAS Newsletter). To help students find us, we rely on NSF to maintain an up-to-date listing of REU programs on its web site and on organizations such as MAA to provide appropriate cross linkages from their own web sites.

Each applicant is asked to provide two letters of recommendation, a list of mathematics courses completed, including the student's final grades and texts used, a list of courses that will be completed in the spring semester before the student comes to our program, and a personal statement of interests. Students for the program are chosen by a committee of program faculty. Our selection process takes into account evidence of the applicant's ability, as evidenced by grades and letters of recommendation, and the applicant's personal motivation and the possibility of growth through the program, as evidenced by the student's personal statement and comments in letters of recommendation. We are particularly interested in students' performance in either (or both) of the first courses in modern algebra and analysis. We try to be particularly receptive to applicants from institutions that do not provide graduate programs or other research opportunities for their undergraduates. We also attempt to make sure that women have access to our programs. Over ten years, eighty students have been supported by NSF in our program - thirty women and fifty men.

To help admitted students decide whether to join our program, we ask one summer's REU students whether they would agree to be contacted by students admitted to the next summer's program. To date, all have agreed. It seems that about a fifth of admitted students make contact with the previous year's participants.

## 4. Research Mentorship.

Most of our REU students have worked 1-1 with advisors, with an occasional team of two students working on a problem with one or more advisors. Advisors typically meet for an hour per day with
each student (or student team), and some advisors hold longer daily consultation periods.

Our experience has shown the importance of daily faculty guidance in undergraduate research. During the REU program, each student's faculty advisor poses a progressive series of problems that are of increasing difficulty and generality and that lead towards the overall goal of the student's program. This strategy allows for positive reinforcement of the student as he or she advances in incremental steps to more difficult and unknown territory.

From time to time, we have involved one of our advanced doctoral students in our REU program, as a closely supervised research mentor. REU students have responded very positively to these graduate students, probably because the age and cultural differences between REU students and the graduate students are less than the corresponding differences between REU students and the rest of us. During our 1997-8-9 grant period, with NSF approval, we expanded from one to two graduate student mentors. We invited advanced graduate students from other universities to join us. We chose graduate students who
a) were expert in mathematics related to our program theme;
b) had been involved in successful undergraduate research projects themselves.

The two outside graduate students chosen in 1997 and 1999 respectively are writing their theses at Berkeley and at M.I.T., and one was a graduate of our own REU program. During their time with us as supervised research mentors, the graduate students received training in the art of being research supervisors, and were also able to collaborate mathematically with members of our matrix and operator theory group.

## 5. Program Structure.

Prior to arrival on campus, the participating undergraduates receive information on the expected background in linear algebra and matrix theory (suggested sections of several textbooks are cited) and students have a sampling of typical REU project areas from previous years.

During the first week of the program, faculty present background lectures on their proposed research projects, so that each student can select an appropriate research topic and advisor. A side effect of these lectures is to introduce all students to all projects, thereby facilitating interactions between students whose projects are similar. Also during the first week of the program, we provide special training in computing tools such as MAPLE and MATLAB that are useful for research
exploration in matrix analysis, and in the use of the internet and other library resources in mathematics. At an early stage of the program, students are introduced to the use of one of the standard technical text-processing languages, e.g., LaTeX. (During their time at William and Mary, REU students have access to a Pentium-equipped Windows NT laboratory, and a special departmental laboratory equipped with Linux computers. Both laboratories are equipped with MATLAB and MAPLE and are located in the mathematics department's building.)

Individual research, rather than in-class learning, is the heart of our program. Nevertheless, we do schedule seminars for our REU students. From time to time during later weeks, selected faculty members present brief talks on useful techniques for conducting research. Other seminars are presented by visitors to the department. We obtain visiting speakers at no cost to the REU program by judiciously arranging the normal trips of research visitors to the department. We run an appropriate social program involving the speaker, faculty, and REU students in conjunction with the seminar series.

Another type of talk focuses on applications of matrix analysis in other disciplines. In recent years, faculty members from economics, computer science, and physics have presented seminars on applications of linear algebra in their disciplines.

Finally, we offer a special seminar with the graduate program director from a Group I Ph.D. department to discuss what to look for in a mathematics graduate school, the application process, various kinds of financial support available to mathematics graduate students, etc. In the last three years, students have found these sessions to be very helpful. In the summer of 1999, we experimented with a virtual visit, using an internet linkage, and student response was good.

After four weeks, students present reports on their research topics and describe their preliminary findings to the entire group. In the eighth week, students present a final oral report and a final written report on their work. For many students, this report will be the basis of a journal article that (previous experience shows) students work on in the weeks after the REU program ends, collaborating with faculty mentors by e-mail.

In most years, our REU program includes a visit to one of the federal research facilities in the area, where students can be introduced to mathematical scientists and see at first hand the dynamics of ongoing research programs. NASA/Langley Research Center, ICASE, the Virginia Institute of Marine Science, and the Jefferson Continuous

Electronic Beam Accelerator Facility are sufficiently close for a day's excursion.

To help students feel at ease with each other and with program faculty, one day each week we schedule a fast food lunch for students and REU faculty. This is an opportunity for extremely informal discussions of small problems, progress of students and faculty on problems of current interest, and non-mathematical topics. In addition, we encourage all faculty and students to attend informal coffee and cookie breaks held from time to time during the program.

## 6. Program evaluation and follow-up.

One very important measure that we use to judge success of our REU program is the extent to which our students discover new mathematical knowledge concerning substantial unsolved problems. To make that evaluation, we rely on the judgment of our faculty research mentors, something that we trust because of our faculty members' own research records and their many years of successful experience as undergraduate research advisors. But some quantitative measures are also available, and we use them. For example, we are interested in the percentage of REU students who eventually become co-authors of refereed mathematical papers with their faculty advisors. Since 1990, over $40 \%$ of our REU students have become co-authors with their research mentors of refereed journal articles.

Our students' REU experiences have been recognized by outside groups other than mathematical journals: for example, based upon his research in our 1995 REU program, one of our REU students won the University of Maryland's Dorfman Prize for the best research by an undergraduate in the Computer, Mathematical, and Physical Sciences. Other REU projects have become part of undergraduate theses at Harvard that have won highest honors.

To evaluate our success in encouraging students toward graduate school in mathematics, we have followed students after they completed our program. Projections from surveys of the 56 students from the 1990-96 REU programs suggest that perhaps $90 \%$ of our REU students pursued graduate study in the mathematical (or related) sciences. To obtain better response rates to our surveys, we decided to adopt a more aggressive follow-up strategy starting in 1997. We have complete data on the 24 students supported during the 1997-8-9 grant period and can report that every one of those students is either still an undergraduate or is enrolled in graduate school, with one being a doctoral student in finance and the others being students in mathematical sciences.

In addition to the numerical data above, we survey our students at the end of their summer REU program, collecting comments about the success of various aspects of the program, and getting new ideas for improving future programs. We ask students to complete a brief anonymous questionnaire, reporting on:
a) whether, in retrospect, we sent them adequate and accurate information on the mathematical content of our program, and on housing, meals, travel, airports, etc.;
b) the adequacy of housing, office space, computer access, and library materials during our program;
c) the various seminars during the program;
d) interaction with the research mentors (Was the relationship friendly?

Did they get to spend enough time with their mentors? Were the mentors helpful in guiding research?);
e) what impact the program had on their plans for studying more mathematics and attending graduate school in mathematics.
Student responses have been very favorable. Combined with impressions gathered by faculty mentors from conversations with advisees, student responses from one summer help us plan the next.

We also maintain e-mail contact with former REU students and follow their progress. Initially this contact is often part of a collaborative effort between student and advisor to prepare an article for submission to a journal. But the personal relationship between advisor and REU student often continues over time as students pursue their mathematical studies in graduate school and the REU program director has not been shy about contacting former REU students.

## 7. Publications resulting from previous NSF REU support.

In the past, our REU students have often become co-authors of refereed journal articles. In the following listing, REU students are marked with an asterisk.
C. Cates*, J. Drew, C. Johnson, and C. Tart, Characterization of supercommuting matrices, Linear and Multilinear Algebra 43 (1997), 35-51.
S. Chang*, C-K Li, A special linear operator on $M_{1}(R)$, Linear and Multilinear Algebra, 30 (1993), 65-75.
S. Chang*, C-K Li, Certain isometries on $\mathbf{R}^{n}$, Linear Algebra and Applications, 165 (1992), 251-261.
G.S. Cheon, S.G. Lee, C.R. Johnson, and E. Pribble*, The possible number of zeros in an orthoganal matrix, Elec. J. Lin. Alg. 5 (1999), 19-23.
A.L. Cohen*, L. Rodman, and D. Stanford, Pointwise and uniformly convergent sets of matrices, Siam J. Matrix Analysis, to appear.
S. Fallat, H.T. Hall, and C.R. Johnson, Characterizations of product inequalities for principal minors of M- and inverse M-matrices, Q.J Math (Oxford) (2) 49 (1998), 451-458.
S. Ferguson*, C. Johnson, and T. Shalom, Information requirements for determining the inverse of a persymmetric matrix, in preparation. M. Gelfand ${ }^{*}$ and I Spitkovsky, Almost periodic factorization: applicability of the division algorithm, Advances in Math. Sci., 184 (1998), 97-109.
S. Gottleib*, C. R. Johnson, and I. Spitkovsky. Inequalities involving numerical radii, Linear Algebra and its Applications, 37 (1994), 13-24.
G. Hartless* and L. Leemis, Computational algebra applications in reliability, IEEE Transactions on Reliability, 45 (1996), 393-399.
J. Helton, D. Lam*, and H. Woerdeman, Sparsity patterns with high rank extremal semipositive definite matrices, SIAM J Matrix Anal. Appl., 15 (1994), 299-312.
C. Johnson, C. Jones*, and B. Kroschel, The Euclidean distance completion problem: cycle completability, Linear and Multilinear Algebra, 39 (1995), 195-207.
C. R. Johnson, M. K. Kerr*, D. P. Stanford. Semi-positivity of matrices, Linear and Multilinear Algebra 37 (1994), 265-271.
C. R. Johnson, J. S. Miller*. Rank decomposition under combinatorial constraints, Linear Algebra and its Applications, 251 (1997), 97-104.
C.R. Johnson, J. Pitkin*, and D. Stanford, Line sum symmetry via the DownEig algorithm, Computational Optimization and Applications, to appear.
C. Johnson and G. Whitney*, Minimal rank completions, Linear and Multilinear Algebra 28 (1991), 271-273.
C.R. Johnson, S. Lewis*, and D. Yau*, Sign patterns that allow given line sums, Linear Algebra and Applications, to appear.
Yu. Karlovich, I. Spitkovsky, and R. Walker*, Almost periodic factorization of block triangular matrix functions revisited, Linear Algebra and Applications, 293 (1999), 199-232.
D. Keeler*, L. Rodman, and I. Spitkovsky, The numerical range of 3-by-3 matrices, Linear Algebra and its Applications, 252 (1997), 115139.

A-L Klaus* and C-K Li, Isometries for the vector(p,q) and induced (p,q) norms, Linear and Multilinear Algebra 38 (1995), 315-332.

C-K Li, and P. Metha*, Permutation invariant norms, Linear Algebra and its Applications, 219 (1995), 93-110.
C-K Li, J. Lin*, and Rodman, L., Determinants of certain classes of zero-one matrices with equal line sums, Rocky Mountain J. Math., to appear.
C-K Li, P. Mehta*, and L. Rodman. Linear operators preserving the inner and outer c-spectral radii, Linear and Multilinear Algebra 36 (1004), 195-204.

C-K Li, P. Mehta*, L. Rodman. A generalized numerical range: the range of a constrained sesquilinear form, Linear and Multilinear Algebra 34 (1994), 25-49.
C-K Li, S. Shukla*, and I. Spitkovsky, Equality of higher numerical ranges of matrices and a conjecture of Kippenhahn on hermitian pencils, Linear Algebra and its Applications, 270 (1997), 323-349.
C-K Li and W. Whitney*, Symmetric overgroups of $S_{n}$ in $O_{n}$, Canadian Math. Bulletin 39 (1996), 83-94.
D. Quint*, L. Rodman, and I. Spitkovsky, New cases of almost periodic factorization of triangular matrix functions, Michigan J, Math., 45 (1998), 73-102.
A. C. M. Ran, L. Rodman, J. E. Rubin*. Direct complements of invariant lagrangian subspaces and minimal factorization of skew-symmetric rational matrix functions. Linear Algebra and its Applications, 180 (1993), 61-94.
I. Spitkovsky and D. Yong*, Almost periodic factorization of of certain block triangular matrix functions, Math. of Computation, to appear.
D. P. Stanford, J. Urbano*, Some convergence properties of finite matrix sets, SIAM J. Matrix Analysis and Appl. 15(1994), 1132-1140.
B. Wainberg* and H. Woerdeman, The maximum row sum nonsingularity radius, Linear Algebra and its Applications, 247 (1996), 251-264.

Perhaps a dozen additional REU articles are under consideration by journals or are in preparation.

College of William and Mary, Williamsburg, VA 23187-8795
E-mail address: lutzer@MATH.WM.EDU

# The College of Wooster Applied Mathematics Research Experience 

John Ramsay

## INTRODUCTION

The College of Wooster Applied Mathematical Research Experience (AMRE) is an eight week summer research program which joins student teams and a faculty advisor from The College of Wooster with a local business, industry or government agency (client) in order to apply a mathematical science perspective to problems found in a "real world" setting. Though first and foremost an educational endeavor for the student researchers, AMRE benefits all parties involved. Students are provided a summer research opportunity that expands their knowledge in their major field of study by giving them a setting in which they may apply the theory they have learned in the classroom. In addition, these students receive invaluable practical experience in a mathematical sciences career setting. Faculty have the opportunity to be involved with highly select students in a summer activity, while expanding their own knowledge in and contributing research to fields of applied mathematics or computer science. Clients have the opportunity to tangibly support education, and at low cost, obtain solutions to problems that would likely not be addressed internally.

## HISTORY

Modeled in part upon a similar program at Harvey Mudd College and designed to provide consulting and problem-solving experience in the context of the mathematical sciences, the Mathematical Sciences Department created The Wooster Mathematics Clinic, subsequently the Applied Mathematics Research Experience during the summer of 1994.

[^2]That first summer included projects with the Finance Department of The City of Wooster and Smith Dairy Products Company. Each subsequent summer has involved between seven and twelve students working on either three or four projects.

It is worth noting two significant differences between the AMRE program and most mathematics or engineering practicum experiences at other colleges and universities. First, the program is completed as a full-time endeavor during the summer, as opposed to part of an academic year student course load. Second, the students receive wages for their work, not academic credit. The reason for this was to be sure to attract our best students into the program. Our best students don't need more academic credit but most do need summer jobs.

## PARTICIPANT AND PROJECT SELECTION

In order to seek potential clients, solicitation letters are sent to area firms by the AMRE program director and potential projects are subsequently reviewed. Following negotiation with individual firms, contracts are signed, and a client liaison for each AMRE team is identified. The faculty advisor works with the client Liaison in the spring to determine the scope of the project and formulate a project statement. AMRE students are selected from among applicants to the program, usually rising juniors and seniors, and their majors have included: mathematics, computer science, physics, economics, chemistry, religious studies and pre-engineering.

## DESCRIPTION OF ACTIVITIES

A team of (usually three) students and one faculty is assigned to each project. Each team is provided with an on-campus office and has access to college computing resources. In most cases the teams' time is primarily spent in their campus office with travel to the client location made as needed for such activities as data collection and computer usage. During the initial weeks of the program, the advisor's involvement is heavy as the student team becomes familiar with the project definition. Student teams give weekly progress reports in the form of oral presentations to the AMRE group, in addition to periodic presentations to their respective clients. Each team gives a final oral presentation and written report to the client.

In addition to the work done on the project, a number of lectures are presented by the faculty advisors as part of the AMRE program.

Lecture series have been given in Applied Statistics, Operations Management and Neural Networks. Colloquia are also given on a variety of topics including Group Dynamics and Consulting, Technology Usage, Production Planning, Oral Presentation, and Communication in the Corporate Workplace. These colloquia are given by individuals from academia, local corporations and by professional consultants.

## PROJECT EXAMPLES

Two specific examples will explain more fully the nature of the service and product that AMRE offers: 1) In 1995 one of the AMRE teams analyzed an inventory cost problem for LuK Incorporated. A typical problem in product assembly is that of determining appropriate batch sizes for the various parts to be run through the system before assembly line and machine shop equipment is reset to begin work on a second product. Since the demand for products is fairly constant, producing a large quantity creates a "holding" cost as the demand slowly reduces the inventory. On the other hand, smaller batches require more cost in the frequent "set-ups" necessary on the equipment. LuK was using a commercial program which gave a least cost batch size determined by a standard Economic Order Quantity model. The AMRE team first designed a model that produced the same results as the old system and then improved the model by taking into account "in-process" inventory that was accumulating due to a bottleneck within the production line.
2) One of the 1999 AMRE teams worked for Goodyear Tire and Rubber Company. The project compared multiple artificial neural network models based on their ability to categorize carbon black aggregates into one of five pre-defined categories. Each model accepted an aggregate in the form of an input vector consisting of twenty-three measurements automatically recorded from microscopic images. An indication of the carbon black category for each aggregate accompanied its input vector. The team members used a portion of the available data to "train" each model and then tested the model on the remaining portion of the data. The most promising model utilized a Bayesian approach to provide a probability estimate for each carbon black category indicating the likelihood that the given aggregate belonged to that category. As an extension of the project, team members have integrated a Bayesian model into a new artificial neural network paradigm that is capable of both classification and regression estimation using a relatively simple four layer topology.

Following is a list of the titles for some of the other projects completed over the past five years: Strawberry Crop Volume Prediction,

The J.M. Smucker Company Therapy Material Application: Overlay Creator, The Prentke Romich Company Production Scheduling Analysis, Vermeer Manufacturing Examination of Plastic Extrusion Molding Process, Rubbermaid Home Products Computation of Stress and Deflection in Banners and Backlight Faces, Metromedia Technologies Cycle Counting and Inventory Accuracy Analysis, The Gerstenslager Company XML Parts Information Display System Development, Bell \& Howell Publications Systems Turbine Blade Design, LuK Corporation.

## FUNDING

The Applied Mathematical Research Experience was initiated by a seed grant in 1994 from The College of Wooster's Hewlett-Mellon Presidential Discretionary Fund for Institutional Advancement and continued support from this fund was provided through 1997. Additional funds have come from the College's Office of Undergraduate Research and the Office of the Vice President for Academic Affairs. A fee from each of the program's clients has been an increasingly important part of the funding. Fees in the first two years were small (one or two thousand) and increases have been made each year as the program gained recognition. Currently, each project costs approximately $\$ 16,000$. Half of this cost is covered by the client fee and half by the college. The college's contribution is mostly absorbed through room and board provision, the Office of Undergraduate Research and indirect costs, but some additional funding from the Office of the Vice President for Academic Affairs is still necessary.

## EVALUATION

The evaluations received have been positive from all perspectives. From an educational point of view, the students find the teamwork and "hands on" experience particularly worthwhile. This is reflected both in the student evaluation forms that are completed at the end of the program but also in the newspaper articles that have been written about the program. Here are a few samples: "We've been bombarded with all this math in class and it's nice to see how it all works." Scott Meech, The Wooster Daily Record, 1994 "I enjoy computer programming in general, but creating something that is directly applicable to helping other people is a big thing for me." Ben Adair, The Wooster Daily Record, 1996 "The experience I gained here I had no idea I would.

Working on the AS400 system platform is going to be very helpful.", Charles Nussbaum, The Akron Beacon Journal, 1999

Though the educational opportunity provided for our students is of primary importance, the financial feasibility of the program depends very much on the value of the work done for the clients. In every project client liaisons have indicated that they have been very pleased with the results presented them by the AMRE team. Liaisons have been very impressed with the students, identifying their enthusiasm and quality as the greatest strengths of the program. In all program evaluation forms received, clients have indicated that the results of the project was worth more to them than the fee charged. However, probably the two greatest endorsements for the program are first, almost half of the projects clients have hired one or more of the students to continue working for the remainder of the summer or in the following fall and second, several of the companies have already sponsored multiple projects.

## CONCLUSION

In the modern workplace it is becoming increasingly apparent that employees need more than just the technical skills necessary to perform their jobs. Critical thinking, problem solving, interpersonal relationship skills and both oral and written communication skills are vital if employees are to continue to make valuable contributions in their workplace. Many businesses have undertaken the task of training their employees in these areas. As a liberal arts college, The College of Wooster places the development of these qualities foremost in the training of its students. By establishing connections between our students and business and industry, AMRE provides an environment where students can begin to practice the use of these skills, while providing business with an opportunity to contribute to the development of individuals who can come into the workforce equipped with these vital qualities.

The College of Wooster, Wooster, OH 44691
E-mail address: jramsay@ACS.WOOSTER.EDU

# Cornell REU Program (1994-1999) 

R. Strichartz, Director

Our program is designed to engage undergraduate students in research work that is meaningful to the mathematical community and interesting to the students. It is expected that some, but not all, of the research will be published, either in joint papers with the faculty advisor, or in papers written by the students alone. All participating students will gain the experience of working on a challenging unsolved problem, and will be encouraged to continue working on their research projects after the program ends.

In order to meet these goals, we choose projects that are within the grasp of undergraduate students, and yet present real opportunities for new discoveries. These types of projects fall into two broad categories:

1) Computer related research. Students write programs, or use existing software, to work out examples, generate conjectures, or do computer experiments in areas that the faculty advisor is actively interested in. The students get a crash course in the area, and often have to dive in without understanding all aspects of the problem. But they are responsible for translating mathematical algorithms into real programs, and participate actively in deciding strategies and adjusting to the realities of computational limitations. Depending on their abilities and interests, they may also participate in the more theoretical aspects of interpreting the results and trying to prove some of the conjectures that emerge.
2) Geometric and combinatorial problems. There are some problems in geometry and combinatorics that do not require a great deal of background knowledge to understand and work on, and so are suitable for talented undergraduates. Cornell has strong representation in these areas among its faculty and visitors.

Received by the editor September 16, 1999.

Secondary goals of the program are to expose students to the broad range of research areas in mathematics, and to develop skills in communicating mathematics.
(1) Nature of Student Activities: Students come to Cornell for an 8 week period during the summer. Usually there are 10 students and 3 high school teachers. The students are grouped in 3 research teams of $3-5$ students under the direction of a faculty advisor (Strichartz and 2 others, chosen from among Cornell faculty and visitors). (There is some flexibility to allow students to switch teams, or work on more than one project, according to the evolving interests of the students.) Each team meets regularly (usually every day) with its faculty advisor. All students attend a lecture series twice a week, the Smorgasbord Seminar, in which Cornell faculty and visitors representing many different areas of mathematics give talks that give a taste of research in these areas. The Cornell Mathematics Department has an especially broad spectrum of interests, including probability, statistics, logic, combinatorics, and numerical analysis, as well as the standard areas. The students also meet for a Jam Session once a week in which they give progress reports on their work to each other. At the end of the program, the students give public talks on their work at the Undergraduate Research Forum. They are also encouraged to give talks at MAA meetings and at their home schools. There is also one meeting with the Graduate Field Representative to discuss the application process for Graduate School.

Students live in a coop just off campus. They receive free rent in addition to a stipend of $\$ 2750$. Students meet each other the first day of the program, and attend a campus-wide pizza party early in the summer where they can meet students attending other research programs at Cornell. Students receive library privileges and the same access to campus facilities as regular Summer School students.

A classroom is reserved for the program for meetings. Students doing computer work use the department's Instructional Computer Lab, directed by Allen Back, a mathematics Ph.D. who is extremely knowledgeable and helpful about software support.
(2) The Research Environment: R. Strichartz has supervised research for 36 undergraduate students in the past 9 years. He has 10 papers coauthored with these students accepted for publication, and 2 more in preparation. He is active in research in harmonic analysis and fractal geometry, and is on the editorial board of Journal of Fourier Analysis and Applications. He will select the other faculty advisors from among the faculty and visitors of the Cornell Mathematics

Department. To date, the following advisors have participated: R. Durrent, L. Wahlbin, K. Bezdek, R. Connelly, K. Pilgrim, R. Ehrenborg, J. Hubbard, J. Kigami, A. Epstein. In addition, we usually have graduate students and postdoctoral visitors informally involved in the work.
(3) Student recruitment and selection: We create a poster that is mailed to every mathematics and computer science department in the northeast U.S., and is sent along with a letter to about 200 individual mathematicians across the country who know the faculty advisors. The poster contains a description of the research projects and information the students need to apply for the program. We also maintain a Web page, and the NSF lists all REU programs. Usually this process brings us over 100 applications.

Students are asked to write a letter describing their interests and experience, and indicate their first and second choice of projects. We require 2 letters of recommendation and a college transcript. We encourage email submissions. The deadline is the end of February. (We are flexible about late applications, and notify students if their files are incomplete.)

All files are read carefully by the faculty member in charge of the student's first choice project. Special attention is paid to the student's letter, to see if the interests and background of the student are well suited to the needs of the project. In this way we select a group of about 30 top candidates. All 3 faculty members read the top files and meet to make the final decision. Students are accepted for a specific project, in most cases their first choice, and are notified by email, with a (flexible) 1 week deadline. We make 10 initial offers, and make subsequent offers to replace students who decline. We usually have to go through 20 offers to fill the 10 places. We usually have at least $75 \%$ of participants coming from outside Cornell, but the number varies from year to year.
(4) Projection evaluation: Students are asked to fill out a detailed evaluation form at the end of the program, in which they comment about all aspects of the program. These comments are used to make improvements on the program from year to year. Students are encouraged to keep in contact with their faculty advisors. Often they are involved in writing papers, or they seek letters of recommendation and advice on graduate schools.
(5) Research Methodology of R. Strichartz: I have been working with undergraduate research assistants, in part through the REU program, for the past 9 years. The students are all involved in writing
computer programs to study examples, test conjectures, and explore mathematical problems. To the extent that they are interested and able, the students also participate in the process of interpreting results, devising algorithms, trying to prove conjectures that arise from their work, and writing up the results. These collaborations form a vital part of my research work, and enable me to participate in the development of the new methodology of experimental mathematics.

The way I organize my research team is as follows. Before the summer begins, I write an outline of some problems with explicit algorithmic procedures, and send this to the students, along with some relevant reading material. When the students arrive, I try to see what interests them, and how they will feel most comfortable working. Some students prefer to work alone, and others do better in small groups. I get the students working right away, even while they are trying to learn background material. I meet with the students every day to give them feedback on their work, and so that the students can help each other. I choose projects that have a very clear beginning, but not necessarily a clear conclusion. I allow the students a lot of flexibility to explore and pursue interesting leads as they arise, and to change direction if they seem stuck. Frequently we find that the most interesting discoveries were totally unanticipated.

Cornell University, Ithaca, NY 14853
E-mail address: str@math.cornell.edu

# Research and Education Activities of the Mathematical and Theoretical Biology Institute at Cornell 

Stephen Tennenbaum

The Mathematical and Theoretical Biology Institute (MTBI) for undergraduate research was established in 1996. Its activities have been supported with a three-year grant from the National Security Agency (1996-98); the National Science Foundation (1996-98); a threeyear grant for Cornell graduate school fellowships from the Sloan Foundation (1997-2000); and with substantial moral and financial support from the Office of the Provost of Cornell University. MTBI's director, Carlos Castillo-Chavez, received a Presidential Mentoring Award in the Sciences and Engineering in the Fall of 1997 in part, for his mentoring activities as director and founder of MTBI. The focus of MTBI has been to support research opportunities mostly for underrepresented minorities who may have expressed interest in conducting research in applied mathematics or related fields. MTBI provides legitimate research experiences in applied mathematics, particularly in computational and mathematical biology (documented in seventeen technical reports with fourteen more in preparation in just three years). These research experiences are directed mostly to undergraduate students who have had no prior research experiences and who have completed their sophomore or junior year of college in a mathematically related discipline. MTBI has supported, at different levels, nearly 70 US students over the last three years. MTBI's main objectives include a sustained effort to increase the number of underrepresented minorities who apply and are accepted in some of the top US graduate programs in applied mathematics or

Received by the editor September 16, 1999.
related fields. MTBI's role does not end when a student enters graduate school. In fact, MTBI's makes it part of its responsibility to see that MTBI's graduates survive graduate school.

MTBI activities and staff continuously support those who plan to or have already entered graduate school. MTBI does it by partially involving MTBI graduates as a resource to its programs or as participants in MTBI activities.

Efforts of this kind are expensive by some standards. However, a MTBI believes that they will be clearly justified in a few years after one begins to see the large number of MTBI students who complete their Ph.D's (some examples will be provided later on). MTBI selects, as a matter of policy, a significant percentage of its US students from non-selective colleges and universities across the nation and from Puerto Rico and this is a critical aspect of its policy of really increasing the number of underrepresented minorities who are attending graduate school.

MTBI's first year focused mostly on bringing US Latino and Native American students to its research programs. However, MTBI later established as policy not to provide a program exclusively oriented to underrepresented minorities. This shift in MTBI's policy resulted from the trivial observation that the world in which MTBI students must compete and succeed, which as its first step involves attending a US graduate program, is highly international, competitive, and heterogeneous. Therefore, MTBI has experimented with a variety of educational, academic, and research environments to develop and test suitable supportive models. MTBI believes that underrepresented minorities must be involved in research programs with the highest academic standards, that is, programs that provide the tools and training needed to carry out a significant piece of research in a short period of time; programs that increase students' confidence by letting them measure their talent, creativity and training against a high level of competition within a nurturing and supportive environment. Only direct and successful experiences in programs of the highest academic standards will give underrepresented minorities the drive to apply, be accepted, and successfully compete over many years in the top US graduate programs in applied mathematics or related sciences.

MTBI provides a competitive, nurturing, and heterogeneous research environment through a balanced mixed of students, faculty, and staff with different backgrounds and research experiences within a culturally-sensitive environment. The presence of international students, mostly from Latin-America, supported with non-federal funds, has played a fundamental role in setting a critical component of MTBI's
academic environment. Some Latin-American students are older (midor late-twenties) and often have had some direct research experiences particularly, in the biological sciences. These international students are indeed a fundamental resource to the small research groups consisting of three-to-five students, groups that are self-formed during the fourth-week of the summer experience. These groups main objective is to develop a joint research project in just three weeks (during the first four weeks they receive appropriate mathematical training). The background and training brought by international students greatly enhances the nature and quality of the resulting group projects. The experience and desire brought by international students from similar cultures (Latin-America) often becomes the glue that makes the group go forward. The following example may help illustrate this point. This year (1998), MTBI brought a marine biologist Ignacio Mendez from Ensenada, Baja California where he works as a research scientist after having completed a BS in biology and a specialization in applied statistics. Ignacio works in the field of conservation biology where he wants to complete someday a Ph.D. (Ignacio married young, has two children, and is in his late twenties). He works as statistical support person and does research in conservation biology. His interests are clearly focussed. Ignacio wants to know what is the role that harvesting has on schools of fish particularly, on schools of the pacific tuna. He got a group of US-MTBI participants (two from California and one from Puerto Rico) interested in addressing this question using dynamic models. He contributed to the project with the question, his knowledge of biology, his access to data, his enthusiasm and his knowledge of statistics. His collaborators (two juniors and a sophomore, two attending universities in California and the other Cayey, Puerto Rico) developed a nonlinear differential equation model (three equations) that followed the dynamics not of individuals or of fish biomass as it is quite common in fisheries models, but rather of schools of fish (a novel twist).

MTBI has set a unique housing arrangement. A sorority house becomes a living research institute as it belongs exclusively to MTBI during nine weeks each summer. This residence houses all the students and some of its faculty MTBI's summer institute, as we call it, is equipped with a MTBI-supported computer laboratory, whiteboards, study rooms, comfortable living areas, and it is served by two cooks. MTBI divides its summer research experiences into various phases: first, it brings the most promising students back for a second summer
for deeper mathematical training (advanced topics: nonlinear dynamical systems, systems of partial differential equations, stochastic processes, methods of simulation and modeling); second, returning students work as mentors and assistants (about 10 hours a week) and as role models ( 24 hours a day); third, MTBI monitors the application process to graduate school and follows the progress of each student; fourth, MTBI brings some students for a third summer as regular teaching assitants; fifth, MTBI brings back former students who feel isolated in graduate school to reduce the likelihood that they quit. For example, Sharon Lima, class of 1996, who graduated from Loyola Marymount in California, felt isolated at Purdue where she had just become a graduate student on a teaching assistanship (not a good situation) in the department of mathematics. MTBI noticed the difficulties that she was experiencing and determined that her likelihood of success at Purdue was minimal at best. MTBI brought Sharon back for a second and third summer. These visits help her keep and/or regain her motivation. She did not drop out of Purdue while she pursued other options; MTBI recommended her for a multi-year graduate school fellowship to the department of mathematics at the University of Iowa where she enrolled last August. MTBI now has three students with multi-year fellowships including the one awarded to Sharon Lima at the University of Iowa (Joaquin Rivera, from the University of Puerto Rico, Cayey at Puerto Rico; Brendaliz Acosta, from the Universidad de Puerto Rico, Cayey; and Sharon Lima from Loyola Marymount, CA. Gina Fernandez (Dominguez Hill in CA and a 1996-MTBI graduate) declined a fellowship offer from the University of Iowa as she needs to remain near her family in California). None of the four students, three women, who received six-year fellowships offers from the University of Iowa, come from selective schools. A second and quite different example is given by Julio Villarreal, the son of a janitor, an average student at best at San Diego College in CA. Julio has now become the best student in the graduate field of biometry at Cornell. Julio came back for a second and third summer to MTBI. The second time he made a quantum leap in his mathematical training and confidence and his report, where he analyzed nonlinear systems of ODEs and a hyperbolic system of PDEs, is part of the enclosed second volume of MTBI research. The third time he came (because he demanded it) as a part-time assistant and 24-hour a day role model, after having successfully completed his first year of graduate students at Cornell. Julio demanded to be part of MTBI, demanded to be the mentor of the two 1998 high-school students, and has now become a force within MTBI. Julio shows students in a unique
and powerful way that graduate school is a realistic and wonderful option for them. A third example is provided by an international student, Ricardo Saenz from Ciudad Juarez, Mexico. He was recommended by Javier Rojo (a native from El Paso, Texas and a Mexican-American) a professor of mathematics and statistics at the University of Texas El Paso. Ricardo, because of his economic background, was charged statetuition at UTEP. Nevertheless, he could not afford it and a professor of mathematics eventually help him pay it. Ricardo walked across the bridge that joins El Paso with Ciudad Juarez everyday. We brought him to MTBI where he became a great contributor, he even gave a couple of lectures on number theory. He returned for a second year and his report, where he analyzed a hyperbolic system of PDEs, is part of the enclosed second volume of MTBI research. He returned for a third year and served as a TA (still a senior in college) for the summer program in Mexico. He accepted a fellowship from the Mathematics Department at Princeton University that included a $\$ 2,000$ signing bonus with a matching clause (Princeton would match any other offer). MTBI students live in an institute (sorority house during the regular year) that includes study rooms, a computer lab (with 12 computers and two printers), two cooks, dinning facilities, white boards, etc. Computer rental (about $\$ 4,000$ for 8 weeks) was not part of the funds requested in the initial grants. However, thanks to the Office of the Provost of Cornell University, MTBI was able to rent the computer equipment in 1997 and 1998. In 1996, MTBI bought, with NSA funds, two computers for about $\$ 4,600$ that became obsolete within a year. MTBI's policy to rent twelve computers (all fast Pentiums) for about $\$ 4,500$ for nine weeks keeps the equipment modern and does not leave a large number of computers unused during the regular year. The funds to rent these computers and to pay for the additional software were provided again by the Office of the Provost of Cornell University in 1997 and 1998.

MTBI's location in Cornell University, which houses some of the country's top-ranked programs in biometry, ecology and evolutionary biology, applied mathematics, mathematics, and computer science, as well as one of NSF's supported supercomputing facilities, provides MTBI students with the exciting research-oriented atmosphere that is commonly found at most major research institutions. CUSSP program has been carefully designed by theoretical and mathematical biologists, under the direction of Prof. Carlos Castillo-Chavez, to provide a well thought-out, intensive, and serious research experience to undergraduates who will complete their sophomore or junior year during the school year previous to entrance into the program. Students majoring in mathematics, biology, or related fields, who have had at least
one year of calculus and who have shown the desire and the ability to work with quantitative methods, are eligible to apply. The 1997 Summer version (as well as the 1998) of CUSSP was organized again under the auspices of Cornell University and the Society for Advancement of Chicanos and Native Americans in Science (SACNAS). CUSSP has been held thanks to the support, and encouragement of the Office of the Chief of the Mathematical Research Division of the National Security Agency (NSA), the Office of the Provost of Cornell University, with grants from the National Security Agency (1996-98), the National Science Foundation (1996-98), and with substantial financial support from the Office of the Cornell Provost (1996-98). Dr. John Alderete, the President of SACNAS has provided strong support for this project. SACNAS's specific contributions to CUSSP include making accessible its database for the recruitment of students (over five hundred addresses); providing partial financial support for the participation of nearly eighty MTBI students (some have attended more than one national meeting) to the 1996, 1997, and 1998 SACNAS annual conference; providing a forum for MTBI student research. An hour session was reserved at the 1996 and 1997 SACNAS national meetings to highlight, via three oral presentations, the results of the research carried out by the 1996 and 1997 members of CUSSP.

SACNAS has provided a forum for six research posters by MTBI students during 1996, ten in 1997, and eight in 1998. SACNAS provided a two-hour slot at its 1998 national meeting for a session in applied mathematics that brought five outstanding mentors to its national meeting (Simon Levin, Ecology and Evolutionary Biology, Princeton University; Lisa Fauci, Department of Mathematics, Tulane University; Jim Schatz, NSA; Denise Kirschner, University of Michigan, Medical School; and Moss Sweedler, NSA and Cornell University). All expenses incurred by the five speakers were paid by SACNAS. SACNAS has provided a publicity booth for MTBI at its 1996, 1997, and 1998 annual meetings. SACNAS included MTBI's director in the relevant planning stages for the SACNAS 1997 annual meeting. SACNAS has established an MTBI reception as part of its 1996, 1997, and in 1998 SACNAS program (the 1998 reception included eight poster presentations). The objectives of MTBI's Research Programs for Minorities are to encourage and facilitate the access to and the successful completion of graduate studies by mostly Latino, African-American, and Native American students in the mathematical sciences via an undergraduate research training program that exposes them to the elements of scientific research via a large pool of projects that address relevant questions in theoretical biology. We measure our success directly by the quality
of our technical reports, by the acceptance of our students to graduate school (a list will be provided later in the report), and by their success in graduate school. MTBI's goals are achieved through: training and mentoring minority undergraduate students during a sevenweek summer research program offered to selected college juniors and seniors and an eight or nine-week program to second-year MTBI students who are more likely to enroll in graduate school as measured by their participation on MTBI's prior seven-week program; establishing undergraduate research opportunities at the interface of mathematics, statistics, and their applications to theoretical biology as many underrepresented minority students are likely to remain in mathematics only if they experience realistic applications of mathematics (our projects, now selected mostly by the students themselves, clearly reveal their interest in problems that have some connection to society); additional efforts to recruit students who attend universities designated by NSF as minority institutions and/or students from underrepresented groups who come from economically disadvantaged families. The PI travels to Puerto Rico, Texas, and California for this purpose; monitoring student progress for at least five years after their participation in CUSSP to ascertain their success in graduate and/or professional schools, thereby also documenting the successes of MTBI. MTBI successes are monitored via its published technical reports and its web page with address, http://www.biom.cornell.edu/MTBI/mtbihome.html; hiring faculty composed of first-rate minority and non-minority researchers to teach, advise, and serve as role models to MTBI students. Last year we had two women faculty (Asian- and White-American), and three male faculty (African-, Mexican-American, and a Mexican). All of them with active research programs; expanding the visibility of the research carried out at MTBI by minorities via the inclusion of student research in the Biometrics Unit Technical Report Series at Cornell University. MTBI has now produced 17 technical reports and has 14 under preparation; supporting MTBI's Web Page and linking it to the Web Pages of. scientific associations such as the American Mathematical Society. Networking minority students with well-known mathematicians, mathematical biologists, and theoretical biologists, as most minority students will probably be mentored by a non-minority advisor in their graduate studies (help has been requested from the Society for Mathematical Biology and other professional organizations). We have just co-organized a special session in applied mathematics at the annual meeting of SACNAS where students were able to interact with them, at the MTBI reception, during lunch, and during the session with the
five speakers: Simon Levin, former President of Ecological Association of America and the Society of Mathematical Biology (SMB); Lisa Fauci, member of the board of directors of the SMB and an expert on numerical analysis and biofluidynamics; Denise Kirschner, a member of the board of directors of SMB and an expert on computational pathogenesis; Jim Sachtz, Chief of the Division of Mathematical Sciences at NSA; Moss Sweedler, now at NSA and an expert in mathematics engineering and computer algebras; expanding MTBI's publicity efforts to its intended communities to guarantee that promising undergraduate minority students in the country apply to MTBI; inviting some of the best known mentors in the fields of mathematical and theoretical biology and applied mathematics to MTBI's Summer Colloquium Series (the list is provided later on); supporting an environment in which high quality undergraduate research is recognized and expected (MTBI technical reports provide the best examples); supporting a research staff capable of generating exciting, innovative research projects to motivate undergraduate students and lead to the achievement of significant results within a summer. This year we plan to hire Steve Wirkus as a member of MTBI's staff. Steve is completing his Ph.D. in applied mathematics and has been a TA at MTBI for the three summers; supporting a computer environment and facilities that make it possible for undergraduate students to become junior research members of a first-rate research institute. MTBI has its own laboratory within the Biometrics Unit. It is used by faculty, TAs and advanced students during the summer. It is equipped with several computers, printers, and a scanner. It includes two computers donated by the INTEL corporation. Software has been purchased via a variety of grants including an INTEL grant.

## Comments from Students

"Working in groups for research projects, in addition to being fun and stimulating, was very demanding on dedication and quality, without the competitive stranglement too often stressed in college."

Ariel Rodredguez-Herrera, Univ. of Puerto Rico. MTBI 1996/1997
"The best summer of my life. It was an opportunity that I was desperately seeking for; a chance to realize my strengths and my potential...But most important to find who I am, what I am capable of accomplishing and what I really want."

Erika Camacho, Wellesley College.
MTBI 1996
"I came into the program with a fair amount of uncertainty, about the program, about the people I would meet, my ability to meet the requirements - I'm sure you know at least some of these feelings. I also did not really know for myself if my plans to attend graduate school were for any particular reason or if it was just the norm.... This is the first of the things that I will mention that I gained from you guys. I had been a research assistant at OSU on different projects, but to collaborate on a new project, one in which we were the decision makers and the doers was a totally different experience. It was one that set my mind on fire."

Mark Muktoyuk, Oregon State University.
MTBI 1997/1998
"The project has been one of the most valuable research experiences that I've ever had. The quality of all of our projects was outstanding.... It's so exciting to think that I was given the opportunity to be a part of it!"

Anonymous student evaluation. MTBI 1999
"This is the most intense program in which I have ever participated. It was great!"
"I've never had a better opportunity; this program not only exposes students to new subjects, subject fields, research; it also heads us in the direction of graduate school. Before I wasn't sure of what I wanted to pursue and lacked the confidence to apply anywhere- now I'm looking forward to it. It also taught me the how hard I could work- and work towards a goal. It made me feel tremendously important to have such distinguished professors and advisors listen with interest to my ideas. Thank you!"
Anonymous student evaluation. MTBI 1999
Cornell University
E-mail address: set1@cornell.edu

## Galois Theory at Davidson 1998

John R. Swallow

During the summer of 1998 Davidson College hosted its first summer undergraduate research program in mathematics. The program, "Galois Theory at Davidson," was designed to fulfill several purposes: first, experience for two or three undergraduates; second, to advance the director's own research in the Inverse Galois Problem; and third, to offer the host department the opportunity, through participation in a small program, to consider whether and how best a larger program might be put in place at Davidson College.

History and Funding. The program was originally envisioned for two undergraduate women at Davidson College who, after finishing an abstract algebra course in the fall of 1997 and registering for a Galois Theory course for the spring of 1998, expressed interest in pursuing summer research with their instructor. The instructor (an REU participant himself in 1988) was being supported under a grant from the National Science Foundation (NSF) Career program; having not previously requested funds to support undergraduate researchers, he applied for additional support under an REU Supplement for the two students. Over the Christmas break, however, the two women were offered paid summer positions overseas by a Christian organization with a local chapter on campus, and both accepted. After a program director at the NSF confirmed that the awarded support for two students could be used for students recruited externally, what had been conceived as a research experience for two locally enrolled individuals became a small REU program recruiting nationally. The program was included on the NSF's list of REU programs, and advertisements were sent to colleges and universities across the country, specifically seeking applications from undergraduates interested in working as a

Received by the editor August 26, 1999.
part of a small research group. The program encouraged applications from women in keeping with the intent of the original proposal, and in May 1998 two undergraduates accepted offers, Judith Baum from Mt. Holyoke College and Nathaniel Thiem from Macalester College. The director also sought funding for an additional student participant through the Carolinas-Ohio Science Education Network (COSEN), a program funded by the Pew Charitable Trusts to promote research experiences, especially among minorities and women, at a group of colleges and universities in the Southeast. The proposal was accepted, and though one student expressed interest, she was finally unable to accept the position.

Goals and Implementation. The primary goal of the program was to provide an authentic research experience for the two students by offering a chance to participate fully in the development of a new theoretical method for the solution of certain Galois embedding problems. To implement the method, it was necessary to solve several fundamental Galois embedding problems, and the director anticipated that well-qualified undergraduates would be able to make substantial contributions to their solution. In the event that the research group was unable to make progress on the original problems, the director had also prepared several related problems, requiring less preparation and connected with explicit construction of solution fields to the embedding problems. The program spanned ten weeks, the first two of which were devoted to an introduction to the mathematics. During the first week of the program, the director presented a series of lectures, covering the theorems from Galois theory necessary to understand the context of the problems, followed by an introduction to the theory of quaternion algebras and tensor products. The week concluded with the assignment of several "warm-up" exercises which the students could use to assess their own comprehension and aptitude for the main problems. After working on these problems, one student elected to attempt the fundamental embedding problems and the other elected instead to investigate the related problems. For the remaining weeks, the two students essentially worked individually, with advice from the director, although they periodically discussed their respective progress on an informal basis. During the last two weeks each student wrote an article explaining the motivations the problem they chose, together with their results, and each student made a thirty-minute presentation before the Department. A secondary goal of the program was to give the students an opportunity to assess their own suitability for graduate study in mathematics.

The group met several times and discussed the skills and motivations necessary for graduate study and the process of selecting a graduate program. Also, the students took part in a weekly lunch meeting of summer student researchers in science, funded through another department's summer research program. These meetings raised issues related to science research in general and offered a forum for presentations by student participants. Finally, the Department of Mathematics sponsored a summer colloquium series, introducing the two students to a variety of mathematical topics outside of the theme of the summer program. The program finally sought to create a social environment for the students, both to encourage the exchange of ideas and to guard against a perception that mathematics and science must necessarily be a lonely apartments with other summer research students he knew personally. These other students were able to provide off-campus transportation in the Charlotte area and formed a limited society for the participants. The two students were also occasionally invited to the director's home. Davidson faculty hosting summer students met and planned a rafting trip to western North Carolina, but unfortunately the trip did not take place. The lack of additional off-campus social activities was a shortcoming of the program.

Student Outcomes. At the end of the summer, Nathaniel Thiem intended to apply in the fall to graduate schools in mathematics and to pursue an honors project at his home institution expanding his results from the summer. During the ensuing year he did both, completing a substantial honors project at Macalester College and accepting an offer, with support, to the doctoral program in mathematics at the University of Wisconsin. The work from this honors project forms a portion of an article, Quadratic corestriction, $C_{2}$-embedding problems, and explicit construction, jointly written with the director and now submitted for publication. Judith Baum, on the other hand, decided against graduate study immediately following graduation. After pursuing several options, including work as an curricular assistant in the mathematics department of a liberal arts college, she accepted employment with an insurance company.

Assessment. In an interview with the director at the end of the summer program, the students concurred that the summer program
succeeded in providing an authentic research experience in mathematics and in encouraging them to consider their own aptitude and motivations for graduate study. Their main recommendations for improvement focused on the size of the program and the extent of organized social activity. Both participants agreed that a larger program in mathematics would have offered more opportunities for group work as well as for planned social activities. The director believes that the students' assessment is accurate. A more successful program will require a larger group of students, as well as additional faculty, not only to offer a variety of mathematical topics, but also to share in the administration of the program. Interaction with other student summer researchers in science, while beneficial, should community divided into several closely knit research groups.

Davidson College, Davidson, NC 28036
E-mail address: joswallow@davidson.edu

# Cooperative Game Theory REU 

David Housman

The Drew University REU in Cooperative Game Theory was held during the summers of 1990-1993. The program brought together groups of six students (three in the final summer) for a first-time immersion in mathematical research. The program was designed to create a community of scholars where inquiry was valued and questions of career and life aspirations could be discussed in an atmosphere of care and trust. The major components of the program are described below.

Students. I sought mathematically capable students for whom the program would have a major impact on their career decisions. Operationally, this meant students who had (1) completed at least two courses in which they had written proofs, (2) strong letters of recommendation, (3) expressed openness to, but uncertainty of, graduate studies, and (4) no substantial research experience. Since more applicants fit these criteria than the available positions, final selections were made to ensure a diversity of mathematical, collegiate, and social backgrounds. About equal numbers of students came from baccalaureate colleges, masters institutions, and doctoral universities.

Faculty. I was the program director and sole mentor. I met with each student individually from one to five hours weekly. This usually meant a discussion in my office, but we also had discussions with each other over meals, in their house, between talks and in the dorms at conferences, in vehicles on our way to and from conferences, and on exploratory walks. Students also interacted with faculty visitors and with faculty at conferences we attended as a group. Students found close formal and informal interaction with faculty and students to be the most beneficial aspect of the program. Through these interactions the major gains were made in each student's level of mathematical

Received by the editor September 1, 1999.
maturity, ability to express mathematics, amount of self-confidence, and desire for a mathematical career.

Topics. The general area of research was axiomatic and algorithmic aspects of values for cooperative games and their applications. Students were introduced to (1) games in coalitional function form and how this model could be modified in various ways, (2) several standard solution concepts, (3) several properties games and solutions could possess, (4) some known interrelationships, and (5) some interesting unanswered questions. With consultation, students were then free to choose a topic. By selecting different topics, each student felt ownership and developed expertise. With one general area of research, the students were able to understand and support the progress of each other. By providing a framework for the research area, students were empowered to ask their own questions as well as making conjectures and proving results.

Seminars. I started out each summer with four lectures and group discussions that introduced cooperative game theory and the area of research. There were weekly seminars where each student talked about her or his progress and answered questions for 10 to 30 minutes. Many seminars included a visitor who listened and reacted to student talks, gave a talk of his or her own, and discussed career and graduate school over a meal. The small and supportive audience provided a friendly environment for students to gain confidence in and improve their speaking skills. At the same time, valuable suggestions were raised and students were able to clarify the direction of their research.

Professional Trips. During each summer, we traveled as a group to Rutgers University, to an international conference on game theory, and the summer MAA/PME meetings (expect in 1990). The meetings provided opportunities formal student presentations and for interaction with faculty having common interests.

Reports. Students wrote both an interim an a final report. The interim report forced students to clearly describe their research topics and allowed me to assist student development of proper style early in the summer. The final reports described the problem considered, background literature read, approach(es) taken, results obtained, and questions motivated by the results.

Residence. During the first three summers, students shared an on-campus house with separate bedrooms, adequate kitchen facilities, and a large living room and outside deck. The house provided pleasant space for both private study and group interaction.

Social Events. I favor informal events where conversation flows easily, and so there were several luncheons and dinners in restaurants
and my home. We also went on a one day excursion to Manhattan each summer.

Continuation. My last official communication with each student included written comments about their final reports and a draft recommendation letter. I have found that the second item is particularly valuable for the participants because it gives them written confirmation of their strengths, while the first item tends to point out areas needing improvement. I have remained in somewhat irregular contact with these 21 students as well as students whom I have mentored at other institutions. I know that five have received doctoral degrees and five are in their dissertation stage of doctoral studies in a variety of mathematical sciences programs.

```
Goshen College
E-mail address:dhousman@goshen.edu
```


# Hope College REU Program 

Tim Pennings

Introduction. Undergraduate student research has long been a priority at Hope especially in the sciences, with the biology and chemistry departments maintaining a continuous string of REU awards since the inception of the program. Faculty/student collaborative summer research has continued to flourish with 80-100 students now involved in research each summer in the Science Division including NSF-REU sites in Biology, Chemistry, Geology, Physics, Mathematics, and Computer Science.

Hope mathematics faculty are actively engaged in faculty-student research. This research has been - and continues to be - supported internally by faculty development grants and faculty-student research grants, and - except for one year - since 1991 by NSF-REU grants. In 1999, for example, ten students were involved in mathematics research at Hope College, of whom six were part of the REU program. Summer research projects supported by the NSF and Hope College have been recently completed by students in point-set topology, dynamical systems, geometric group theory, non-commutative geometries, and computations in semi-group rings These have led to numerous published papers and talks given by students at regional and national meetings.

The goals of summer research program at Hope College are to help talented and motivated students develop as mathematical researchers, to promote mathematics research as a career, and to have the participants achieve significant mathematical results in partnership with a faculty mentor.

Along with advanced undergraduates who have chosen to major in mathematics, we hope to attract and encourage some students who are still in their first two years of higher education and have not yet committed themselves to a particular discipline. We have found from

Received by the editor June 24, 1999.
past experience that some of the brightest and most talented students spend time exploring varied interests until they generally are obliged to choose a major in their junior year. Providing them with a positive research experience at the appropriate level can serve to attract some of these students by showing them an appealing side of mathematics they would not otherwise see. Thus some of our projects will be tailored for students who have not yet taken advanced undergraduate courses.

Nature of Student Activities. The program will begin on the first Monday in June, and its duration will be eight weeks. Students will work in groups of two with one of three faculty members. Students will initially learn about their own projects and those of the other students through introductory lectures given by the three faculty participants.

Student research begins by reading background material both provided by the mentor and found through library research. The faculty will also provide suggested problems whose solution will contribute to an understanding of the research questions to be considered. The student pairs will then be given their own project or problem to solve. The faculty mentors serve as consultants and partners on the problems, and as the summer progresses, student-mentor meetings evolve into discussions of conjectures and open questions, along with suggested strategies for the solution of these.

Generally speaking, research projects have been inspired by problems in the faculty members' own research programs. It is anticipated that each should lead to some original results, in order to achieve our primary goal of introducing students to the nature of mathematical research. Past experience has indicated that it is best to design projects so that a range of partial outcomes are possible, and the projects have been designed with this in mind.

Developing skills in writing mathematics is also a goal of the research program at Hope. Students are expected to write their results, initially for distribution at Hope, and in many cases for eventual publication. Beginning the writing during the summer allows for feedback from the faculty and from other students. Instruction and support in the use of word processing on $\mathrm{EAT}_{\mathrm{E}} \mathrm{X}$ is given.

We also work to develop students' abilities in presenting mathematics orally. Students take turns presenting their results during the later weeks, finishing with a summary talk during the final week.

We believe that it is important that students have the opportunity to present their work at professional meetings. Following the completion of the project, participants will be strongly encouraged to present
their results at regional or national mathematics meetings. The budget for the proposed project has funds for student travel to meetings. Also, Hope College will work with the participants' home institutions in seeking funding for travel, and will supplement the travel funds for the REU participants from Hope College. Last year (1998), all six of our REU students presented the results of their research at the Joint Meetings in San Antonio.

Since one of the primary goals is to promote mathematics research as a career, we will invite a leading research mathematician from one of the areas major research institutions (e.g., University of Michigan or University of Chicago) to spend a day with us giving talks on his/her own research in particular and the nature of graduate mathematical research in general. Conversations will continue informally over lunch and dinner. This will serves as a midcourse break from the routine, as well as an inspiration for continued work.

Recreational activities are also planned for many of the evenings and weekends. These include summer theatre, visits to the beach (Lake Michigan), hiking, volleyball games, trips to Chicago, ultimate frisbee, swimming, and picnics. Especially during the first few weeks, faculty members take turns hosting meals and evening socials in order to help the students become acquainted.

With scores of undergraduate students from Hope College and the country living on campus while doing summer research, many of these activates are either organized or done spontaneously with students working in other disciplines. For example, this past summer students organized late night capture the flag on campus, and a scavenger hunt.

We have learned from experience that all of these social events and recreational activities comprise an essential component of the summer research program. First of all, such activates help foster good student relationships and friendships. Since students spend considerable time each day studying together, the increased intensity and enjoyment that comes from working with friends is as essential to a productive summer of research as are good research projects and mentoring. Secondly, since students are coming with few possessions and often no car to a new place for only eight weeks, it is imperative that we help get them acquainted quickly with the area and with each other. Providing the students with a wide variety and constant stream of activities for their free time helps ensure an enjoyable and memorable summer.

## Examples of Research Projects

Discrete Dynamics on the Unit Interval - Tim Pennings

Project 1: Given $\delta>0$, a $\delta$ - pseudo-orbit is a sequence $\left\{x_{n}\right\}_{n=0}^{\infty}$ such that $d\left(f\left(x_{i}\right), x_{i+1}\right) \leq \delta \forall i \in \mathbb{N}$. A function $f$ has the shadowing property if $\forall \varepsilon>0, \exists \delta>0$ such that given a $\delta$ - pseudo-orbit, $\left\{x_{n}\right\}_{n=0}^{\infty}$, there is an $x \in X$ which satisfies $d\left(x_{n}, f^{n}(x)\right)<\varepsilon \forall n \in \mathbb{N}$. In this case, we say the $\delta$ - pseudo orbit is $\varepsilon$-shadowed by the actual orbit. In previous undergraduate research we have found necessary and sufficient conditions for an increasing continuous function on the unit interval to have the shadowing property (published in Real Analysis Exchange), and necessary and sufficient conditions for any continuous function to enjoy the strong-shadowing property (where the initial points of the orbit and pseudo-orbit coincide). Left to consider is finding conditions for which arbitrary continuous functions on the unit interval have the shadowing property. (Background Needed: A year of undergraduate analysis.)

## Continuous Dynamics on a Surface - Tim Pennings

Project 1: Discover/create a continental divide which is a fractal. That is, consider a surface where each point is colored either red or green depending on whether a drop of water (ball bearing) beginning at that point and acted upon only by gravity will eventually travel to the east or the west. Find a surface so that the collection of all red (green) points has fractional Hausdorff dimension. (Background Needed: Multivariable calculus, linear algebra, and computer familiarity.)

Project 2: Given a putting green, how many ways is it possible for a ball to be putted into the hole? Given a ball acting under the influence of gravity and (a constant) retarding frictional force, how many initial conditions (speed and angle) will result in the ball going into the hole (a point) assuming the speed of the ball is zero when it gets to the hole. Can greens be designed where each point on the green allows 1 ) only one, 2) $n \leq \infty, 3$ ) infinitely many choices possible to sink a putt? Can a green be designed where for any given integer $n$ there exists a point on the green where there are exactly $n$ successful initial conditions? What can be said if the hole is modeled as a disc instead of a point and the speed of the ball needs only be below a certain value as it passes over the hole? (Background Needed: Multivariable calculus, linear algebra, and computer familiarity.)

Noncommutative Geometries - Darin Stephenson
This research will focus largely on the interplay between noncommutative ring theory and noncommutative algebraic geometry. The basis for this work is the theory of noncommutative projective schemes which was first introduced in the early 1990s by work of Artin, Tate and Van den Bergh [ATV1, ATV2]. Since this field is relatively new,
there are many unsolved, computational problems which are ideally suited to undergraduate research involvement. This field is also quite complex, and its study requires students to have had a solid course in abstract algebra. Therefore, I will develop student research projects at various levels which relate to noncommutative geometry either directly or indirectly. In this way, younger students without a background in abstract algebra will participate in this challenging research program, while more advanced students will have the opportunity to be work on unsolved problems in the rapidly expanding field of noncommutative geometry.

The following is a list of some potential student research projects, progressing from the one requiring the least background and mathematical maturity to the one requiring the most.

Project 1: What is the expected area of a random triangle of perimeter 1? There are many ways that this question could be interpreted, and thus many possible correct answers. Perhaps the most natural interpretation is that two of the triangle side lengths, $X$ and $Y$, are chosen randomly and uniformly from the region of describing all possible choices (defined by $X<1 / 2, Y<1 / 2$ and $X+Y>1 / 2$ ). In this case the correct answer is $\pi / 105$. Students will be led to explore the solution to this and other interpretations of the problem, and then towards generalizing to polygons. Students will learn some classical geometry and probability theory, and they will be introduced to the process of doing mathematics including experimentation, conjecture and proof. (Suggested background: Calculus II.)

Project 2: Complete the classification of noncommutative projective three spaces which embed quantum projective planes of weight $(1,1,2)$. This project would follow up on the success of students I directed in the 1998 Hope College Mathematics REU. In that project, students began with a certain family of examples of weighted quantum planes from $[\mathrm{S}]$ and produced natural quantum three spaces via a Veronese embedding. They then found the 'points' of these noncommutative spaces. The algebras they started with comprised only one of three families of quantum planes of weight $(1,1,2)$, and thus a similar project can be undertaken starting with the other families. Students will learn a good deal of algebraic geometry and commutative algebra, and will gain experience in comprehending a solution given by others to a problem and then adapting it to solve a new, more difficult problem. (Suggested background: A course in abstract algebra.)

Participants. Participants will be chosen on the basis of their academic record, professional objectives, and estimated potential to pursue independent work. We will give preference to students with no previous research experience. Moreover, as mentioned in the Introduction, we plan to include 1-2 students/year still in their first two years of higher education (typically before they have chosen a major and have taken upper level mathematics courses) who show considerable mathematical promise. Providing students at this stage with a genuine research experience at their own level can help them make an informed decision about continuing in mathematics. (For example, a 1998 Hope graduate now a graduate student in mathematics at the University of Washington confirmed his interest in mathematics when he participated in our REU program after his first year of college.)

The recruiting procedures as outlined below are designed to ensure a strong group of applicants. The state of Michigan is home to a large number of colleges and universities with strong and active undergraduate programs in mathematics. Undergraduate institutions such as Hope, Calvin, Albion, Kalamazoo, Alma, University of MichiganFlint, University of Michigan-Dearborn, Northern Michigan University, Adrian, and Aquinas have sought a variety of opportunities for their majors and their departments, through the activities of the Michigan Section of the Mathematical Association of America and other cooperative ventures. Most of these institutions have been participants in the Lower Michigan Mathematics Competition, a team problem solving competition for advanced undergraduates.

A number of other public and private institutions most likely to be familiar with Hope College will also be targeted for special recruitment effort. These include members of the Associated Colleges of the Midwest, the Great Lakes Colleges Association, and the Pew Midstates Science and Mathematics Consortium. Also included in the target group are a number of women's colleges and institutions with significant minority enrollment which are located outside the region and are known to have strong mathematics programs and/or an interest in undergraduate research, based on participation in undergraduate research conferences and student publications. (See Section I. for a copy of the recruitment brochure and a listing of these schools).

The colleges and universities described above will receive a full information packet describing the Hope Undergraduate Research Program, with program posters, application forms, and project descriptions. In addition, program announcements and an invitation to request the information packets will be sent to a second larger list of (approximately 150) colleges and universities.

A strong effort will be made to include women and members of under represented minority groups among the pool of applicants and final list of awardees. Since 1994 at least a third of our participants have been women, and our two female faculty mentors effectively model the potential for women to succeed in graduate school and to do quality mathematical research. Concerning under-represented groups, minority affairs officers of institutions in the first category described above will receive a program announcement with a request to submit the names of potential candidates, who will then receive an information packet and application form. Hope College also has been establishing divisional and institutional ties with schools such as Clark Atlanta University and Howard University with large minority populations.

Prospective participants will submit an application form, transcript, and two letters of recommendation from college faculty members. Requirements will be the completion of appropriate coursework as indicated in the individual projects listed above. Hope students will also be invited to submit applications. No more than one half of the participants will be chosen from Hope College.

Each faculty supervisor will work with two research students. Individual discussion with participants will be utilized along with application materials to match participants with supervisors.

Project Evaluation. Participants will be contacted in the fall semester following their summer of research to allow them to evaluate the research program. (See Section I.) Also, participants will be tracked to determine their choice of graduate study and/or career, their progress through graduate school, and their eventual choice and placement in a profession.

Less formal but more frequent follow-through procedures occur with students through their continued involvement in the writing and submission of research papers for publication. Also, our objective of having the students give talks at major meetings provides opportunity for contact both at the meetings and in the preparation process.

Institutional Commitment. Hope College has a long-standing demonstrated commitment to the involvement of undergraduates in research. Faculty and students have worked together on a large number of research projects. Financial and other institutional support has been - and continues to be - committed to these projects.

Hope College will grant the P.I. $1 / 4$ release time during the spring semester to give time to adequately advertise, select participants, and prepare for their arrival. This support is $\$ 3000$ per year, for a total of $\$ 12000$.

The college will pay one half of the cost of housing for the summer research participants. Thus the college's committment is $\$ 4800$ ( $\$ 1200$ per year). To minimize cost to participants, housing includes kitchen facilities, although food service is available. Services normally available to Hope College summer students are provided to summer research participants at no cost. These include recreational facilities, library services (including inter-library loan), and computing services. In addition, the college provides office space, classrooms, supplies, and clerical support at no cost.

Hope College will also pay all postage, telephone and printing costs from advertising the project - estimated at $\$ 200$ per year. Clerical work is also provided by the college. Finally, travel funding for students, estimated at $\$ 500$ per year, will be supplemented by Hope College to allow students to give talks at national and regional meetings.

General Assessment and Student Comments. For the most part, our recruitment procedures appear to be effective. We have been pleased with the quality and national representation of applicants. We successfully found strong, enthusiastic participants while giving special consideration to students who had not had such a research experience. Our success at attracting underrepresented groups were mixed. Although we targeted schools with significant minority enrollments and sent additional recruitment materials to their minority affairs offices, we were unsuccessful at attracting minority applicants. We were successful at attaining a good gender balance - one third of our participants have been women.

Overall evaluation by student participants have been very positive. Many respondents indicated that they were more likely to pursue graduate study in mathematics because of the program. Some examples:
"The REU was a good experience for me. I had already considered graduate school in mathematics, but the program gave me more confidence and direction with that decision. I learned skills which helped me greatly during my first year of studies here at the University of Texas at Austin. I also made friends whom I still keep up with to this day."
"The REU program was the one experience that solidified my goals to pursue a higher degree. This was my first exposure to academic research. The experience was so enjoyable and challenging that I knew I wanted to seek a position that would keep me close to the study of mathematics."

Another wrote that an earlier NSF sponsored mathematics enrichment program had helped a great deal to awaken an interest in mathematics, leading to applying for an REU position. The REU program convinced the student to go to graduate school, and helped to give an idea of "what it will be like once I'm there."

This past summer's students were especially enthusiastic including comments such as, "I'm more likely to pursue a career in mathematics." "It was one of the most interesting and most fun summers I have ever had . . . This REU was conducive to thought, creativity and play - which is an excellent combination." "I would come back anytime!!!" "This experience convinced me that I will definitely follow a career in research mathematics. This was a tremendously great experience. I cannot imagine a better way for a mathematics undergraduate to spend a summer. The program was excellently run and the advisors did a great job of advising us. . . . I hope others get the chance to do an REU in math here at Hope College."

Faculty participants believe that the goals of the project have been achieved. We are pleased that students have been engaged in the entire research process from reading background material and solving a unknown problem, to organizing the results into a "publishable" paper and then presenting it. The fact that the presentations include the regular sessions of the joint AMS/MAA annual meetings, and that the publications include the AMS Transactions give us both pride and confidence that good things are happening.

Hope College, Holland, MI 49422
E-mail address: pennings@gauss.physics.hope.edu

# THIRTY-THREE YEARS OF MATHEMATICS RESEARCH EXPERIENCES FOR UNDERGRADUATES AT INDIANA UNIVERSITY 

Daniel Maki

## INTRODUCTION

Mathematics REU programs at Indiana University - Bloomington have existed for most of the last 33 years, but they have changed with the times and with changes at NSF. We survey these changes, and describe some of the forces which brought about the changes. We also discuss the types of students who have been in our program, the sort of projects they have worked on, and what has happened to some of them.

THE EARLY YEARS: $1966-1973$
The Indiana University program to provide research experiences in mathematics to undergraduates began in 1966. It was directed by Professor George Springer, and it was primarily funded by the National Science Foundation, under the title Undergraduate Research Participation Program, URPP to all local participants. Additional funding was provided by the Honors Program at Indiana University. The program ran in the summers for eight weeks, and the basic plan for each summer was to have the same number of faculty as students, so that each student would work on a project one on one with a faculty member. With only a few exceptions (in which case, one faculty member worked with two students), the plan was followed for all of these years. The average number of students was 12 per summer, with a minimum

Received by the editor September 29, 1999.
of 10 students and a maximum of 15 . Many of the students in these early years are now full professors around the country, including Carl Cowen (1966, now chair at Purdue), Eric Bedford and Darrell Haile at Indiana University, Steve Zucker at Johns Hopkins, Andrew Sommese at Notre Dame, and Greg Zuckerman at Yale. Several of the faculty who directed student research in those early years are still on the Indiana University faculty, including Maynard Thompson, Joseph Stampfli, and Daniel Maki.

## THE ENERGY YEARS: $1974-1979$

Following the oil embargo in 1973, both the National Science Foundation and our URPP program changed focus. NSF became very interested in renewable natural resources. The guidelines for mathematics URPP programs asked for projects which were interdisciplinary in nature and which were, in some sense, related to renewable natural resources. The program at Indiana University now involved faculty from many different departments and from different schools. The key faculty in the mathematics department were Professors Springer, Thompson, and Maki, and the key departments which joined with mathematics were biology, geology, and geography, along with quantitative business analysis in the School of Business. The nature of the IU program, one faculty member $=$ one student, and the size and duration, 10 to 12 students for 8 weeks in the summer remained the same as it had been during the early years.

As an example of the sort of projects carried out during these years, we list the table of contents of the book produced at the end of 1976.

## Student Reports <br> Undergraduate Research Participation Program Summer 1976

I. Discrete Time Dynamic Programming Applications in a Finite Resource System, by Mary Ann Bauer, (Professor Wayne Winston, advisor).
II. A Stochastic Analysis of Earthquake Intensities, by Kenneth Constantine, (Professor Robert Blakely, advisor).
III. Investigation of Logic for Question Answering, by Erica Flapan, (Professor Stuart Shapior, advisor).
IV. Problems in Stochastic Constrained Optimization, by Thomas L. Graff, (Professor Maynard Thompson, advisor).
V. A Mathematical Model of the Regulation of Nitrogen Fixation in Microorganisms, by David C. Harris, (Professor Gary Sojka, advisor).
VI. Various Problems from the Theory of Differential Equations, by Carson Hinds II., (Professor George Minty, advisor).
VII. Differential Topology with Applications to Thom's Theory of Structural Stability and Morphogenesis and the Inexact Sciences, by Joseph F. Johnson, (Professor Suresh Moolgavkar, advisor).
VIII. Some Results on the Convective Dispersive Equations, by NaiHang Kwong, (Professor Robert Glassey, advisor).
IX. A Brief Study in the Theory of Games and an Application to Oil Pricing, by Michael A. Marval, (Professor Daniel Maki, advisor).
X. Batched Searching of Indexed Sequential Files, by Ron Olsson, (Professor Victor Goodman, advisor).
XI. The Application of Dynamic Programming to Energy Problems, by Elizabeth Pratt, (Professor Wayne Winston, advisor).
XII. A Model of Bidding Systems for Offshore Oil, by Andrew Rich, (Professors James Patterson and Robert Winkler, advisors).
XIII. Derivation of Optimal Age Schedules for Fecundity and Survivorship, by Michael D. Stefano, (Professor John Emlen, advisor).

## THE DARK YEARS: 1980-1989

The change in administration from the Carter years to the Reagan years brought a sudden end to all NSF sponsored site programs for undergraduate research experiences in mathematics. One remnant of the program remained as individual investigators with NSF or NIH funding were invited to ask for small supplements to fund one or two undergraduate student projects. At Indiana University this option was exercised by both Professors Bedford and Maki, and they supported
and sponsored projects during several summers. Since travel funds for students were not available during this period, the students were all from Indiana University.

## THE RECENT YEARS: 1990-1999

At the end of the 1980s, the National Science Foundation revised programs to provide research experiences for undergraduate in mathematics, and the Indiana University site program was also revised. The key faculty who have been involved during the 90s are Professors Maki, Edmonds, Haile, Goodman, and Stampfli. However, many other faculty members have also been involved. The programs has also benefited greatly from the support of Donna Fink, the administrative assistant for the Department of Mathematics.

The Nature of the REU program in the 90s has been the same as the URPP program was in the 60s and 70s, however, in recent years we have included many more guest speakers in the summer program. For example in summer 1999, there were 17 lectures, which were not directly related to student projects. As part of the guest lecturer program, each summer Professor Charles Livingston has given two lectures on knot theory. To illustrate the nature of the projects in the 90s, here is the table of contents of the 1996 book of final projects reports.

Student Reports
Undergraduate Research Participation Program
Summer 1996
I A Catalog of Behaviors for Two-Dimensional Leslie Matrix Models with an Exponentially Damped Survival Term, by Aaron Archer, (Professor Maynard Thompson, advisor).

II Computation of Global Attractors, by David Braithwaite, (Professor Michael Jolly, advisor).

III Derivative Complexes of Cubical Complexes, by Lisa Friedland, (Professor Laura Anderson, advisor).

IV Classification of Simple Branched Coverings of the Twice Marked Punctured Torus: The Double Conjecture, by Lydia Hadden, (Professor Allan Edmonds, advisor).

V The Space of Binary Matrices and Global Maxima of the Determinant Function, by Christopher Kennedy, (Professor Bruce Solomon, advisor).

VI Speech Recognition Via Wavelets, by Heather Lehr, (Professor Daniel Maki, advisor).

VII Anti-Admissible Sets, by Jacob Lurie, (Professor Jon Barwise, advisor).

VIII Do Treasury Bonds Follow a Markov Process?, by T.J. Mather, (Professor Victor Goodman, advisor).

IX Complex Dynamical Systems, by Andreea Nicoara, (Professor Eric Bedford, advisor).

X Invariant Subspaces of the Shift Operator, by Eric Pessagno, (Professor Joseph Stampfli, advisor).

XI Topological Invariants of Curves in Surfaces, by Gail Potter, (Professor Alan McRae, advisor).

XII Single Server Capacity Problem, by Rachel Schutt, (Professor Victor Goodman, advisor).

## Colleges and universities represented in our program in the 90s

The list below includes all 55 of the home colleges and universities for the students in our REU in the decade of the 90s. Total number of students from that school is in parentheses.

## LIST of SCHOOLS

University of California, Berkeley (3)
Brown University (3)
Northwestern University (3)
Boston University (1)
Harvard University (8)
Columbia University (5)
Indiana University (9)

University of Pennsylvania (2)
Pomona College (1)
Rice University (1)
University of Chicago (9)
University of Washington (1)
New Jersey Institute of Technology (1)
Washington University (1)
University of Texas, Austin (1)
Cornell University (1)
Princeton University (1)
North Carolina State University (1)
University of California, Santa Barbara (2)
Harvey Mudd College (4)
Hope College (1)
Stanford University (4)
Oberlin College (3)
University of Michigan (1)
CUNY, Queens College (1)
Texas A\&M University (1)
Iowa State University (1)
Brandeis University (1)
Oregon State University (1)
Bowdoin University (1)
University of Wisconsin (1)
Yale University (1)
Saint Olaf College (2)
Occidental College (1)
Kenyon University (1)
John Hopkins University (1)
Montana State University (1)
University of Rochester (1)
Morgan State University (1)
State University of New York at Buffalo (1)
Vanderbilt University (2)
University of Dayton (1)
University of Notre Dame (1)
Duke University (1)
Berea College (1)
Purdue University (1)
University of California, Santa Cruz (1)
Wake Forest University (1)
Massachusetts Institute of Technology (1)

University of Toronto (1)
New York University (1)
University of California, Los Angeles (1)

## Follow up on Students of the 90s

During the recent years of our REU program, we have tried to keep in touch with our alumni and to follow their careers. The following list was current as of fall 1998. The students are listed by years in our program, starting with 1991, through 1998. All students who were in the program in summer 1999 are still undergraduates at their home schools.

Indiana University, Bloomington, IN 47405
E-mail address: maki@indiana.edu

# The LSU Mathematics REU 

James J. Madden and Neal Stoltzfus

The Research Experiences for Undergraduates program at Louisiana State University was designed to give able college sophomores and juniors the opportunity to engage in research on topics of recognized mathematical importance in a supportive professional environment. As much as possible, we treat student participants as full collaborators. We recognize that as beginners in the research enterprise they have special needs. We aim to meet those needs with a carefully tuned program of activities, which we describe at length in the main body of this essay.

The REU program at LSU was initiated in 1993 with a grant from the Louisiana Educational Enhancement Support Fund (LEQSF), with the authors as co-principals. This contract provided for an REU program with 6 students in Summer 1993, expanding to 8 students in summer 1994. Professor Robert Perlis functioned as a third co-principal in the summer of 1994. During the summers of 1994 and 1995, the National Science Foundation Grant number DMS-9322278 supported a total of twelve students - 2 in 1994 and 10 in 1995. LEQSF funding was renewed in 1996 with a two-year award supporting a total of 18 students in the summers of 1996 and 1997. In both summers, Madden, Perlis and Stoltzfus were co-principals. In summer 1998, two students participated in a limited program. In 1999, the program had 12 students with Stoltzfus, Perlis and Hoffman as faculty mentors. In general, the level of support obtained from LEQSF has been somewhat higher than NSF, averaging about $\$ 6,900$ per student for the 32 LEQSF-supported students prior to 1999 as opposed to $\$ 5,000$ from NSF for the 12.

Participants were recruited nationally. We sought to include groups that are under-represented in the mathematical sciences and regularly

[^3]achieved a good gender balance. We found it difficult to attract minority students, but a small number have participated. A total of 58 students have participated. About one-third of these have entered graduate school in mathematics,others have entered graduate school in related subjects, while another third are still undergraduates.

## 1. Nature of Student Activities

We begin this section with a comment on our philosophy. After this, we describe the time line for student activitieas. Next, we give a general account of the type of mathematical work that goes on. Finally, we give a vignette intended to help the reader form a realistic picture of student work. A 1995 participant in our program, Shelley Harvey, wrote a description of her REU experience in our program which was published in the Notices of the American Mathematical Society, Volume 45, Feb. 1998, 267-268.

Although a lot of mathematical thinking is done privately, research in mathematics is an essentially social activity, since its goal is the creation of ideas to be shared and integrated into the mathematical corpus. For this reason, we have always devoted special efforts to fostering a strong intellectual community. Mathematical communication, in all its forms and settings, is central to the REU, and we take advantage of every opportunity to help students to improve communication skills. On a larger scale, it is communication between research communities and the great currents and traditions of mathematics that raises mathematical work above the simple drive to satisfy curiosity. For this reason, we strive to give the participants a sense of how their work fits into broader intellectual frameworks: how their work contributes to a senior researcher's overall research program, how this program fits within the discipline and the place of the discipline in mathematics as a whole.
1.1. Time line. A fairly strict time line has evolved. In practice, it works smoothly and naturally. Since it was established in 1995, we have had no need to depart from it by more than a day or two. During the 8 week program, the undergraduate participants take the following steps.

Survey two or three specific research areas selected by the project directors. This takes place during the first week and a half of the program. Each of the directors delivers a series of 5 lectures on a research topic in which he is actively involved and gives out "lab work" designed to help participants get a feel for the lecture area. Perhaps the greatest challenge the directors face lies in finding a way to make an advanced research topic accessible. Our strategy will be described in
the following subsection, The mathematical work. In general, students work in only one area, but he time spent learning about other areas pays off in enabling the students to understand one another's work.

Plan a project in one of these areas. Participants complete this phase by the end of the second week. In order to do this, they work individually with the directors to fill in and extend their understanding of the lectures, bring a specific problem into sharp focus and prepare an oral report. In detail, what happens is as follows. The first 8 days of the program alternate between "lecture days" and "conference days". On conference days, each student meets for 45 minutes with one of the directors. The first two conferences ensure that each student exchanges thoughts with each director. After a student has chosen an area, she/he meets mainly with the director of that area. On the last two days of the second week, each student has several meetings with her/his mentor. The student works out a research plan and report. On Monday and Tuesday of the third week, the participants present their plans in the form of short seminar talks. This is more or less the point at which they "leave the nest" and take off on their own.

Carry out the plan. This occupies the third, fourth, fifth and sixth week. Each student chooses the working style that suits her/him best. We do not need to impose much structure or schedule, because the work carries them along. The directors remain available at all times, initially providing additional guidance, and later, as the students gain independence, becoming a critical yet approving audience. During this period, maintaining a cohesive community is crucial to achieving the goals of the program. By sharing ideas, challenges, the excitement of the quest and the joys and disappointments that accompany hard intellectual work, the participants maintain high morale and a sense of mission. We have been very fortunate in finding a simple formula-a gimmick, almost-that keeps the group together and prevents anyone from becoming isolated. Afternoon tea is held every working day of the program at 3:30 p.m. in the mathematics lounge. Undergraduate metabolism being as it is, perfect attendance is normal - all get hungry at that time of day. Participants inform one another about what they are doing, comment on each other's work, and share suggestions and encouragement. On most days, the lounge blackboard is filled by the end of tea time. The directors join in the discussions, offer their own perspectives and sometimes even give impromptu lectures. Often, a discussion ends with a director and a couple of students planning a morning meeting for the following day, or rushing directly to an office or to the computer lab to test out an idea. Another benefit of tea is that it keeps the directors extremely well-informed about the progress of each
participant. Given the freedom that students have to design their own schedule, it is hard to imagine a more effective way of accomplishing this.

Report on results. During the seventh week, participants consolidate what they have done and prepare a lecture, which is delivered at the end of the seventh week. Mathematics faculty from outside the REU are invited to attend. (In the future, we plan to advertise these lectures around the university to encourage engineering, science and education faculty to attend.) During the eighth and final week, participants prepare written reports that are kept permanently in the REU archive. At the end of the program, participants and directors frequently make plans to stay in touch and continue joint work. We have no formal mechanisms for this, because we feel it is not something that ought to be forced.
1.2. The mathematical work. Here, we will comment on the general qualities of the mathematical work that takes place in the REU. For a discussion of the specific content of selected projects, see the vignette which follow.

Experience has shown that REU participants have the ability, time and motivation to make meaningful contributions to real research, provided that we carefully plan an entryway and formulate problems in terms that participants can assimilate. A solid course in linear algebra, some abstract algebra and some experience writing proofs are the only course requirements for admission to the REU. Whatever we present must be built directly on this foundation.

Prior to the start of the summer program, faculty mentors devote significant effort to developing material for the introductory lectures. In selecting topics, we use several criteria. A problem suitable for the REU must have genuine scientific interest and be richly connected to other mathematical topics. In addition, we must be able to pare the problem down to an elementary formulation without cutting away whatever it is that makes it significant, and we must know that progress with the problem in the pared-down form will really advance our understanding of the original.

Let us illustrate with an example from the early years of the program. Madden's research at that time required a three-dimensional generalization of some results from Zariski's theory of complete ideals in regular local rings. Zariski himself posed the problem of generalizing the theory in the 1938 paper where it was first elaborated, and in modern times there have been several attempts. For a review, see [S. D. Cutkosky, Complete ideals in algebra and geometry,

Commutative algebra: syzgies, multiplicities and birational algebra, W. Heinzer, C. Huneke and J. Sally, editors, Contemporary Mathematics 159, American Math. Soc. 1994.]. Actually, Zariski's work was an improved (and rigorous) formulation of a 19th-century theory that was presented by F. Enriques, O. Chisini, in their well-kown textbook, Lezioni sulla Teoria Geometrica delle Equazioni e delle Funzioni Algebriche, Nicola Zanichelli (ed.), Bologna, 1915. One of the 19th-century formulations is entirely in terms of the behavior of polynomial coefficients under iterated quadratic coordinate transforms. This interpretation of the theory enabled us to formulate some specific, concrete questions about the algebra of polynomials in three variables that we felt could settle the issue that had originally arisen in Madden's work. It was these problems that we handed over to the REU participants. What they accomplished was totally unexpected. They did not find complete answers to the questions we posed, but they did find a wholly unexpected class of examples that completely change what we can expect a three-dimensional generalization of Zariski's theory to look like. In a way, we got far more than we had hoped for. (Phil Bradley, who was the main contributor to this work, went on to graduate studies in mathematics at MIT on an NSF graduate fellowship.)

Not every project, of course, is this successful. But we use our greatest successes as models for the future. This project had the elements we try to duplicate: a significant problem in a respected tradition, a gateway via elementary mathematics, an opportunity for the REU participants to move a larger project a step ahead.
1.3. Project vignette. The following is a description of the work carried out in Summer 1997 by Katie Evans (participating between her Sophomore and Junior years at Morehead State University, Kentucky), Becky Mathis (participating between her Junior and Senior years at Hanover College, Indiana) and Michael Konikoff (participating between his Junior and Senior years at New College of the University of South Florida) under the direction of Professor Madden and with the assistance of Madden's graduate student, Gretchen Whipple. This work has been incorporated into a paper, which has been submitted. A preprint is available at http://math.lsu.edu/~preprint/index.html, item number 1999-1.

The research area was real algebraic geometry. The research problem, originating in the study or real singularities, concerned the structure of totally-ordered commutative artinian algebras over the real numbers. Since the convex ideals of such an algebra form a finite
totally-ordered commutative monoid, it is reasonable to begin by attempting to make a structure theory for such monoids. This produces a problem that is ideal for REU students, because these are fairly concrete objects, and because very little is known about them.

During the semester preceding the REU, Madden and Whipple did an extensive search of the literature, from Clifford's 1957 review [A.H.Clifford, Totally ordered commutative semigroups, Bull. Amer. Math. Soc. 64 (1958), 305-316] to the present. This turned up no references treating the points that were critical for the intended application. We had particular interest in finding examples with a particular property, which we will not describe here. We'll call them NFI-monoids (for "non-formally integral" - see the preprint of Evans et al. referred to above. The only references in the literature to monoids of this type occur in papers of John Isbell on ordered rings. In the semigroup literature, such examples are almost invariably excluded from discussion. At the beginning of the summer, Madden and Whipple had strong suspicions that that no two-generator NFI-monoids existed, they had one example of a three-generator NFI-monoid, several examples on four or more generators (coming from Isbell) and little understanding of the general situation.

Madden prepared a set of notes on the algebraic theory of ordered commutative monoids for the use of the REU participants, and introduced them to the theory in a course of five lectures at the beginning of the summer. At the same time, he prepared a Mathematica notebook consisting of several hundred lines of Mathematica code that included many tools students could use to search for examples of NFI-monoids and analyze their structure. Evans and Mathis rapidly became adept at the use of the notebook, and by studying the programs there, they were even able to begin writing their own Mathematica routines. By the middle of the summer, we had a large collection of examples on three generators, but still very little theoretical understanding of them.

Up to this point, Konikoff had been grappling with some of the finer theoretical points that had come up in the lectures. With a catalogue of examples finally available, he started asking questions about a geometric interpretation of the monoids that Madden had suggested (but not pursued) in his lectures. With some assistance from Madden, he worked out a classification scheme based on a method of assigning a rational convex cone in 3 -space to each isomorphism type of three generator "NFI-monoid." This cone is actually the the dual to the positive cone of a total order on $\mathbf{Z}^{3}$ that is associated with the NFI-monoid's order, and the points in it correspond to submonoids of the natural numbers that are, in a sense, approximations of the NFI-monoid.

Using Mathematica graphics, Evans and Mathis displayed their catalogue of examples as a collection of Konikoff cones. Several interesting features stood out:

1. Many of the cones occurred in pairs that shared an edge
2. Some cones were properly contained in others
3. Most, but not all, of the cones were simplicial
4. Cones were concentrated in certain regions in space.

These facts helped to elucidate features of the catalogue that had been been noticed earlier. We were able to explain 1) and 2) in purely algebraic terms, but the meaning of 3) and 4) remained elusive.

This was the state of the work at the end of the summer. Evans, Konikoff and Mathis announced their intention to speak on their work at their home institutions. Madden succeeded in proving that there are no two-generator NFI-monoids. Whipple used the work as a starting point for her Ph.D. dissertation, which was completed in summer 1999.

## 2. Recruitment and Selection

In the first four years, recruitment utilized national mailings of up to 600 packets containing application forms, information sheets and an attractive poster. This proved inefficient, as, with rare exceptions, our applicants learned about the LSU REU through announcements that appeared in professional notices or on the World Wide Web. Experience indicates that when the LSU REU is included with the other NSF-funded math REU's in published lists, we can expect 50 to 100 applications. We have a World Wide Web page (at http://math.lsu.edu/~stoltz/REU/ann.html) that includes detailed information on the program and provides email application forms. We see to it that pointers to this page are present at mathematics hubs on the Internet.

By far the most popular method of communication between the REU and potential participants is electronic-mail. Between one and two hundred electronic mail inquiries about the program are typically handled. Each inquiry receives in return an electronic information pack that includes a description of the program, instructions for application by email, detailed descriptions of mathematical work and a list of all the other mathematics REU opportunities in the USA.

We request that applicants provide an academic record supplemented by descriptions of the advanced math courses they have taken, two letters of reference, and a personal statement explaining their interest in the REU. In selecting students for the program we look for evidence of ability in mathematics, ability to commit to and complete

| Year | Total | Men | Women | Black |
| ---: | ---: | ---: | ---: | ---: |
| 1993 | 6 | 5 | 1 | 1 |
| 1994 | 10 | 7 | 3 | 1 |
| 1995 | 10 | 6 | 4 | 0 |
| 1996 | 9 | 5 | 4 | 0 |
| 1997 | 9 | 5 | 4 | 0 |
| 1998 | 2 | 0 | 2 | 1 |
| 1999 | 12 | 11 | 1 | 0 |

Table 1. Table of Participants: Minority and Women
a project and ability to interact well with others. As mathematical background, we require linear algebra, some abstract algebra and experience writing proofs. Faculty letters have been the most informative and helpful in identifying successful participants.

We have always made a special effort to attract participants from segments of society historically under-represented in mathematics, and have been successful in achieving gender balance. LSU is situated within a region with a high percentage minority population from which we do actively recruit (but, unfortunately, with little success). For the last two years, the LAMP program at LSU has offered research experiences specifically for minorities. In 1996 and 1997, we accepted a mentoring role for a total of 3 students from this program. Our experience with them was positive, but entirely different from the mathematics REU. Part of this was due to the fact that they were planning careers in engineering rather than pure math. In the future, we plan to work with the directors of the LAMP program to search for candidates for the math REU.

## 3. Evaluation

Initially, we gauged success in terms of the research results obtained and by measuring particpant attitudes. For the first, we relied mainly on our own sense of mathematical significance. During the formative stages of the REU, participant attitudes were certainly the most important, for the success of everything else depends upon an environment that promotes positive feelings. At the end of each summer, we distributed an evaluation sheet, and arranged a meeting between the participants and a faculty member who had not been associated with the REU. After the first summer, these instruments revealed a few things that needed attention. These were quickly resolved, and subsequent evaluations were high in every category.

At this stage, it is clear that we need a more mature evaluation procedure. It is no longer appropriate for us to rely on our own judgement regarding the quality of the mathematical work. The best solution here seems to be to attempt to get more of the work coming from the REU published, so that it can be evaluated publicly. We definitely plan to do this, but this is a slow process. For more immediate feedback, additional involvement of faculty outside of the program (e.g., as reviewers of student reports) will be valuable. Good tools for evaluating the learning that takes place in the REU are sorely needed if, as we hope, the REU becomes a prototype for graduate and undergraduate training during the academic year.

```
Baton Rouge, LA 70803-4918
E-mail address: madden@marais.math.lsu.edu
E-mail address: neal_stoltzfus@math.lsu.edu
```


# THE SUMMER UNDERGRADUATE MATHEMATICAL RESEARCH INSTITUTE 

Dennis Davenport

In 1996 the National Security Agency (NSA) held its fourth Invitational Mathematics Meeting. The main purpose of the meeting was to find ways to increase the number of minorities with advance degrees in the mathematical sciences. Because of these discussions, Miami University in Oxford, Ohio developed the Summer Undergraduate Mathematical Science Research Institute (SUMSRI) for mathematically talented undergraduate students. SUMSRI was first held in the summer of 1999. We received funds form the NSA $(\$ 104,479)$, the Wedge Corporation ( $\$ 15,000$ ), Miami University $(\$ 59,166)$ and $\$ 5,000$ in private donations. The Principal Investigator was Dennis E. Davenport, Associate Professor at Miami University and he will also be the PI this year. The project is directed towards, but not limited to, African Americans, women, and other underrepresented minorities in the mathematical sciences. SUMSRI will provide approximately 15 nationally selected students an intensive research experience. The primary goal of the institute is to convince participating students to pursue advance degrees in the mathematical sciences. By doing so, we hope to help address the shortage of African Americans with doctoral degrees in the mathematical sciences. Between July 1973 and June 1996 only 172 mathematical sciences doctoral degrees were awarded to African Americans. This is approximately $1.45 \%$ of all mathematical sciences doctoral degrees awarded to U.S. citizens during this time. A recent survey conducted by Dr. John Alexander and Dr. William A. Hawkins titled "Survey of Minority Graduate Students in U. S. Mathematical Sciences Department," recommends creating programs like SUMSRI to encourage minority undergraduate students to pursue

Received by the editor September 16, 1999.
advance degrees in the mathematical sciences. We will also encourage participants to attend national meetings (for example, the AMS-MAA Winter Meeting, the Conference of African American Researchers in the Mathematical Sciences (CAARMS), SIAM meetings, NAM Mathfest, MAA Mathfest, or Pi Mu Epsilon meetings) and give talks. The program will run for seven weeks in the summer. During these seven weeks students will participate in problem seminars in mathematics, statistics, or computer science. At least one seminar will be led by an African American who is an active researcher. At least one hourly colloquium talk will be given each week. The majority of the talks will be given by well-known African American, women and other underrepresented minority mathematicians. We will also have panel discussions on information about graduate school, fellowship sources, and career opportunities in the mathematical sciences. The program will also include a technical writing seminar, a GRE preparation workshop, and a seminar where algebra and real analysis will be taught. The institute seeks mature students who have completed at least two years of undergraduate mathematics with distinction. An important goal is to recruit African American and women students. SUMSRI will provide each student with continued group support and valuable role models.

SUMSRI is designed to prepare participants for the rigor and pace of graduate school. We feel that this preparation will allow the participants to successfully complete and compete in graduate school. The Institute aims to aid participants by intervening in their learning development at a crucial stage. The main goals are:

- Address the shortage of African American and women mathematicians by producing minority research mathematicians.
- Provide the students with a research environment and improve their research abilities.
- To improve the student's ability to work in groups and give them a long term support group.
- Provide role models.
- Improve the students' technical writing skills.
- Give the participants an opportunity to give a talk and to write a technical research paper.
- Familiarize them about graduate school and inform them about available financial aid for graduate school.
- Make the students aware of career opportunities in the mathematical sciences.
- Prepare the students for the GRE.

The Institute will operate for seven weeks. During these seven weeks, students will participate in problem seminars in mathematics, statistics, or computer science. Also, at least eight colloquium talks and two short courses on algebra and real analysis will be given.

Seminars. The program consists of seminars where professional mathematicians and statisticians contribute problems. Before arriving at Oxford the students are mailed a card listing each seminar area and prerequisites. They are asked to rank the topics and return the card. They are then assigned an area based on their choice. We hope to give each person his or her first choice. Last year we were able to do so. Students are only required to attend their assigned seminar. During the first four weeks of the program, three professional mathematicians, statisticians, or computer scientists will present four lectures each on their area of specialization and assign problems. At least one of the seminar leaders will be an African American; the other two will be Miami faculty from either the Department of Mathematics and Statistics or the Department of Systems Analysis. During the lectures each seminar leader gives research problems for the students to consider. It is very important that these problems be challenging and at the same time easy enough for a very good undergraduate student to get partial results. Each student chooses a problem to work on and consults the appropriate professional. The seminar leaders are asked to meet with their students every day except Friday during the first four weeks and at least twice a week during the last three weeks. We strongly encourage students to work in groups. At the end of the program the students give an oral presentation on their results and write a paper. The paper will be included in a journal published by the Institute. Last year the research seminars were in algebra, linear programming, and statistics, directed by Dennis Davenport (Miami University), Earl Barnes (Georgia Tech), and Vasant Waikar (Miami University) respectively.

Colloquium. An important part of the program will be the colloquiums. Research mathematicians, statisticians or computer scientists will give at least eight one-hour colloquium talks. The majority of the presenters will be minorities or women. The talks will introduce the students to several advance topics in the mathematical sciences. Some of the talks will relate closely to the seminar topics. Seminar leaders will assist in these talks by either selecting or conferring with the speaker. In the summer of 1999, talks were given by Carol Wood of Wesleyan University, Fern Hunt of the National Institute of Standards and Technology, Neil Hindman of Howard University, Scott Williams
of SUNY at Buffalo, Leon Woodson of Morgan University, Kimberly Kinateder of Wright State University, and Kyoungah See, Emily Murphree, and Vasant Waikar of Miami University.

GRE Preparation. Since most national fellowships and top ranked universities require the GRE, the Institute will conduct a GRE preparation workshop. For the first four weeks, each Friday morning will be devoted to preparation for the GRE. The analytic, qualitative, and subject tests will be emphasized. One of Miami University's Putnam team coaches, Charles Holmes, will lead these sessions. Last years students found these sessions to be very informative. Their mock exam scores showed a significant improvement from week one to week four.

Recreation. The students will have access to Miami University's state-of-the-art recreation center; the center includes an Olympic size swimming pool, an Olympic diving pool, an indoor climbing wall, an indoor jogging track, weight training area, and racquetball courts. SUMSRI also plans several outings for the weekends. Last summer, the students were taken to an outdoor play about the interactions between Native Americans, escaped slaves, and white settlers during colonial times called Blue Jacket. They spent time at the newly opened Newport Aquarium and the King Tut exhibit. They also spent the Fourth of July at Kings Island Amusement Park and the last weekend white water rafting. Next year we hope to include a Cincinnati Reds baseball game.

Projects. Several sources will be used to find research projects for the students; such as the Mathematical Association of America's Monthly, professors' research interests, Math Horizons, and a web site of unsolved problems. To give an idea of projects, we mention ones from last year. In the algebra seminar the students worked on two projects. One was a conjecture posed in the American Mathematical Monthly. The conjecture says that any $k \times k$ submatrix of the addition table of $Z_{n}$ contains a latin transversal, where a latin transversal is a collection of $k$ entries no two of which are in the same row or column and no two entries are the same. Although they were unable to prove this conjecture, the students were able to find several partial results. The other project was a little known result from semigroup theory which says given any semigroup $S$, if $S$ has a unique left identity $e$ and each element of $S$ has an $e$-right inverse, then $S$ is a group. The students were asked to determine why uniqueness was important and to prove the result. They wrote a joint paper on the conjecture from the Monthly. In the linear programming seminar two individuals
and one group worked on variations of the traveling salesman problem. Suppose a company wants to build a warehouse that will make deliveries to several customers on a regular basis. The cost of delivering the product from the location of the warehouse could play a role in the construction site. Hence, the company may want to know the cost of supplying the clients from several potential sites. The following problem is motivated by this practical example. Let $\left(a_{i}, b_{i}\right)$ be fixed points in the plane for $i=1,2, \ldots, n$. Let $H=(x, y)$ be a movable point in the plane. For each position of $H$ find the shortest route that starts from $H$, visits each point $\left(a_{i}, b_{i}\right)$ and ends at $H$. Now, color the points in the plane so that two points get the same color if their routes visit the points in the same order. The problem is to study the properties of the colored regions that emerge. The group of two students worked on this problem for three fixed points. One student considered the problem with four fixed points. And one student considered the problem with four points in three dimensions. It was interesting to note that in all three cases there were some surprising differences. The students in the statistic seminar worked on two projects. One group used the unrelated-question randomized response method to determine what percentage of students taking math courses at Miami University during the summer cheat on tests using graphing calculators. They then did a similar study for students taking chemistry and physics and compared the results. The second group used estimations of density to determine the change in the number of four-letter words in the English language. Using Minitab, they generated a random sample of 100 page numbers for two Webster dictionaries, one from 1950 and the other from 1986. They then counted the four letter words on the randomly selected pages. They then estimated the number of four-letter words using the population density method.

Students. We seek fifteen undergraduate students who have completed at least two years of undergraduate mathematics with distinction. Each student should have taken and received top grades for an upper division course requiring exposure to proofs. To apply, we request two faculty letters of recommendation, a math course list including the textbook used for each course, an official transcript and a statement on career plans.

Following are excerpts from the survey used last year.

- Would you now consider pursuing a graduate degree? 15 answered yes or definitely
- What did you like best about this summer's program?

Opportunities to learn (short-courses) advance mathematics and do research.
The short courses.
Learning how to do research.
The way we all got along and worked together.
The exposure to graduate work, and hands on research projects.
The relaxed and positive atmosphere.
The short courses and GRE prep.
Being in an environment where you can learn from the others interested in mathematics.
Seminar/Short Courses.
The other SUMSRI participants.
I now know what I want to do.
The preparation for graduate school and the opportunity to meet other mathematics majors.
The people-both participants and professors (and grad. students too). Meeting professors and speakers, meeting students with similar interests, grad school and career info, weekend trips.

- Do you believe you will keep in contact with other SUMSRI participants after leaving the program? 15 people answered "yes" or "definitely"
- By participating in SUMSRI do you believe you have improved your (check any that apply)
8 overall social skills
3 library research skills
7 technical writing skills
5 understanding of different cultures
4 web research skills
11 critical thinking skills
11 presentation (public speaking) skills
15 knowledge of graduate school programs and financial aid
14 awareness of career opportunities in mathematics
14 preparation for the GRE exam

For more information about SUMSRI visit our web site at http://jewel.morgan.edu/~sumsri.

Miami University, Oxford, OH 45056
E-mail address: davenpde@casmail.muohio.edu

# The Michigan Tech Probability REU Program 

Anant P. Godbole

## 1. Project Objectives and Overview

The current REU Program at Michigan Technological University, Discrete Random Structures, followed on the heels of the three previous REU projects, The Probability Theory of Patterns and Runs (1991), Discrete Probability and Associated Limit Theorems (1992-93), and Probabilistic Methods in Graph Theory, Combinatorics, and Number Theory (1994-96). I hope to further consolidate my involvement with undergraduate research by leading eight talented students in a series of intensive research endeavours during each of the Summers of 19972001. The present project shares with its predecessors the hypothesis that Mathematics is a science with intricate interconnections that must be recognized and exploited; thus, the selected students seek solutions to problems that emanate in combinatorics, number theory, graph theory, geometry, algorithms, and molecular biology, but by employing contemporary and powerful probabilistic techniques (martingale difference methods, isoperimetric inequalities in product spaces, Janson's inequalities,...) in tandem with the kind of classical analysis (inequalities, estimating sums,...) that surely forms an integral component of any mathematician's repertoire. The scope of the present project is far wider than during the years 1991-1996, both through an increase in the level and depth of the probability used, and by means of an overall broadening of the nature of the problems considered, to encompass most of the areas within the domain of discrete mathematics. I believe that the view that mathematics is a discipline with undeniable links between its various compartments rubs off on the student participants, and that, at the end of the summer, they have a clearer sense of why (or

Received by the editor March 13, 2000.
even whether) they should pursue research careers in the Mathematical Sciences.

Eight students are selected to form the research team each year after a nationwide search, and the project commences around June 15, running for two months. As in the past, I strive to form teams that are as diverse as possible. At the end of the research period, the team members typically travel to a meeting such as the MAA Summer Mathfest to present preliminary versions of their work, while the final completed research is usually presented either at the Annual Meeting of the AMS/MAA, or at a carefully targeted special topics meeting, such as the International Conference on Random Structures and Algorithms, or the Southeastern Combinatorics and Graph Theory Conference. Since 1995, we have had a joint research symposium with Joe Gallian's REU group from Duluth, Minnesota; this midsummer get-together is held at a time when no student has obtained definitive results, but serves, nonetheless, as a wonderful forum at which to exchange notes, give talks, and establish contacts.

A typical investigation is conducted jointly by two students. Our REU research topics have grown somewhat harder over the years they are not any more demanding technically, but do require that students be able to continually draw on previously garnered knowledge and rapidly acquire new wisdom - so that two heads are likely to function far more efficiently than one. It is likely, moreover, that a student will work with several partners over the course of the summer and/or that (s)he might work individually on a particular problem. At least twelve problems are set aside for the group to work on each year; this yields an average of three problems per person per year, which I consider to be a safe number, given the fact that several problems lead nowhere, while others are found to have unexpectedly difficult or intractable solutions. Each problem worked on by REU students is a genuine research problem; never do I "know" the "answer" to the question at hand. The team members' investigations are no longer as closely linked as was the case in previous years, but it is often the case that different groups end up using similar probabilistic techniques (as was the case in 1993-1999, when the Janson exponential inequalities and the Hoeffding-Azuma inequalities were employed successfully by several groups working on rather different problems).

Each team is required to turn in a project report, written (using some version of $\mathrm{T}_{\mathrm{E}} \mathrm{X}$ ) in the precise format of a refereed journal targeted as being the most appropriate for submission during the coming academic year. There is no guarantee that any particular investigation will ultimately lead to a published paper, but I believe that every effort
must be made to involve students in the writing-and-publishing aspect of the research process, with all its joys (complimentary referee reports, acceptance) and frustrations (multiple revisions of manuscripts, rejection...).

## 2. The Research Environment

2.1 The Daily Schedule: The first two days of the program are spent in describing the twelve or so problems that have been targeted for attack by the group. The pace is deliberately kept leisurely, so that students can ask questions, understand the notation, and get into the right frame of mind, be it probabilistic, combinatorial, numbertheoretic, or whatever. The various directions that the project might take are indicated. At the end of this period, each student will be asked which particular problems (s)he found attractive, and for what reason. Based on this information, about four to six student teams will be formed. Such a format has been used by me since 1994. Students get to work on the problems they like, and seem to delight in having such a choice. And yes; there are problems that almost everyone seems to be interested in (e.g., the "no three in a line" problem from 1997, or the "Trivial=Optimal" problem from 1999. Such questions are set aside as auxiliary "learning" problems, i.e., investigations for everyone to sink their teeth into, and to use as test cases to try basic probabilistic techniques on. As the project period progresses, new teams are formed and the 12 problems have each been seriously worked on. Often, more is undertaken.

Six years of REU site direction have convinced me that my students work best in two three-hour sessions each day. This might reflect the fact that the projects I assign are technically demanding and require the use of high-level thought processes. I have found that 8 a.m. to 5 p.m. work days (as I used to conduct in 1991-92) are self-defeating and lead to unnecessarily critical self-evaluation on the part of the students. Moreover, long formal work sessions tend to decrease the participants' efficiency, particularly since the students are actively thinking about and discussing their problems with each other even after hours.

The work day commences around 9 a.m. with a brief review of what was accomplished the previous day. I often like to give a short, unstructured pep talk at this time, typically in the students' office. Several things can be accomplished during this period: Gaps in the students' background need to be filled in ("what is a $k$-uniform hypergraph, again?"); everyone's mood needs to be lightened by a short session of mathematical humor (Q: What's yellow and equivalent to the axiom of choice? A: Zorn's lemon); newly acquired knowledge needs
to be put in focus ("why exactly do we need to write this sequence as a martingale?"); the students' misgivings need to be addressed ("you can do research without reading all these books in their entirety first!"); students need to be reminded of what they were doing a bare 18 hours ago (for 20-year olds do forget very fast); and concerns need to be voiced ("you are expecting too much of me, slow down!"). I try to keep the mood very light during this meeting; I am, after all, trying to establish a bond between myself and the students and to get students with different personalities to work together - for I believe that the best joint work is done if the principal characters are friends first. Occasionally, part of the morning session is taken up by a student volunteer giving a short lecture on a topic of common interest, such as the Lovász local lemma, or the notion of conditional expectation as a function. This volunteer is, more often than not, a student who happens to have grasped the concept first; (s)he is usually thrilled to be able to enrich the understanding of the others in this fashion.

Following this meeting, the first half of the work day begins. It has been my observation that my help is most needed during this period, when I have to continually rush from one group to the next, shifting mental gears all the time. The morning research period inevitably appears to set the tone for the rest of the day, with a "depressed" student staying that way till evening, in the absence of intervention by me. It is imperative, therefore, that I talk with each student, in each group, during this time, for the morning discussion period reminds each young researcher of where (s)he is headed, convinces him/her that (s)he is on the right track, and reinforces in her/his mind the intrinsic worth of the research problem at hand. If this is not done, the resulting lack of focus is often difficult to combat. I will usually have spent at least a few hours each night thinking about the students' research and the direction in which to lead it; the morning meeting, therefore, also gives me the chance to fine-tune (or change) the focus of the students' investigations. The team-members are constantly reminded that solutions to their problems are not known; that my instincts could be way off target; and that several strategies will have to be tried out, and that too without any guarantee of success. Some students seem to have an instinctive understanding of the research process, and I like to enlist their help in explaining to the others what (I consider to be) the whole point of the REU experience: To learn how to conduct real mathematical research that students can be proud of (and not consider inconsequential) 10 years down the line; to expect major difficulties before any breakthrough is made; to learn how to accept defeat; to learn that an eight week period is not sufficient to completely wrap
up a problem; and to accept that a lot of work will have to be put in during the coming academic year before a young researcher can fully comprehend and appreciate what (s)he has accomplished.

The group breaks for lunch at noon. I like to join the students every now and then, but I realize that they have several things to do during this period (sending e-mail, jogging, designing the perfect REU t-shirt, adding to the REU website....), and try not to impose too much. Mealtime evolves occasionally into another forum for mathematical discussion, but every effort is made to keep the mood light, with my focus being instead on getting to know the students better, on offering advice concerning choices of graduate schools, etc. It is also important to keep track of how well group members are interacting; if each person on a team is being adequately challenged; or if a particular person is being left behind by the sheer force of another's mathematical personality. These questions find easy answers (and solutions) in the relaxed forum of an hour-long lunch break.

After lunch, the group returns to Walker Hall for about three hours of concentrated work. I have found that most results are obtained during this period, when students are feeling far more secure, and when they tend to be contemplative and totally engrossed. Once again, I divide my time among the groups, but my role is a little more passive at this time. I try to resist the temptation to spend more time with the more vocal members of the group, making sure that adequate time is spent with the ones who are quieter, or self-reliant to the extent that they never ask for help.

At 4:00 p.m. or so, most students are tired, and looking forward with anticipation to the evening's activities. A brief chat at the end of the day helps put the day's work (and accomplishments) in focus, and students can be told what aspects of their work need to be thought about, possibly before the start of the next working day. No formal "homework" is ever assigned, but the students get the message.

Variations in the above schedule occur for a variety of reasons: Friday afternoons are often devoted to student talks; we might have a visitor on campus (recent medium-term visitors have included Dan Isaaksen, Emily Puckette, Paweł Hitczenko, and Krishna Athreya); I may be away at a conference; or we may have an NSF-CBMS conference in town (Probability Theory and Combinatorial Optimization in 1995, and Statistical Inference from Genetic Data in 1999.)
2.2 Results from Past REU Awards: More details concerning results from this REU site can be found in the "Survey of Programs" document prepared for this meeting. Here we give a brief summary:

Papers from the program have been published in a variety of journals including Advances in Applied Probability; Combinatorics, Probability and Computing; Discrete Mathematics; Electronic Journal of Combinatorics; Statistics and Probability Letters; and Journal of Applied Probability. Papers have been presented at a variety of regional, national and international meetings. Many students have gone on to graduate school. Several have been awarded NSF/DoD Graduate Fellowships.

## 3. Nature of Student Activities

3.1 Project Philosophy: The last ten years have seen a considerable change in the way I view the REU mission, and in the role I play towards its successful implementation; some aspects of my current project philosophy will be outlined in this section.

Students are made aware, soon after they are recruited, of what they can expect at Michigan Tech. They are told of the serene and peaceful geographical location; the excellent computer lab; the (relative) lack of social/cultural activities; the key difference between the MTU site and some others (which might, for example, have students working one-on-one with individual professors); the nature of the problems they will be working on; the fact that analyzing messy expressions will be unavoidable; and the fact that research in probability often requires the use of techniques and ideas from several areas of mathematics. Students must be informed, in great detail, of what to expect. I send recruited students copies of papers written by previous REU team-members (or indicate how these may be downloaded or viewed on the web), titles of books to review (and monographs to browse through), and lists of personal items they might need at MTU.

I believe that the applicant pool for each of the 20 REU sites is representative of some of the very best mathematical talent in the nation. The recruitment process has become very competitive, with the best students typically receiving several offers. The intrinsic talent of our recruits is beyond doubt. Given these facts, I believe that it is imperative that REU students be given a taste of contemporary, cutting-edge mathematical research. The work done by the students must be of interest to a wide mathematical audience, and must cause several more members of our community to sit up and take notice. Consequently, I will rarely assign problems that are not "important" in some sense. The REU mission is no longer, I believe, hampered by the existence of critics who are skeptical of the whole process, and who are convinced that the research done by REU students is essentially "trivial". However, a continued effort on the part of REU directors to involve
students in important work that the students themselves can consider significant a decade later will enable the REU movement to gain the broad-based admiration it richly deserves, and lessen comments such as the one I read in the October 1999 AMS Notices, "Let us continue entertaining our students with multimedia presentations and gimmicks such as undergraduate research (enthusiastically supported by NSF!), and we will keep hiring scientists seriously trained abroad."

One of the important goals of an REU site is, to quote from the REU Program Announcement, " to lead the participants from a relatively dependent status to as independent a status as their competence warrants". I agree totally, and feel that several things can be done to ensure that this occurs: The problems that the students work on need to be extremely well-motivated. They need to be open-ended, in the sense that their solution must lead to several more questions. The students need to be made aware of the fact that the work they are doing is truly important; it helps to have a noted mathematician interested in the outcome of their project (for example, Chalker and Radcliff's work on sphere of influence graphs is well-known to most of the researchers in the area, even before publication; Paul Erdős was quite impressed by the work done by Ghosh and Revelle on ErdősRényi laws for palindromes; and the work of Graziano, Lamorte and Sandquist on the problem of Zarankiewicz has drawn compliments from Jerry Griggs and Andrew Thomason.) Each young researcher should be encouraged to ask questions, and allowed to pursue lines of investigation that I realize have little chance of success. Each student needs to feel comfortable accepting help from me. (S)He needs to be made aware that the degree of help offered will decrease radically, even in the course of a short eight-week period. And help should be given to the students in varying forms and to different degrees: some students are slow starters, and need a lot of careful attention and prodding in the beginning - but can invariably be left on their own at later stages of the project. Others are diametrically opposite, and need to be slowed down, to prevent a reckless rush towards the "truth". These students need more help later on, when their quickly perceived proofs need to be rigorized, legitimized, and written down.

It is difficult to do research in an excessively formal atmosphere. Most students in REU programs are already contemplating going to graduate school, and I feel that a friendly, non-intimidating research climate can irrevocably tilt the balance in favor of such a decision. A few students conclude, at the end of the REU project, that they do not want to pursue a career in the Mathematical Sciences - and we feel that they, too, are able to make a well-informed decision on the basis of
the honest, work-intensive and fun-filled two months we strive to give them at MTU.

Publishing quality papers is considered by many to be the natural culmination of any meaningful research endeavour. I agree, in a guarded way. Several papers have been written by my past students, and the process of preparing these papers for submission has helped the students enormously. In most cases, the work done by the students is of a sufficiently high quality to satisfy a referee (who is often unaware of the fact that most of the authors are undergraduates!). I have come to believe, however, that the publication process needs to be slowed down, and will no longer put pressure on the students to finish a paper by the end of the following academic year, for example. At the end of the project period, students will be required to turn in a report, typeset in some version of $\mathrm{T}_{\mathrm{E}} \mathrm{X}$ and written in the precise format of an appropriate mathematical journal. The students will have been made aware of what more needs to be done if their paper is to stand a chance of being published in a refereed journal - alongside the work of established mathematicians. They will have been told (i) that the above journal is where the paper will be submitted for consideration eventually, provided that the additional work mentioned above can be done, and (ii) that their report will be used as the basis of the submitted manuscript. No time table is set for this, however, and no paper is submitted until the time is right. I also use this interim period to make some contributions of my own, for I feels that project direction and idea-generation are, by themselves, insufficient to warrant co-authorship. To summarize, a paper written by (or with) REU students needs to be crafted over a long period, and will typically contain far more than what was accomplished in the two month project period. It is far more important, in the short run, for REU students to prepare their work for presentation at professional meetings; I describe this aspect of my project philosophy next:

Each year, I target a set of meetings for attendance by the participants; it is my conviction that the entire REU experience can be put in wonderfully sharp focus through the presentation of results at a professional meeting. Paper presentation develops several fundamentally important skills, from abstract preparation and time management, to the ability to field unexpected questions and think on one's feet. More importantly, it is impossible, in my view, to give a talk at an AMS meeting, for instance, without having complete and absolute command over one's work. First, of course, there is the informal undergraduate research symposium with members of the Duluth REU team. This event is in July, with the location alternating between Houghton and

Duluth. Most of my students have obtained no more than very preliminary results at this time, but the meeting leads to long-term contacts and wonderful mathematical interaction, while providing the participants with an opportunity to organize their thoughts and give twenty minute talks to an audience of their peers. Secondly, at the end of the REU session, several team members travel to the MAA Mathfest, or to a regional AMS meeting. In many cases, travel to such meetings can be combined with the students' trip back home, and expenses are usually quite minimal, especially with the travel support generously provided to most undergraduate attendees by the MAA and the Exxon Corporation. These meetings give students the opportunity to present preliminary versions of their summer research. Finally, after four more months, when the students' research has been solidified; the proofs of all their lemmas have been written down in excruciating detail; and many of their papers are ready for submission, the team travels to an important national or international meeting to present their completed work. We usually attend the Annual AMS/MAA Meeting or a carefully selected "special topics" meeting, such as the Southeastern Combinatorics Conference in Baton Rouge or Boca Raton, or the International Conference on Random Structures and Algorithms, held every two years in either Atlanta or Poznań, Poland. Attendance at the latter kind of meeting is far more fruitful than at the annual math meetings, since there are fewer parallel sessions, and the members of the audience are all conversant with the nature of the problems investigated by the REU students. Though it is not always possible to adjust the students' schedules and find the funding to go to such meetings, every effort is made to do so.
3.2 Nature of Student Activities: I have, over the last few years, become increasingly interested in the Probabilistic Method, as pioneered by Erdős and Turán, and admirably described in the recent book by Alon and Spencer. The "method" consists of a large group of standard and novel probabilistic techniques that are used to deduce the existence of a certain kind of combinatorial, graph-theoretic, number-theoretic or geometric structure. The basic idea is to put a probability measure on the set of all structures, and then to show that the desired property is satisfied with positive probability. The fundamental idea behind the method is so simple that it can be explained readily to most undergraduates, who seem to appreciate its complete tranparency. More importantly, the method has been applied successfully to so many diverse problems that it serves as an admirable vehicle for introducing students
to important problems in Ramsey Theory, Combinatorial Design Theory, Combinatorial Number Theory, etc. A group of coherent problems, whose solution is likely to employ the probabilistic method forms, in my opinion, an excellent base for a successful REU site. Among the methods described by Alon and Spencer are the second moment method, the Lovász local lemma, the method of alterations, and the bounded martingale difference method. My students use all these techniques. In addition to the probabilistic method, I have employed the Stein-Chen technique extensively over the past $7-8$ years. This method does more than just prove the probability that a certain strucure exists is positive; when applicable, it actually exhibits the fact that the number of structures with the desired property has (approximately) a Poisson distribution. The Stein-Chen method has been described admirably in the recent book by Barbour, Holst and Janson and has been, and will continue to be, used successfully by REU students. In addition, the recruits can be expected to learn and employ a wider array of methods during their stay; these include Talagrand's isoperimetric inequalities in product spaces, the Rödl "nibble", branching process methodology, and Stein's method for normal approximation. The central theme behind these techniques is the concentration of measure phenomenon; applications to combinatorial situations have been very recent, with excellent surveys being written recently by Spencer and Steele.

Over the years, students have used the above philospohies to research questions in "random versions" of the Sidon property, sum-free sets, sphere of influence graphs, the Zarankiewicz property, cordiality and bandwidths of graphs, the Ramsey and van der Waerden theorems, nearest neighbour graphs, palindromes and word patterns, etc.

## 4. Student Profile and Recruitment

We adopt a nationwide recruitment policy. No special quotas are set aside for MTU students and/or for students from nearby universities. I am particularly proud of the diverse group of schools from which I have been able to recruit students; these include large state-supported schools such as the Universities of Michigan and Arizona; selective Liberal Arts Colleges such as Swarthmore and Carleton; small public schools such as Youngstown State University and Cal Poly (San Luis Obispo); and research powerhouses such as Harvard and Cornell. I am convinced that both the students and I work best when thrown into the midst of such an eclectic mix.

The increased involvement by NSF in the recruitment process is of enormous help; I have found that at least one half of the applications to my site are in response to NSF-generated announcements
and websites. Moreover, the overall quality of the applicants appears to have gone up steadily, and there are now as many as 10-15 candidates on my short list of the very best applicants. In 1999, I received 80 completed applications, about $45 \%$ of which were from women. The two basic methods of recruitment are (i) The MTU REU web site (http://www.math.mtu.edu/home/math/anant/reu) and (ii) Announcements and forms that are sent by electronic or (if necessary) conventional mail to the appropriate person at each College/University offering an undergraduate degree in Mathematics and/or Statistics. More encouraging follow-ups are sent to predominantly minority schools nationwide. This contact person is asked to encourage potential applicants to check out the REU website and/or seek further information from me by regular or electronic mail.

The candidates are selected based on several criteria. I believe that each of the following components is of fundamental importance to the selection process; the criteria have not been ranked in any particular order:

- The student's academic qualifications; his/her previous coursework in project-related mathematics;
- The student's involvement with problem-solving and math competitions; his/her previous research experience;
- The quality of the student's two letters of recommendation;
- The depth of the student's statement of purpose;
- The variety of the student's background; his/her ability to work with others;
- The student's self-identification as a member of an underrepresented group;
and
- The student's willingness to spend time on follow-up research and paper presentation after the culmination of the project.
$45 \%$ of my students have been women; in 1993, my top six list contained five women. I fully expect to continue to successfully recruit a large number of women for my program. In a similar fashion, I will endeavour to seek out minority students for participation.

Michigan Technological University, Houghton, MI 49931
E-mail address: anant@mtu.edu

# Summer Research at Mount Holyoke College 

Alan H. Durfee

The summer REU program in mathematics at Mount Holyoke College started in 1988. It has continued every summer until the present, with the exception of the summer of 1991. During this time 130 students have participated. This report will attempt to give an informal account of our program; for more details visit our web site at www.mtholyoke.edu/acad/math/reu.

The majority of the funding for the program has been provided by the National Science Foundation. In earlier years we also had funds from NECUSE (The New England Consortium for Science Education) and more recently from the College itself and our own department funds (about $\$ 10,000$ per summer).

The students come here for eight weeks during June and July. Each summer we have two groups, each with five to seven students. Each group is directed by a Mount Holyoke faculty member. The faculty vary from summer to summer; over the years all but two of our nine faculty in mathematics and statistics have participated.

Mount Holyoke College is very pleasant in the summer. The setting is idyllic with many outdoor activities available. There is also a rich variety of cultural opportunities, including a resident summer theater company with a new play every week and a music workshop for young string virtuosi (Musicorda) which puts on free concerts. Other departments run summer research programs as well, and interdisciplinary contact is frequent and productive.

The above paragraph if from our most recent grant proposal. The students sometimes have other opinions. According to a review of our program on the Harvard College Mathematics Club web site, "One big drawback is that Mount Holyoke is in the middle of nowhere. The

Received by the editor December 7, 1999.
campus is beautiful, with lakes, trees and hiking paths galore (just avoid the mosquito-infested swamp that I stumbled into once), but the town of South Hadley has a population of maybe 1000 in the summer and there's just nothing there." (N.B. The actual population of South Hadley, summer or winter, is 19,000 .)

We attempt to work on significant mathematical problems, not just artificial problems created for the students. The problems are always closely connected with the faculty member's current research. For this reason, directing a group of students is a major undertaking and a fulltime occupation for the two months the students are here. A detailed list of topics can be found in Appendix A. In the beginning we set the following criteria for a good topic:

- It should admit extensive computer experimentation. Thus each project should deal with an area of mathematical research where experimental evidence will illuminate and possibly overcome an impasse in purely theoretical approaches. We found that students have a natural affinity for programming and experimentation, and can become familiar with a problem by starting on something that they can carry out immediately.
- It should deal with mainstream mathematics or statistics. We hope to ensure that material learned by students in carrying out the projects will be of subsequent value to them as users of mathematics, either as research scientists or as professional mathematicians. This criterion is important, since if the topic proves totally resistant to attack, at least the student will have learned some significant mathematics.
- It should have a reasonable expectation of producing results which will be of interest to the larger mathematical community. We feel strongly that the projects should be significant enough to result in publication if all goes well. Also, the students become quickly aware that they are not working on 'toy' problems, and this is a significant source of pride for them.
- It should be accessible to students with no more background than courses in calculus and linear algebra together with a course in the general area of the project such as abstract algebra or real analysis. One of our goals is to show students that the research frontier is sometimes not far away. Also their experience in research can help them decide whether mathematics might be a possible career.

One surprise which has occurred over the last eleven years is that students at this level often are capable of formulating and proving abstract results, sometimes in very original ways. For instance, Durfee's 1988 group was working on the number of critical points of a real polynomial. He explained to them the technique by which Milnor proved the invariance of the degree of a smooth map, thinking that this technique would be of no use to them. However one student used it to prove a result which has remained central to the theory of this topic, and since then no better proof has been found.

We can choose topics which would not be suitable for investigation by a graduate student starting research for a PhD. Of course, a graduate student starts knowing more, and has more time to learn background material. On the other hand, a graduate student should tackle a topic which is moderately safe, with the hope of actually finishing at some point. Undergraduate topics, however, can be more risky and hence much more interesting, even though they may not work out in the long run.

For instance, Peterson's 1999 group investigated models of turbulence. This was an adventurous project, since it was not clear ahead of time that the group would be able to do anything of interest. As they were learning the history of the problem during the first week, though, they realized that the many desktop computers around them were actually more capable (in terms of both hardware and software) of simulating turbulence than the supercomputers of 1970 which initiated this research. At this point the group began generating solutions to the Navier Stokes equation using the so-called pseudo spectral method, searching for turbulent solutions, and investigating their statistical and geometrical properties. These solutions are still quite mysterious, and the group found it exciting to realize that this problem was actually accessible to them.

Choosing a good topic is not an easy job. Faculty often wait for a suitable topic to present itself before committing themselves to a student research group. When Durfee was in the process of looking for a topic for his first group in 1988, he happened to ask V. I. Arnold for suggestions. Arnold suggested a number of topics, among which was the appealing question "Fix a degree $d$. What are the possible numbers of maxima, minima and saddles that a real polynomial of two variables and degree $d$ might have?" This turned out to be a remarkably good topic for student investigation, and resulted in a paper in the American Mathematical Monthly.

Our method of recruiting students has changed over the years. We started by sending flyers to all the colleges and universities in New

England. With the advent of the web, though, we stopped doing this, and now we count on our web site and the fact that we are listed in various publications.

We look for students between their junior and senior year of college, though we often take younger ones who are well prepared. As mentioned above, we expect them to have taken the calculus sequence, linear algebra, and at least one course beyond these such as abstract algebra, real analysis, number theory, and so forth. Although we often are lucky enough to get students with more preparation than this, our topics are chosen so that this background suffices.

We usually get over one hundred applications for our ten places. The applicants usually fill out the form on our web site and return it by email. Most of them are well qualified, and it is hard to decide whom to take. They are selected based on the courses they have taken, their grades, the recommendations from faculty members, and the applicant's statement of interest. We try to form each group so that it has a variety of talents. In particular, we make sure that each group has one or more students with good computer experience. We also try to balance our groups with students from different parts of the country and from both large and small institutions; in fact, students from more than 41 different institutions have participated in our program. Lastly, and very important, we try to balance the number of men and women in each group. This is important for us since we are a women's college (the oldest in the country) with a strong historical commitment to the sciences. Over the years we have succeeded quite well at this; of the 130 students who have attended, a little over half (more precisely, 69) of them have been women.

Each summer we also take one or two Mount Holyoke students who are qualified and wish to attend. Over the years, 25 have participated. Mostly, however, we encourage our students who are interested in summer research to enlarge their horizons and join a group on another campus. Our often talented international students are limited in obtaining funding, so if we can support them, we are happy to have them join one of our groups.

The program runs for eight weeks, starting in early June. Each group is assigned a room equipped with a large table, blackboard, comfortable chairs, desks, a library of relevant texts and papers (which tends to reside on the large table), and dual-boot pentium Windows/ Linux computers and other machines (SPARC Stations, MacIntoshes, etc.). Other rooms are available for study and quiet. All groups share a common room with a refrigerator, microwave and coffee-making facilities. The entire area is air-conditioned.

We structure our program as much as we can. Each group begins the morning by meeting with the faculty advisor to plan the day's activities. The daily schedule might begin with a presentation by the faculty member of new material, a presentation by students of their own progress, a discussion summarizing what has been done, or a restatement of the project's short or long term goals. The faculty advisor is in contact with the group throughout the day. The day ends with afternoon tea in the common room. Of course, this description is of theoretical day. In practice, each is different.

Once a week each group gives a formal progress report. The inexperience of undergraduates is never more painfully apparent than in their first presentations. We have learned over time how valuable it is for them to have repeated opportunities to say what they are doing, and in the process to clarify for themselves and their friends in the other groups the details of their problem. In the course of the summer everyone speaks and hence comes to grips with the material. There is also considerable repetition from one week to the next, so the other groups eventually learn the topic at some level. There are also visiting speakers, most of whom are paid for with Mount Holyoke funds.

In recent years we have had a weekly seminar where the students talk about subjects of interest to them, subjects not related to the research project. These have turned out to be quite popular.

Visits with other undergraduate research sites are arranged during the course of the summer. In the past we have visited groups in Williams College, Worcester Polytechnic Institute, Amherst College, the University of Massachusetts and Boston University. These visits have been wonderful days of getting acquainted, talking and listening, comparing and mildly competing. Combining formal and intense mathematics with pool side barbecues and mountain top picnics brings students to a closer understanding of what we, as faculty, experience with our subject.

The students last summer investigated possible schools for graduate study in mathematics, including Harvard, MIT, Rutgers, Princeton and Dartmouth. At each site they met with faculty and students. In the course of their visit, they managed to mention the nice things the previous schools had done for them and thus managed to get free meals and other goodies. They also organized a concert at Mount Holyoke. They followed this up with another one at the Mathfest in Providence with students from other places participating. This was in addition to giving talks there on their summer work.

Despite our efforts to choose a suitable blend of students for a group, occasionally problems do occur. Sometimes a student feels that he
or she is hopelessly less talented than the rest of the group and is completely out of place. In this situation we do our best to reassure them that talent comes in many forms and that they certainly have something to contribute.

A post-summer student evaluation once contained the following: "Perhaps a bit more control on the inter-relations among members of the group [would be good]. There was one rather dominant personality who seemed to be leading the way for much of the start of the summer. Although this person was a reasonably good mathematician it was frustrating that it seemed that we in the rest of the group were stuck doing little more than clean-up work on the results that he had already established." We had hoped we had solved this problem by dividing the students into groups and getting them to work on different topics, but evidently the problem persisted.

During the summer the working relationship evolves from that of teacher and student into one of research colleagues. In the beginning, even the best students will carry out pages of computations but will be unable to recognize patterns or make conjectures about what they have done. Many also are unwilling at the outset to think about a topic from more than one point of view. For instance, in 1994 one student in Durfee and O'Shea's group carried out an algebraic computation while another produced some computer pictures of the outcome. However the first student did not want to look at the pictures produced by the second student, saying that they would confuse him!

In the beginning most of the students are unable to recognize when an argument is really correct, and not just correct enough to get a check mark from their professor. But by the end of the summer, they are familiar with the long process of questioning, correcting, revising and shortening associated with mathematical research.

Each project produces one or more final written reports on their work, and every student is required to do this. Work on these occurs throughout the summer as the students struggle with making their ideas precise.

If a final paper looks ready for formal publication in a journal, the advisor can help draw the results together into a paper. Of course there is no guarantee that a project will result in a paper which can be published, and this is one of the risks choosing topics in mainstream mathematics. (In fact, we once lost a student we were recruiting to another REU site because the other site seemed more certain that the students would be able to publish something at the end of their stay.)

A typical story of what happens to REU research is the following from Robinson: "My 1992 REU got scooped. They did do work
that was publishable but someone else, C. Y. Lin (Taiwan), got it published first. However, Jan Denef (Leuven) has used our 1992 paper to introduce his grad students to local zeta functions. The 1995 group's paper, on the Igusa local zeta function for the Fermat surfaces: $X^{p^{n}}+Y^{p^{n}}+Z^{p^{n}}$, was never published. It could have been but I have never pulled it together. But David Reuman did write his senior thesis at Harvard about this the next year. The 1997 group had one result that is interesting but maybe not enough for a paper. The 1999 group will submit their paper on the Igusa Local zeta function for the different reduction types of an elliptic curve for publication this year."

In the earlier days of our project we were unsure of what to do with student papers which were interesting and represented a lot of work, yet were not quite suitable for formal publication. With the advent of the web, this problem has been solved, though. We now publish all the reports on our web site, where they are easy to find. At this moment, there are seventeen of them there.

Recently our students have presented their results at national meetings, both the summer Mathfest and the annual joint meetings. The advent of special sessions for research by undergraduates has facilitated this process. Last summer two of them won a prize for their talk at the Mathfest. One used the prize money to buy phone cards for the others so that they could call him in Israel where he is now spending the year.

Since the students often produce many interesting mathematical examples and counterexamples, and often lay the groundwork in a particular area, their work can also appear (with ample recognition) in later papers by their faculty mentors and other mathematicians. In fact, the effect of the sumer projects upon our own research can be enormous:

- Durfee's first summer project ("Counting the critical points of real polynomials", 1989) changed his research area; before, he had primarily worked in local problems in complex algebraic geometry, but afterwards he spent more and more time on real algebraic geometry and global problems.
- O'Shea became interested in computational algebraic geometry as a result of his 1988 REU topic, which he continued in 1990 and 1994. Part of the impetus for the book Ideals, Varieties and Algorithms (David Cox, John Little and Donal O'Shea; Springer, 1991) and its sequel Using Algebraic Geometry (Springer, 1998) was to provide a source for background material for his REU students.
- The work of Davidoff's 1993 REU group inspired Peter Sarnak and his thesis student Michael Rubenstein to produce a general solution of the problem Davidoff's group worked on.
- Peterson's 1988 group initiated work on the topic of singular Laplacian growth. Peterson felt that there were some nice results waiting to be discovered, and continued this investigation with his 1992 group. However, the second summer did not work out. Peterson abandoned the topic, only recently to return to it, when he discovered that the nice results were indeed there. He will be presenting this work at a meeting on mathematical physics in Les Houches, France.

Our efforts at evaluation have been sporadic, alas. Several times we have had the students fill our questionnaires at the end of the program, and we have used their comments to improve what we do. Our current plan is to have an exit interview at the end of the program, then to have them answer a questionnaire in the middle of the following school year, when most of them are seniors, and another during the middle of the year following that, when most of them are in graduate school or are employed.

A student in our first summer wrote the following in a report on her experiences: "After the summer program ended, I talked to many people...about my research. I was amazed and delighted when I saw how interested people were in what I had been working on. I have met a number of algebraic geometers who are working on similar topics. I have talked to some people in computer science who are writing software which would be useful if I were to continue working on these problems. I feel very much a part of the mathematical world." (Sara C. Billey, Summer research at Mount Holyoke College. In: Models for Undergraduate Research in Mathematics, Lester Senechal (Ed.), MAA Notes Number 18, Math. Assn. Amer. (1990), 99-101.) Another report by a student on her experiences last summer will appear in Focus.

We have not made any formal effort to follow our students after they have left our program, though many of us stay in touch. For this reason we are not sure how many students have gone on to do graduate work in the mathematical sciences (though we estimate this number to be $80 \%$ ), how many have obtained advanced degrees in mathematics, or how many are now teaching mathematics.

We recently added an ethics and graduate school survival component to our summer, but we have to admit that it has not been a success so far. In 1997, our students joined the Mount Holyoke summer research students in biology and chemistry for a two-day workshop
on this subject. Unfortunately it was mainly oriented to the other sciences and did not have much relevance for our students. The result was that the next year half our students refused to attend. In the following year (this past summer), a member of the biology faculty gave a lecture to all the summer science research students on how to give short talks and poster sessions. In this he said that "It is really important to publish your results; the whole point of research is to publish, and it is worth nothing if it is not published". However, our students insisted that they liked to do mathematics for the pure joy of doing so, and held publishing to be of lesser priority. When the faculty member insisted on his point of view, our students walked out. So we are still working on this one.

In the future our ethics component will consider the use of mathematics, for good or bad, in industry.

More detailed information about our summer program can be found on our web site at www.mtholyoke.edu/acad/math/reu. This contains, for each past project, a list of the student participants, a description of the project and a description of the results produced by the students. It contains their reports and papers in down-loadable form as well as references to formal publications by the students or resulting from their work.

## Appendix A: Topics

Here is a list of topics investigated by our groups during the period 1988-1999 (some of them more than once).

Bootstrap estimation of eigenvalues in principal components analysis Limits of tangent spaces
Time dependent evolution of planar domains
Counting critical points of real polynomials in two variables
The red blood cell shape
Extremization in function spaces
Curvature and limits of normals to surface singularities
Steepest descents with Sobolev norm
Igusa's local zeta function
Laplacian growth
Topics in comparative number theory
Varational methods for solution of PDE's
Difference sets in groups via representation theory
Limiting tangents and normals to singularities of algebraic surfaces
Curvature of algebraic singularities
Weight distributions of certain generalized codes
Algorithmic proofs of the Quillen-Suslin theorem
Codes and Kloosterman sums
Polynomial representations of knots
Models of turbulence

Appendix B: Selected publications

- D. O'Shea, Computing limits of tangent spaces: singularities, computation and pedagogy. In: Singularity theory, Trieste 1991, ed. D. T. Le, K. Saito and B. Teissier, World Scientific 1995, p. 549-573. (O'Shea REU 1988 and 1990)
- A. Durfee, N. Kronenfeld, H. Munson, J. Roy, I. Westby, Counting Critical Points of Real Polynomials in Two Variables (Amer. Math. Monthly 100 (1993) 255-271. (Durfee REU 1989).
- J. Ferry and M. Peterson, Spontaneous Symmetry Breaking in Needle Crystal Growth, Phys. Rev. A, 39, no. 5 (1989) 27402742. (Peterson REU 1988)
- M. Peterson, Nonuniqueness in Singular Viscous Fingering Phys. Rev. Lett., 62, no. 3 (1989) 284-287 (Peterson REU 1988).
- A. Durfee, The index of grad $f(x, y)$. Topology 37 (1998) 13391361. (Durfee REU 1989, 1992)
- A. Durfee, Five definitions of critical point at infinity. In Singularities, the Brieskorn Anniversary Volume, V. I. Arnold et al, ed (Birkhäuser 1998) 345-360 (Durfee REU 1989, 1992)
- D. O'Shea, L. Wilson, Computing normals to real surfaces. In: Mathematics of Computation 1943-1993: A Half-Century of Computational Mathematics, W. Gautschi, ed. (Proc. Symp. Applied Math 48, Amer. Math. Soc. 1994), 349-353.
- D. O'Shea, C. Teleman. Limiting tangent spaces and a criterion for $\mu$-constancy. In: Stratifications and Topology of Singular Spaces, vol 2, ed. D. Trotman and L. Wilson, Hermann 1997. 79-85. (O'Shea REU 1988)
- J. Alexander, R. Balasubramanian, J. Martin, K. Monahan, H. Pollatsek and A. Sen, Ruling out $(160,54,18)$ difference sets in some nonabelian groups. (Pollatsek REU 1994; preprint, submitted for publication)
- E. Clark, L. Ivanova, A. Koll, J. Ross, Y. Rubinstein, M. Peterson, "Turbulence on a Desktop" (preprint, submitted to American Journal of Physics; Peterson REU 1999)

Mount Holyoke College, South Hadley, MA 01075-1461
E-mail address: adurfee@snow.mtholyoke.edu

# The National Security Agency's Director's Summer Program 

Patrick Flinn

The elite REU of the National Security Agency (NSA) is the Director's Summer Program (DSP), established in 1990 and thus having just completed its tenth year.

The number of student participants per year has grown monotonically from eight to twenty six. The top 150 scorers on the Putnam exam who are U.S. citizens are invited to apply, but application is open to any U.S. citizen. (Being a citizen of the United States is required because one must be granted a security clearance before receiving an offer of employment from the DSP. This greatly complicates our selection process, for the clearance procedure is lengthy) In a typical year, thirtyfive to forty students are chosen in October from an applicant pool of 150. Our Personnel Department mails a package, including security forms, to these students, and arranges an interview in Maryland for them once the completed forms are returned. During the interview the applicants must pass a polygraph test in order that the background investigation may proceed. The students also spend several hours with the DSP technical directors. The technical directors discuss mathematics with the students, but can say little about the summer's actual problem set, because it is not determined until April or May!

So at this point, the students have had to apply early in the fall, fill out a long security form, spend a couple of days in Maryland for an interview at NSA, during which they have been told next to nothing about what they might do during the summer. Furthermore, we cannot tender an offer until the clearance has been granted, often in late April or May. As many of these students have wide ranging opportunities for the summer, it is amazing that most of the selected applicants stick through this entire process!

[^4]After the student interviews have ended, the technical directors (usually three in number) begin planning the summer's problem set. This is done at the last minute to ensure that the problem set consists of current important problems. This opportunity to do mathematics for which the payoff is clear and of great value is one of the unique features of the DSP and stimulates extremely hard work and dedication from both the students and the Agency staff.

The DSP runs about twelve weeks, from early June until late August. Housing is arranged, generally at some large apartment complex where the students can be co-located. This is important for social reasons, but also because carpooling is necessary. As the government cannot pay for student housing (they are government employees after all), we compensate them with higher pay from which they must cover their living expenses in the Baltimore-Washington area. It is always a tough job for the directors to find suitable housing which is affordable, yet in a safe neighborhood reasonably close to conveniences and NSA. Moreover, few apartment complexes are willing to rent to people so young for so short a time.

During the first couple of weeks of the DSP, the problem set is presented and a short course is offered. The short course is designed to help the students come up to speed on the summer's problem set. Mathematicians at NSA are involved with a remarkably broad set of technological areas, and the DSP problem set reflects this variety. The problem set ranges from purely speculative theoretical mathematical questions to ones that are more properly in the realm of engineering, physics, or computer science. In virtually all, however, some computer proficiency is required, since the computer is the experimental laboratory and verifier of the soundness and utility of most ideas. The DSP considers about ten problems, each supported by a subject matter expert from the office of origin. This problem supporter explains the project to the students. After all of the problem supporters have had their opportunity to stir up interest and the background material has been covered in the course, the students choose their project for the summer. Those students making a common selection form a team which gets technical support from the problem supporter and technical directors. Over the course of the summer, an incredible bond generally develops among these people. The divisions between experience and inexperience dissolve. Only colleagues remain, who are united in trying to solve a difficult problem whose importance and timeliness often motivates even greater effort and emotional commitment. With carefully chosen summer projects, most students make some progress, while each year the DSP contributes one or two remarkable solutions.

The last week or so is spent documenting the summer's work. Each participant is responsible for writing up his or her contributions, which may be edited into a project paper. These write-ups help disseminate the progress made during the summer throughout our community. In addition, this documentation exercise gives the participants practice in writing up their research.

There are high ranking people who know of and support the DSP because of its successes, despite its requirements of both physical and human Agency resources. The Director and Deputy Director of NSA both visit the DSP room if they can (and often they do).

Through the DSP, NSA establishes contacts with some of the future leaders of the U.S. academic mathematics community, who in turn are exposed to the applied discipline of cryptomathematics and have the opportunity to work with some of the best young mathematical talents of their peer group. The DSP is not intended as a recruiting vehicle. Rather, it has sprung from and is run by the mathematics community of NSA, the members of which are concerned for the state of mathematics in the U.S., and who use this opportunity to make a small but unique contribution to its health.

National Security Agency
E-mail address: pflinn@zombie.ncsc.mil

# The Director's Summer Program at The National Security Agency Cryptologic Mathematics for Exceptional Undergradute Mathematicians 

Ms. Victoria L. Yates, Program Administrator

The Director's Summer Program is the National Security Agency's premier outreach effort to the very best undergraduate mathematics majors in the country. Each summer we invite 25 exceptional math students to participate in a 12 -week program where they collaborate with each other and with NSA mathematicians on mission-critical problems. Admission to the program is highly competitive and is intended primarily for students between their junior and senior year, but exceptional freshman and sophomores will also be considered. Graduating seniors will be considered too, but they must be enrolled in a mathematics graduate program for the fall. Students will be paid a salary based on experience and year in school. Minimum requirements are one full year of abstract algebra and one full year of analysis, or equivalent. Computer background, especially C or $\mathrm{C}++$, is desirable but not required.

The goals of the Director's Summer Program are to:

- Introduce the future leaders of the U.S. mathematics community to the Agency's mission and share with them the excitement of working on mathematics problems of national importance,
- Provide a deep understanding of the vital role that mathematics plays in enabling the Agency to tackle a diverse set of technical challenges,
- Encourage bright undergraduate mathematics majors to continue their study of mathematics and pursue careers in the mathematical sciences, and, of course, to

[^5]- Provide solutions to current operational problems.

The students participating in the program work on a broad range of problems involving applications of Abstract Algebra, Geometry, Number Theory, Combinatorics, Graph Theory, Probability, Statistics, and Analysis. For the first two weeks of the summer, lectures on modern cryptologic mathematics are given. After the lectures, the students are presented with about ten current problems and choose one or two as the focus for their research. All research is documented in a series of papers written by the students near the end of the summer.

Throughout the summer, students develop mathematical theory, apply what they learn to obtain real-time solutions, and experience the excitement of success built on hard work and innovation. Most students find the work at NSA very exciting and challenging and many decide to return for another summer. State of the art computing resources are available to all students. For the most part programming is done in C in a UNIX environment. Computational algebra packages including MATHEMATICA, MATLAB, MAGMA, MAPLE are available in addition to a variety of statistics packages.

Because of the lengthy security processing required, the deadline for applications is 15 October each year for the following summer. To apply, students simply send a resume, at least two letters of recommendation from faculty members familiar with their work, and a copy of transcripts through the current academic year. ${ }^{* *}$ Students must be U.S. citizens. The Director's Summer Program is an extremely rewarding summer experience! All information should be sent to: Department of Defense, National Security Agency, 9800 Savage Road (SAB 3), Fort George G. Meade, MD 20755-6515, Attn: R51 (DSP).

For additional information about the Director's Summer Program, call Ms. Vicky Yates, Program Administrator at (301) 688-0983 or send e-mail to vicky@afterlife.ncsc.mil.

[^6]
# The REU Program at Northern Arizona University 

Terence Blows

The Department of Mathematics and Statistics at Northern Arizona University has run NSF sponsored REU Programs for the past six years. An application for further support is pending. The program runs for eight weeks in the summer, with students working on individual projects. Some summers we have had 6 students, for others it has been 8 . Selection takes place in early March; interested students are asked to visit our home page at http://odin.math.nau.edu/REU/ to find possible research topics and also to follow the application procedures. The research areas available to the students are combinatorics (including graph theory, topology, algebra and geometry) and differential equations (including dynamical systems, nonlinear pdes and modeling). These two areas are ideally suited to undergraduate research. Projects in combinatorics usually require a background in discrete mathematics and group theory; those in differential equations usually require courses in elementary differential equations and analysis. But prerequisites vary from project to project, and can be adapted to suit individual backgrounds. Although the program is open to all U.S. citizens and permanent residents, in our recruitment for the program we utilize Northern Arizona University's geographic location which admits unique possibilities for attracting Native American and Hispanic students. We are also very in attracting quality students from institutions without graduate programs in mathematics. About one half of the students who have participated in the program to date fall into the latter category. We target possible projects to each student once they have been admitted to the program, and their mathematical background is understood. Projects are typically within the advisor's area of research interest, although they may be slightly peripheral to

Received by the editor September 18, 1999.
the advisor's own research direction. A successful student will often spark genuine interest in their advisor, something that enhances the student's research experience. Each student, with guidance from his or her project advisor, selects his or her own project: One that is interesting and approachable but which will nevertheless challenge, and probably frustrate him/her. We ensure that no student is working in isolation from other students. This encourages the interaction of each student with at least one other in the program. Interaction between students is an integral part of the research process and is considered an essential component of the program. Such student interaction is fostered at the beginning of the program by the advisors. As well as working closely with their advisor and at least one peer, students often interact more broadly with others in the program. It is common, in the first couple of weeks of the program, for a faculty member to lecture for an hour or two each day to his/her students; homework and further reading may also be assigned. This allows students to progress rapidly through pre-requisite content and provides them with sufficient background to select the particular direction in which they will pursue their research. Although the lecture format will be abandoned once student research is underway, meetings with advisers occur on a (more or less) daily basis throughout the program. We believe it to be important that each student be initially nurtured as they come to grips with new ideas, but that they are allowed to achieve a greater independence as their confidence grows. The first seminars, given on the first couple of days of the program, are project descriptions given by participating faculty. This allows all students to have an idea of the research to be undertaken by other students in the program. Seminars continue throughout the program with many faculty from the department (and not just those involved with REU students) talking about their research interests. We also invite faculty and graduate students from nearby institutions to give talks. At the end of the program, each student contributes a one-hour talk. Students are also required to write up their work in a form suitable for publication. This will be carefully read by their advisor and returned to the student post-program for editing. The set of final versions are then bound as a single volume and sent to all participants. When appropriate, students and advisor will work on an article for submission to a research journal. Students are housed, two to a room, in on-campus housing. They receive a $\$ 2000$ stipend, have their travel expenses to Flagstaff paid, and receive a small allowance for food. (The latter depends in part on the total travel costs.) Students earn 6 semester credits (pass/fail) of MAT 485: Undergraduate Research. Northern Arizona University waives tuition
and reduces some other costs. We believe it is also important that the students be able to enjoy themselves. Students will have access to the recreation facilities on campus, and a number of things are arranged for them. Flagstaff is a wonderful place to spend the summer (temperatures are usually only in the mid-80s) and we organize hikes (in the nearby mountains and canyons) and trips (for example, to Sedona, the Grand Canyon and Petrified Forest National Park) on most weekends. Students have also been known to organize their own 'probability seminars' in Las Vegas, and to go skydiving in Tucson. Both these cities are just four hours away. As well as outdoor activities, we also organize potluck dinners, games parties and similar social events. About one half of past participants who have since graduated have gone on to graduate school - past students are currently doing graduate work at (for example) Berkeley, Cal Tech, Michigan, Missouri, Oregon, Princeton, Stanford, UC San Diego, Virginia Tech and Washington. Other participants have typically enjoyed their research experience, but have chosen to pursue other fields (for example, some have gone to medical school, and others have taken employment in the computer or insurance industries).

## Student Statement

My experience in the REU program at NAU was amazing. There is no other way to describe it then just that, amazing. It was, with no exaggeration, the best summer of my life.

Before coming to NAU, I was as lost as most math majors are. Whenever people find out you're a math major they always ask, "so do you want to teach?", and when you answer no, they then say, "so you want to be an accountant then?" And if once again you answer no, they want to know what else you can do with a math degree. Before this summer at NAU I always had a hard time responding to that question, but now I know the answer: Anything.

This past summer I went to a place that I had never been to spend a summer with people I didn't know. I was intimidated by the situation, it's not something I do that often. Then I arrived in Flagstaff, and I loved everyday of my summer spent there. I did amazing math with Dr. Steve Wilson, math I had never done before I went to NAU. I loved the math I did, and I learned so much.

I not only learned about Graph Theory, which was my main field of study for the summer, but we were given seminars in modeling, PDEs, statistics, etc. This gave us all a chance to see what the other students were studying, as well as broaden our horizons to other fields of math.

I never though math reasearch could be so exciting. It's hard to top the feeling one gets after you realize you correctly proved a theorem, or how well you'll sleep after you spend a day wrestling with one particular problem, and the problem won.

But I think most students are going to tell you how much they learned at their REU program, so what makes the one at NAU so special? I don't know for sure since it's the only one I've ever participated in, but I would say it has to do with the people and the location. The other five students participating in the program were great. Of course we had our problems, as any six people meant to spend all of 8 weeks together would, but I made some pretty amazing friends as well. I'd never been in a situation before with people who shared the same love of math that I did, it was wonderful. I am still in constant contact with two of the other five.

Besides the other participants, there were the professors. Never in my life have I been at a school where the professors in one department are not only colleagues, but friends. We had dinners at professors house and the other professors in the math department would come, and not just the professors from the program. Also, each and every professor in the program was friendly, and approachable, and willing to help us, whether the help was needed on a math problem, or directions on how to get to the mall in town.

And last, but certainly not least Flagstaff has so much to offer. Good food, movie theaters, a dance club, and most of all climate and nature. I have never seen anyplace with such natural beauty as I did in Arizona. You can climb rocks, or hike the highest mountain in AZ, or see native American cliff dwellings, or go to the Grand Canyon, or go sky diving. I myself couldn't decide, so I did it all, and it was all incredible. I don't think I could ever forget the people I met in the program, the kindness in which I was treated, the math I learned, the amazing Flagstaff summers, where hot is 90 degrees, or the incredible natural beauty that I experienced.

Like I said earlier, in a nutshell, my summer at NAU was nothing short of amazing.
Quinn Saner
Senior at Boston University

Northern Arizona University
E-mail address: Terence.Blows@NAU.EDU

# The REU Program at Oregon State University 

Dennis J. Garity

## 1. History

There has been a REU Summer Program at Oregon State Univeristy since 1987. Dennis Garity has been the director of the program since 1992. During the past five year period 44 students participated in the program. Of these, 23 were female and 21 were male. There was one Hispanic male student and two Asian American students. The participants came from a diverse group of large and small state universities and private colleges and they came from diverse areas of the country. Each year there were between 80 and 120 applicants to the program.

Each summer, three faculty members from Oregon State University worked with the program and directed student projects.

Although the projects were diverse, there were three common underlying themes. One was the symbiosis between pure and applied mathematics. A second common theme was the use of geometric reasoning and modeling. A third theme was the use of computers in modeling and simulation.

## 2. Philosophy and Structure of the Program

The model that is used at Oregon State University has the following main points.

- A single director who oversees the program and keeps track of how each student and project is progressing,
- A dedicated staff of research mathematicians working closely with students on the projects,

Received by the editor September 21,1999.

- A daily seminar which allows us to present an overview of the profession of mathematics, both in terms of subject matter and in terms of career paths and opportunities,
- Good working conditions including shared offices, a number of planned group activities, easy access to resources such as computers, photocopy machines, and the library, and help with housing and transportation,
- An emphasis on learning the process of research (formulating questions, reading the literature, and working cooperatively with the staff and other students),
- An emphasis on fostering a sense of camaraderie among the participants through carefully selected research and social activities, and
- Choosing projects that emphasize the common themes of our program.
The goals of the program have been as follows.
- Influence a diverse group of student participants, including a significant number for whom our program can make a real difference in terms of career choices.
- Involve active members of the research community with REU programs in general and our students in particular.
- Maintain contact with and promote our students after the program.
- Manage to do all of this with limited resources..

In many ways the hardest item on this list is the last one. It is a challenge to come up with a financial model which makes our program attractive both to students and to the active research mathematicians on our staff who have an interest in undergraduate development but who also have their own research and personal commitments. The program manages to do this by using NSF funds and additional funds contributed by Oregon State University.

## 3. Details of Student Activities

The program lasts 8 weeks, coinciding with the summer term at Oregon State University.

There are two main mathematical activities during the summer. The first is a seminar or combination seminar/small group discussion. This lasst for about one hour each morning. It includes staff talks about possible projects, student talks, talks by other faculty members, discussions of career options and opportunities, and presentations of progress on research projects. The second main activity is the research
project. Students are expected to spend most of their time working on a research project, either individually or together with other students and under the supervision of one of the regular staff members.

The goal of our research projects is to teach students the research process so that they can make more informed choices about career options. We try to make the process as enjoyable and rewarding as possible and encourage the students to seriously consider graduate programs. The students learn to formulate a question, research and read the literature, make conjectures, get initial results, and sometimes discover that something that they are working on has been partially done already. This often leads to the students modifying the original problem and continuing to get results. The staff is here to direct this process and to help deal with some of the inevitable frustration. Our students report at the end of our program that they have a much better understanding of what is involved in mathematical research.

We have observed that it is best to have several students working on a single project or on related topics. This promotes interaction. As our program continued, we found it necessary for staff members to work with groups of three to four students, and in that situation it is best not to have more than two completely different threads per staff member. Thus, control is exercised over the student's research topics. A list of potential projects is sent to the students before they arrive and they are asked to give some thought to which projects they want to work on.

When evaluating the success of a project, we ask whether the students read and understood some of the literature, spent a significant amount of time and effort playing with the question, formulated their own questions, obtained some results, and learned some useful mathematics. Ideally, all of these should happen, and generally they do. Some of the results obtained may seem more like exercises to us, but the process of coming up with the results is very beneficial to the students.

During the program we make the students aware of meetings at which they can deliver papers. These include meetings, MAA and AMS meetings, and special conferences on undergraduate research. We offer assistance to our past students who desire to go to such a meeting to give reports on their projects.

The program is most successful when the students develop a sense of camaraderie. We have tried a number of things to create and maintain the appropriate atmosphere. The most important elements are an enthusiastic and committed staff, sensitivity to problems such as transportation and housing, and a number of planned activities.

## 4. Facilities, Resources and Equipment

REU students share one or two offices in the Department of Mathematics. We use office space vacated by graduate students in the summer. The offices have two to three computers that are connected to the Mathematics Department computer network. We have found that having the students share offices stimulates mathematical activity. The important point is that each student has a desk, easy access to the other students, access to departmental facilities, and night access to the building.

## 5. Student Recruitment and Selection

We try to recruit a diverse group of students. This diversity includes ethnic background, educational background, and gender. Students should come from a mix of major research universities, traditional state colleges and universities, and smaller private institutions. Some should be mathematically mature and irrevocably on the path to research mathematics, but a large number should be students who have shown some interest and talent but who have not completely committed themselves to a career in the mathematical sciences or who have not found their appropriate level.

The type of students for whom we feel we can provide the most benefit are students from smaller and less well-known colleges who are described by their professors as being the best student in some number of years and as actively participating in a math club or similar activities. We also feel we can help obviously talented students from larger state universities. Such students often long for mathematical companionship and individual attention.

One admission policy we have had and intend to continue is that of giving priority to students who are truly juniors. We define this as students who will have between one and three semesters remaining before graduation. Our rationale is that these students are as mature as possible while not having yet chosen a post-graduate path. There are a number of very talented sophomores and even freshmen in the country, and some are mathematically ahead of the juniors we get. We feel that however advanced they are as sophomores, they will know even more as juniors. More importantly, those who show genius early do not lack for opportunity, whereas we can make a real difference to a student who is good but has not already been groomed for a career in mathematics.

We send posters, descriptions of the program, and application forms to most mathematics and computer science departments in the United States that offer undergraduate four-year degrees.

Department of Mathematics, Oregon State University, Corvallis, OR 97331

E-mail address: garity@MATH.ORST.EDU

# REU Summer Program at Penn State 

Moisey Guysinsky and Svetlana Katok

The Penn State Summer REU program started in 1999 as an extension of the MASS (Mathematics Advanced Study Semesters) Program. It is a 6 -week program for undergraduate students from United States colleges and universities. Participants are selected based on their academic record, recommendation letters from faculty and an essay. Prerequisites for the participants include an equivalent of at least two years of college mathematics for mathematics/science majors. Normally a straight A record in those courses is required. Support is provided by the National Science Foundation VIGRE grant.

The goal of the program is to expose the participants to different areas of mathematics and let them experience a combination of advanced learning and research initiation. Any mathematical research usually includes three components: study of the subject, solving of a problem, and presentation of the result. We attempted to include all three components into our program. Thus in addition to the traditional for REU's individual or small group research projects supervised by faculty mentors, the program includes two short courses, a weekly seminar, and a three-day "MASS-FEST" conference.

All elements of the program are specially designed for REU participants. A crucial feature of the program is a high level of interaction between the participants and the REU faculty. A special effort is made to coordinate and mutually reinforce content of the courses and other activities so that various mathematical concepts and methods appear in different settings.

Received by the editor September 15, 1999.

## Participants

The search of the participants was nationwide. We maintained a web-site for the program at: http://www.math.psu.edu/mass/Reu99/reu.html/.

The information about the program was also included into the NSF REU list. We received 52 applications, made 19 offers out of which 8 were declined and 11 accepted. The participants came from SUNY College at Geneseo, University of Notre Dame, Penn State, University of Oregon, Chapman University, University of California, Berkeley, University of Nebraska at Lincoln. They included 1 freshman, 2 sophomores, 7 juniors, 1 senior; 2 females and 9 males.

## Funding

The main source of support was the NSF VIGRE grant $(\$ 35,200)$. Two Penn State faculty members have contributed their individual REU NSF grant funds $(\$ 9,500)$ to the budget; they provided research projects for 3 students and served as their faculty mentors. Each participant received a $\$ 1,800$ stipend, travel reimbursement up to $\$ 500$, and the reimbursement for room and board in the amount of $\$ 900$. The budget for the MASS-FEST conference was $\$ 5,500$ which provided hotel accommodations, per diem, and a modest travel allowance for MASS alumni.

## REU Courses

The two two-week short courses given in the REU-99 were "Symmetry and Representations of Finite Groups" (June 21 - July 2) instructed by Adrian Ocneanu, (Penn State, Professor of Mathematics), and "The Theory of Partitions" (July 7 - July 16) instructed by George Andrews, (Penn State, Evan Pugh Professor of Mathematics). The weekly seminar was instructed by the REU coordinator Misha Guysinsky, (Tufts University, Assistant Professor of Mathematics). The following are short descriptions of the courses.
"Symmetry and Representations of Finite Groups" This course covered the basic ideas of the theory of representations of finite groups and the structure of their tensor products. The material that is usually part of a graduate course was approached concretely by using familiar examples such as the cyclic and dihedral groups. The presentation was made in a manner that encouraged the students to build the whole theory by themselves as answers to a series of simple questions.

Topics covered: Real and complex vector spaces; Direct sums and tensor products of vector spaces; The structure of groups; Groups given by generators and reflections; Geometrical interpretation of the trace, determinant, and characteristic polynomial; Exponentials of matrices; Multiplicative defect for products of exponentially non-commuting matrices; Rotations as imaginary exponentials; Representations of groups by linear transformations; Change of base and irreducible repre sentations; Inequivalent representations; The structure of finite Abelian groups and their irreducible representation; Tensor products of representations and their decomposition into irreducible components; Characters of representations; The McKay graph of tensoring with a distinguished representation of a group; The Peter-Weyl Theorem on the orthogonality and completeness of matrix entries irreducible representations; The order of the group as the sum of squares of the dimensions of its irreducible representations.
"The Theory of Partitions" This course covered the basic topic in additive number theory, the theory of partitions. This topic begins with the fundamental question: "How many ways can you write a given integer as sums of positive integers?" In trying to answer this question the students were led to a variety of intriguing discoveries including some of those found by India's great, enigmatic genius, Srinivasa Ramanujan. The Omega Package (implemented in Mathematica) has been introduced in order to use the power of modern computer algebra to investigate partitions. Topics covered: I. Partitions (Definitions; Infinite product generating functions; Ferrers graphs; The Pentagonal Number Theorem and its applications). II. q-Series (The idea of q-analogs; The q-Binomial Series; Heine's transformation and its iterates; Applications of Heine's theorem to partitions). III. MacMahon's Partitions Analysis (Basic definitions; Basic transformation rules; Triangles with integer sides; Partitions with non-negative higher order differences; Evaluation of binomial sums).

## Research Projects

An advantage of our program is that many participants return to Penn State in the fall for the MASS program. This allowed us to offer the students some research projects which required more than 6 weeks for completion. The research projects in various areas of mathematics were provided by interested faculty of our department. Some projects were directly related to topics included in the short courses. We also tried to involve students in more than one project. The projects included:

1. "Matrices with Non-conjugate Centralizers" (1 student; faculty mentor S. Katok, Professor of Mathematics)
The student who worked on this project found two matrices in $S L(3, \mathbb{Z})$ which have the same eigenvalues, but are not conjugate in $G L(3, \mathbb{Z})$, and therefore their centralizers are not conjugate. The existence of such matrices was guaranteed by the theory based on the connection between classes of conjugate matrices and ideal classes of rings of integers in algebraic number fields. The example was obtained from a totally real cubic field of discriminant 1957 with the use of a package of programs called PARI. This provides an example of interesting new phenomena in the theory of actions of higher rank abelian groups.
2. "A Geometric Spectral Theory for $n$-tuples of Self-Adjoint Matrices and their Generalizations" (2 students; faculty mentor J. Anderson, Professor of Mathematics)

The students studied the general facts about the spectral theory of self-adjoint operators, spectral scale theory and made several explicit calculations of spectral scale including a case with multiplication operators defined on an infinite dimensional space.
3. "Reflection Groups and Their Symmetries" (1 student; faculty mentor A. Ocneanu, Professor of Mathematics)
The problem was to find a canonical construction that allows to find a unimodular cover for any Dynkin diagram. The student had to study the theory of the reflection groups and td be carried out.
4. "Simplices with Only One Integer Point" (2 students; faculty mentor A. Borisov, Assistant Professor of Mathematics)
The students found an effective procedure that allowed to find all classes of simplices with vertices that have only integer coordinates and only one point with integer coordinates inside. Using computers they found all classes for the dimensions 3 and 4 .
5. "The Growth Rate of the Number of Generalized Diagonals for a Polygonal Billiard" (2 students; faculty mentor A. Katok, Raymond N. Shibley Professor of Mathematics)
The students had to study the number of different codings for polygonal billiards. For any trajectory there is an isometry which corresponds to unfolding of the trajectory after $n$-reflections. Students wrote a program that allows to find allpossible codings and corresponding isometries for any convex rational polygon. They used this program to study a regular octagon and found that the isometries that come from codings belong to a certain surface.
6. "MacMahon's Binomial Coefficient Powers" (1 student; faculty mentor G. Andrews, Evan Pugh Professor of Mathematics)
The problem was to find for what non-negative integer numbers $a_{1}, a_{2}, \ldots, a_{n}$ the following product

$$
\left(\frac{N-n+1}{1}\right)^{a_{1}} \cdot\left(\frac{N-n+2}{2}\right)^{a_{2}} \cdots\left(\frac{N}{n}\right)^{a_{n}}
$$

is an integer number for any integer $N \geq n$. The student used ideas developed by MacMahon in the beginning of the century and also Omega-package developed last year. He solved the problem for $n \leq 7$. He also indicated the problems and suggested methods for their solutions for $n>7$.
7. "Triangle Chain Partitions" (1 student; faculty mentor G. Andrews, Evan Pugh Professor of Mathematics)
The student solved a problem from the partition theory about the number of non-congruent chains of triangles with integer sides. He used Omega-package in order to make intermediate calculations. He also tried to solve a similar problem for tetrahedrons and obtained partial results.
8. "New Congruences for the Partition Function" (1 student; faculty mentor K. Ono, Assistant Professor of Mathematics). This project started before the REU program began. Using the theory of Hecke operators for modular forms of half-integral weight, the student found an algorithm for primes $13 \leq m \leq 31$ which reveals 70,266 new congruences of the form $p(A n+B) \equiv 0$ $(\bmod m)$, where $p(n)$ denote the number of unrestricted partitions of a non-negative integer $n$. As a simple example, she proved that $p(3828498973 n+1217716)=0(\bmod 13)$ for every integer $n$. The first three congruences were found in 1919 by Ramanujan, and after that finding new ones was considered as a very difficult problem. The paper of this student has been accepted for publication by the Ramanujan Journal.

## MASS-FEST Conference

At the end of the program we had a 3-day MASS-FEST conference. In addition to REU participants and Penn State faculty and graduate students, several MASS alumni, who are now in different Graduate programs came to Penn State for the MASS-FEST. It featured talks by Professors A. Katok ("Variational Calculus and Closed Geodesics"), A. Kouchnirenko ("Newton Polygons"), K. Ono ("Ramanujan's Congruences and More"), J. Roe ("Are Infinitely Many Dimensions Enough?"), R. Vaughan ("Means of Exponential Sums"),
participants of the REU program and MASS alumni. For the REU participants this was a very valuable experience. It allowed them to interact with other young mathematicians, more advanced in their mathematical careers than themselves, and see such mathematicians in action. It also gave them an insight into the MASS Program which some REU participants will attend in the fall. Among 11 REU-99 participants 2 were MASS alumni and 5 will attend the MASS program in the Fall of 1999.

## Social Events

The REU participants were housed in the same block of rooms in one of the Residence Halls. The first two weeks we organized several social events in order to help them to get better acquainted with each other and Penn State faculty. Social events included pizza party, ice cream party, trips to a lake and a skating ring. During the MASS-FEST we organized a dinner, a party, and a trip to Penn Caves. At the end of the program we presented 4 students with the following awards:
"Excellence in Project Presentation" (2 students),
"Most Industrious Participant" (1 student),
"Most Outstanding Research Results" (1 student).

## Student Comments

I enjoyed the 6 week format because it gave me an opportunity to see some complex and in depth math during the summer and still left time to pursue other interests. The conference at the end was great. I got a feeling for presenting mathematical research in front of an audience.

The program was very well administered and it was clear that the professors and administers alike really cared about the students and that we learned.

Overall this REU was a great and useful mathematical experience!
I believe the program was organized and carried out exceptionally well. The courses and lectures were absolutely excellent. Always try to find professors like them.

Department of Mathematics, Tufts University, Medford, MA 02155
E-mail address: mguysi01@tufts.edu
Department of Mathematics, Pennsylvania State University, University Park, PA 16802

E-mail address: katok_s@math.psu.edu

# The Rose-Hulman NSF-REU Program 

S. Allen Broughton

## 1. History of the Rose-Hulman program

The Rose-Hulman NSF-REU program has had two stages of development in its eleven year history. From 1989 to 1996 Professor Gary Sherman directed the program in the area of computational finite group theory. During each of the eight years he worked with six students for a period of seven weeks during the summer. Following the summer period he worked at a distance with the students to refine their technical reports and encourage them to make a conference presentation. During these formative years, Professor Sherman confirmed his philosophy of undergraduate research and established the traditions and standards of quality of the REU that continues to this day. After the arrival of author at Rose-Hulman, Sherman and the author collaborated to expand the program to include eight students and two faculty each year. The idea was to ensure the longevity and robustness of the program by expanding the number of areas of research, selecting the faculty mentors from a pool of interested faculty on a rotating basis, and reducing the student to faculty ratio. These measures would, increase the appeal of the REU, reduce the likelihood of faculty burnout and slow the pace of "mining out" the areas of undergraduate research. The basic philosophy of the program, which we describe below, and the success of the program was unchanged.

There are now three faculty members in the pool for the current program. Each year the two faculty mentors each select a team of four students from those applicants indicating a desire to work in the mentor's research areas. The research areas have a common core of discrete structures with an underlying algebraic or number theoretic basis:

[^7]- Allen Broughton (4 years involvement) - automorphisms and tilings of Riemann surfaces using methods of computational group theory,
- John Rickert (1 year involvement) - number theory and discrete structures,
- Gary Sherman (9 years involvement) - computational group theory and cwatsets.
Because the research interests of the investigators have a common core the students have a substantial common ground of interests and capabilities, though there is enough variety to make things interesting.

In the future, we will try to include more faculty in the program, especially junior faculty, and to increase the total number of students by including more Rose-Hulman students.

## 2. Goals and philosophy of the program

The goals of the program are:

- for students to achieve independent success at formulating and solving problems that are their own, and that require significant growth beyond "textbook problems";
- to increase their depth and breadth of knowledge in an area of algebra, discrete structures or number theory;
- to develop their collaborative work skills;
- to make significant use of computer tools in solving research problems;
- to develop their oral communication skills by making presentations of their work during the program and at a conference after the program;
- to develop their written communication skills by writing a RoseHulman Mathematical Sciences Technical Report and refining the report into a submitted paper where appropriate;
- to develop a collegial peer relation with one or more professional mathematicians outside their college; and
- to develop a sense of the culture of professional mathematicians, both academic and applied.
The key element of the program philosophy is student ownership of the research work, achieved through four basic ideas:
- a carefully crafted set research problems selected by students from a list posed by the faculty mentor,
- student teams mentored by the faculty member,
- computer based computation to aid the students in their research,
- oral and written communication of the research work.

Because of the design of the problems, students "hit the ground running" and immediately begin to work collaboratively on a problem they choose and are excited about. The problems may be easily understood without too much background and students begin working on concrete subproblems and preliminary examples within a day or two of arriving. It is expected that students will have examined one or more problems in conjunction with their peers and settled on their primary and secondary problems within a week or so. There will be a healthy 5 6 weeks remaining to do focused research and put the results on paper in an organized fashion.

Students are strongly encouraged to work together. Each group of four students is given exclusive use of one of two adjoining classrooms (the Theorodrome) in close proximity to the faculty mentors. The rooms are outfitted with appropriate computing facilities, papers and books, and large tables on which to lay out their work. Students are required to make presentations to the group on their work, so that their is a strong incentive to keep "chipping away" at problems and so that the entire group understands the work of each other more than superficially. Every problem team is also required to submit a draft technical report before the end of the program. These components not only hone their communications skills but gives them a pride of ownership of the problems.

The research methods use computations in finite groups, number theory, and other discrete structures using the software package MAGMA as the primary computational tool, and supplemented by other tools such as MAPLE, MATHEMATICA and MATLAB. The students use the software to initially investigate and organize large number of examples, helping them to see relationships and formulate conjectures. After this initial investigation they settle in on proving or disproving these conjectures. Throughout the program the students have access to Sun workstations (UltraSparc 10) in their workroom.

The relationship of between each faculty mentor and participants in their charge is modeled on that of a senior and junior colleague. It is made very clear that the students are not working individually for the investigator but that every one is working together on related problems. After the first few days there will be there are no formal meetings other that the oral presentations. The investigator visits research room several times daily to "chat it up", supply background information, ask "have your tried this", but refrains from taking the problems away from the students by "pre-solving" it for them. Since the students work, live, eat and play in close proximity to one another the collaboration
is natural. To help cement the relationship among the students and faculty there are social events consisting of a welcoming dinner, a few weekend parties, a "de rigueur" canoe trip and an exit luncheon. All events except the exit luncheon are for the entire group. Without fail, the students form a close-knit group that organizes most of its own social activity.

## 3. Research problem selection

Rose-Hulman is a small, private undergraduate college (1500 students $90 \%$ undergraduate) of engineering, science and mathematics programs with a primary focus on excellent teaching. Solid scholarship and research is also expected, with the view that it will enhance the faculty members capability as a teacher. Therefore, research collaborations with undergraduates are especially valued, making summer programs such as the REU a natural outgrowth of faculty research. Each of the research areas of the REU is also the area of the faculty members own research program. Therefore they are constantly thinking of variations of their own research that are suitable problems for undergraduates. Just as important, the situation ensures continuity and growth in the research area. It is a great builder of self-confidence in students to realize that the problems they are working on are a second step or a variation of a problem on which undergraduates achieved success one or two years earlier. For instance, considerable interest has grown around the concept of "cwatset" a grouplike construction that was invented and developed by Professor Sherman's students [7]. The research area of the author has five main problems areas, three of which he has worked on himself, independent of students, and four of which have been worked on by at least two "generations" of students each. They are described at the website [6]. Finally since the faculty member has a strong professional interest in the subject there is a great incentive for the faculty member to encourage the student to put the technical report in final form, publish the results (perhaps jointly with the faculty mentor) and present them to the mathematical community.

## 4. Student recruitment and selection

Though we still mail notices of the program to $300+$ mathematics departments nationwide, we increasingly use our website [4] to advertise the program and provide application materials. The website also provides a description of the research areas, a description of the scientific and social program, the Rose-Hulman environment, a list of technical reports and published papers of previous participants, and
a subsite [6] giving a fairly detailed documentation of the student research results in one of the three areas. In order to achieve as great a geographically diverse and nationally based representation as possible, not more than two Rose-Hulman students (usually only one) participate each summer, in accordance with NSF guidelines. In all years except one we have met our gender balance goal of at least 2-3 female participants and at least 2-3 male participants.

The prototypical student we look for has completed his junior year, has taken a year of abstract algebra or some number theory, has had experience with programming in at least one high level language or computer algebra system, and will have demonstrated a strong interest and ability in mathematics as well as working well with others. Students submit a letter of interest, academic resume, transcripts, two letters of reference from teachers familiar with their work, and specify a preferred area of research. The key pieces of evidence are the letters of reference from the teachers and any evidence that the student supplies which shows active interest and ability in mathematics beyond standard course work. Each faculty mentor selects the four students that will work with him based on the research preference, the application materials, and telephone interviews.

Rose-Hulman is a small, private school much like a liberal arts college. On the other hand, it has a large mathematics department for a school its size because of its technical focus. Therefore, RHIT is very strongly positioned to provide a nurturing atmosphere to students from liberal arts schools who do not have our strong technical focus and the computing and undergraduate research-friendly infrastructure. Thus we intentionally seek out such students - especially those promising students who have had no previous opportunity for undergraduate research - to achieve a healthy mix of such students along with students from larger state and research universities.

## 5. Departmental and institutional support

The Rose-Hulman mathematics department has a long history of involvement in undergraduate research, outside the REU. As previously mentioned, the focus of Rose-Hulman is teaching, but expects and gets research and scholarly work from its faculty throughout the institute. The institute culture particularly values research, scholarly work and other activities that enhances the student's learning experience at Rose-Hulman. The mathematics department has a particularly strong record:

- We have held undergraduate mathematics conferences [1] for 16 consecutive years, attracting keynote speakers of national stature, (Peter Hilton, Bill Lucas, Fred Roberts, Joe Gallian, Marty Isaacs, Gilbert Strang, Richard Brualdi, Michael Moody, Colm Mulcahy ) typically there are 15-20 presentations by students, attendance is about 100. All speakers except the keynote speakers are undergraduates.
- The Rose-Hulman Technical Report Series [3] is a vehicle for faculty, students, and REU scholars to put their mathematics in written form, and is used as a preprint series.
- We will be inaugurating our Electronic Journal for Undergraduate Mathematics [2] in March 2000.
In addition, the institute has been generally supportive of the program, all the way from encouraging such research through its institutional culture to providing partial summer support and academic year course releases to enable the faculty mentors to work with the students.


## 6. Followup and measures of success

There are four principal measures of the programs success:

- the number of students who have continued on successfully in mathematical careers,
- the improvements in intellectual skills of the students,
- the evidence of research accomplishment during the summer program, and
- the wealth of mathematical accomplishment that a cadre of three P.I.'s and 70 student researchers have accomplished.

Many of the goals of the REU are to foster an increased level of mathematical skill, communication and teamwork among the participants. Therefore, a primary piece of the evaluation of the success of the program are the videotaped presentations and the written technical reports of the students. Also, after the summer program there is a follow-up period in which the technical report is completed and the conference talk is given. As the investigators will be in contact with the students it will be very easy to keep track of the students future career moves. A record of students' eventual career choices is kept, so that we may obtain some evidence of the programs impact on careers. Finally a exit interview and survey is conducted as quality check on the summer program. Students particularly give high marks to the freedom to choose a problem, the quality of the mentoring, the pleasant environment and the development of their communication skills.

Of the 70 participants all who are not still undergraduates, all but 2 or 3 all have gone on to Ph.D. programs at schools such as Harvard, Duke, Illinois, Michigan, Wisconsin, Stanford, Berkeley, Kentucky, and Maryland, Some of these students are now graduating and looking for academic mathematical positions. Two of these students reported that their REU publications created a favorable impression on employers with whom they interviewed.

All of the participants have succeeded at doing original mathematics and have become better oral and written expositors because of the mandatory talks and technical reports (measured by red ink!). From experience it appears that this is where the faculty mentor can make a most valuable contribution by assisting the students in greatly increasing their communication skills. Students have always commented that they found these talks and report writing extremely valuable. Each of the technical reports has generated between 1 and 3 talks at local, regional, and national meetings and conferences. As a measure of success of this particular program, at the January meeting of the AMS in 1995, at a special session for REU programs, approximately half of the speakers had some link to the RHIT REU.

During the first ten years, 42 technical reports have been written by 51 students (with 7 more in various stages from the last two years. There are 15 accepted publications with more in the pipeline [5]. Finally, we are increasingly using our website as an open repository of research results.

## References

[1] Rose-Hulman Undergraduate Mathematics Conference. http://www.rose-hulman.edu/Class/ma/HTML/Conf/UndergradConf.html
[2] Rose-Hulman Institute of Technology Undergraduate Mathematics Journal: http://www.rose-hulman.edu/mathjournal/
[3] Mathematical Sciences Technical Report Series, list of reports: http://www.rose-hulman.edu/Class/ma/HTML/ResearchPubs/MSTechReps.html
[4] Rose-Hulman NSF-REU website:
http://www.rose-hulman.edu/Class/ma/HTML/REU/NSF-REU.html
[5] Rose-Hulman NSF-REU publication list:
http://www.rose-hulman.edu/Class/ma/HTML/REU/REUPubs.html
[6] Rose-Hulman NSF-REU Tilings research page:
http://www.rose-hulman.edu/Class/ma/HTML/REU/Tilings/tilings.html
[7] G.J. Sherman, M. Wattenberg, Introducing... Cwatsets!, Math Magazine, 67, No. 2, (1994) 109-117.

Rose-Hulman Institute of Technology, Terre Haute, IN 47803
E-mail address: allen.broughton@rose-hulman.edu

# SUNY Potsdam-Clarkson University REU Program 

Kazem Mahdavi

I am writing about SUNY Potsdam-Clarkson University joint program: Summer Research Experience for Undergraduates(REU),funded by the National Science Foundation (NSF). Faculty members and undergraduate students engage in joint research in mathematics, working together as colleagues. 'Academia becoming a haven for both students and faculty members,' I call it.

I would like to start this article with the story of my struggle to create a REU at SUNY Potsdam, joint with Clarkson University. I will explain how it started, its impact on faculty and students, and where the program is going.

During the Summer of 1989, I involved one undergraduate mathematics major in learning advanced topics in number theory. I was very pleased with the outcome. I noticed that both faculty and student can benefit. During the Summer 1990 I was on Sabbatical at Carleton University, Ottawa, Canada, where my colleagues told me of the benefit of summer research experience for undergraduates.

During the Summer of 1991, I tried to get internal funding for one undergraduate mathematics major to engage in advanced research. The committee in charge of award rejected our proposal. My students and I worked together without funding. The result was quite good. This student now has his Ph.D. I was more convinced that I should continue the summer program. The next year, I, with the help of three undergraduate students obtained partial funding for the summer program. Students walked into the President's office. He asked the committee for support. The Committee funded my project partially. They awarded me $\$ 1,000$ for the student's salary. The result was positive again. The student and I benefited. Eventually for the summer

Received by the editor December 14, 1999.
of 1993 I got full support. By this time the award committee was convinced that student and faculty would benefit and funded $\$ 2,000$ for student plus free housing and $\$ 500$ for other research related expense. The outcome of the Summer 1994 was great. My student and I were mentioned for our contribution to the important paper of Lee Mosher a mathematics professor At Rutgers University. The student is finishing his Ph.D.

I began searching for external funding. I was rejected by NSF. I invited David Powers from Clarkson University and C. Knickerbocker from St.Lawrence University to join me. They accepted my offer. but NSF did fund our project. Then Clarkson and SUNY Potsdam applied for a grant from NSF, again. I invited two of our new mathematics faculty to join our program. Joel Foisy accepted my offer.

In 1997 we received funding from NSF to involve nine undergraduates and three student leaders in research and learning advanced materials in different areas of mathematics during the summer. In 1999 we received NSF funding to involve six students. The office of Faculty Scholarship and Grants provided funding for three more students. Each student would receive $\$ 2000$. SUNY Potsdam provided free housing for students. The Provost office, Clarkson University, and the office of Faculty Scholarship and Grants provided student leaders' salary. The Provost office helped us in many other ways: the students got their free lunch on Thursdays when we had speakers.

Our philosophy is:
(i) create interest in mathematics in students to the point that they choose to purse mathematics at graduate level,
(ii) help students develop mathematical maturity,
(iii)develop mathematical skills in students to the point that they can write a paper and explain topics in mathematics or mathematical results,
and
(iv) create a proper attitude in students towards mathematics.

We had over one hundred outstanding students applied for the program, each summer.

Our student participants were divided into three groups, being challenged according to their abilities. The students had a unique opportunity tolearn mathematics, expand their horizons, and to do research
in mathematics. We ll-known mathematicians visited our site, talked to our students, and presented very interesting talks suitable for bright undergraduates.

We arranged two dinner parties, one pizza party, a day tour of the city of Ottawa, and two trips to Mt. Marcy in the Adirondacks. Each Thursday prior to a distinguished speaker's address, a luncheon was provided to give students a chance to meet and talk with the speakers.

All of our students and student leaders lived in the dormitory rooms provided by SUNY Potsdam. This provided an opportunity to cook and eat together while discussing mathematics. Students had 24-hour access to the mathematics library, offices and computers which were provided by the Department of Mathematics at SUNY Potsdam.

Our 1997 students presented talks at an MAA Seaway Section meeting and posters at the AMS-MAA annual meeting in Baltimore MD. Our 1999 student are presenting posters and talks at national and local meeting. A group of our student won the best prize for presentation at Clarkson undergraduate Research Syposium, Summer 1999

Our survey of the students before they entered our program and after they finished it indicates the positive impact our program has had on them. They learned advanced topics in mathematics, they became engaged in research in mathematics; they benefited by meeting our visitors and listening to their very interesting talks on Thursdays. Our students indicated that they are now better prepared for graduate school, because of our program

The participants appreciated the contributions of the student leaders, especially the graduates. The graduate student leaders acted as role models, contributed to the productive and nurturing atmosphere of our program, and answered many questions our students had about graduate school. These student leaders also benefited from the program: they learned a considerable amount of mathematics, and they indicated that this program would serve as a launching pad for them towards their Ph.D.

Ten out of twelve student participants who attended our 1997 summer program are in graduate schools: Cornell University, University of Chicago, Yale University, University of California at Davis, and University of Kentucky at Lexington.

Our Summer 1999 participants are indicating that they will go to good graduate schools. We expect a result as good as 1997.

We continue to track the students' achievements and accomplishments. Our yearly newsletter with pictures of our REU students, highlights their achievements, and creates a contact between REU students and us. We have noticed that our student participants form a network
to communicate with each other and have REU reunions to discuss their research, to present their research in mathematical conferences, and to exchange other related information, all of which helps to cultivate a mathematical culture. SUNY Potsdam Mathematics Department is now a very exciting place to visit during the summer months, as well as school months.

Almost twelve years ago I dreamt of such a program. Nobody, including myself, thought that this dream could come through. We are now expanding and improving the program.
(i). We invited Louis Kaffmann, from the University of Illinois at Chicago to join us for the next summer(2000).
(ii). We have applied to NSF for funding for three K-12 teachers or perspective teachers to be included in our program during Summer of 2000.

Our program is transferring the summer months into productive months both for students and faculty, a time when students work in places like MacDonald; serving hamburger or cleaning floors. Now with this program they get engaged in researching and learning mathematics

The Summer of 2000 will be my third summer that I direct a Research Experience for Undergraduate students Program at SUNY Potsdam, jointly with Clarkson. We are looking forward to many good and productive summers ahead.

SUNY Potsdam, Potsdam NY, 13676
E-mail address: mahdavk@potsdam.edu

# Mathematics REU at Tulane An Overview 

Morris Kalka

The Tulane University REU program in mathematics started in the summer of 1995 with 3 student participants and 2 faculty, Morris Kalka and Dagang Yang. At that time it was funded for one summer only. The funding was renewed and expanded, so that for the summers of 96 and 97 the program ran with 8 students and 4 faculty, the new faculty members being Slawomir Kwasik and Jim Rogers. The program expanded again in the next grant extension. It is now funded through the summer of 2000 with 12 students and 5 faculty, with Frank Tipler joining the program in 98 . Our program runs for the 8 weeks ending on August 1. On that date Tulane University requires the student participants to vacate the dormitories, where they live during the program. Each student receives a stipend of $\$ 2500$ in addition to free housing in a dormitory. Our NSF grant is supplemented by some matching funds from Tulane University.

Each year, around Christmas, we send out flyers to a large number of institutions advertising our program. We include an application form. We also post an application form on our department web page. The deadline for application is usually at the end of February. We require a transcript (an unofficial one is fine) as well as two letters of reference. We have found that over three quarters of the applications come in electronically. Increasingly, letters of reference are submitted via email as well. The only paper that we see, in many cases, is the student's transcript.

It was the intention of the program to focus all of the research in the areas of differential geometry and topology. This has the effect of

Received by the editor September 1, 1999.
creating a research environment in which the different groups of students reinforce each other in their research. During the first year the program was entirely concentrated in Riemannian geometry. Topology was added in the second year and in 1998 general relativity was added. We view it as an important feature of our program that the research topics are related and there is lots of interaction among all of the student (and faculty) participants.

Our goals for the students are (i) that they become engaged in a research project that is on a level appropriate to their their background and preparation, (ii) that the problem afford them an opportunity to learn new mathematics and most importantly for us (iii) they gain some measure of mathematical independence.

We structure the program to achieve these goals. Students work in groups of two or three in close consultation with a faculty member. The students are assigned to a particular faculty member in the spring prior to the program. The students are paired based on similarity of background and sometimes indicated preference, i.e. two students will request that they work together. Each faculty member has a slightly different manner of working with the students, but all work toward the goal of helping the students achieve some measure of independence and gain some idea of what it is like to work on a research problem.

Our goals dictate some of the aspects of the way that we recruit students. Because we attempt to match students by background and interest, we tend to admit students in pairs or triples. By this we mean that if a students accepts an offer to participate we seek another students (or two) who is compatible with this student. We do our best in this way to assure that the students working together have similar background. We also make a concerted effort to have students of different levels of mathematical maturity. In any given year we have some groups of students who have seen little if any of the subject matter and some groups consisting of students who are quite advanced. What we require of all students, in the admission process, is a demonstrated interest in geometry and topology along with a willingness to work in a group environment. We also make an effort to have students from different sorts of institutions; each year we have students from small colleges, large state institutions and major private research universities.

In addition to working on research problems we feel that it is important for students to learn to communicate the mathematics that they are doing. Accordingly, we ask the students to give talks describing their results and we encourage the students to write up the results of their work. We find that the talks also make for a social occasion;
we normally try hold them at the end of the working day on a Friday. They are followed by a social time where refreshments are served. With regard to social activity, on occasion we have organized some social events. One year we organized a soccer match between our students and those of an REU at Louisiana State University, a short drive away. We find that in general the fact that our program is held in New Orleans, with its wealth of cultural assets make our efforts at organizing such activities unnecessary. Each year the students have found many of the things that the city and surrounding area have to offer by themselves. They routinely sample the jazz clubs for which New Orleans is well known. They also frequent some of the local restaurants. We take no credit for this. The fact that the students work together in groups and tend to socialize and explore the city together in groups makes for a great degree of comraderie and cohesion in our group. This is something of which we are quite proud and which we nurture.

Of course we feel that our program is successful. We measure success a number of different ways. One way, and this is the only quantitative measure that we know, is to look at published papers resulting from the work at the REU. We certainly have a number of these, and we post them on our webpage (http://www.math.tulane.edu). We also look at the fact that most of our participants choose to go on to graduate school in mathematics and are admitted to some of the best graduate mathematics programs in the country. A number have won NSF Graduate Fellowships and similar awards (we make an attempt to keep in touch with the students after they finish the program). While our program cannot take complete credit for such success, we do feel that we contribute to the students' motivation to continue in mathematics. Among the recipients of this year's NSF Graduate Fellowships are two of our past participants (Andrei Gnepp and Kathy Paur).

[^8]
# NSF Undergraduate Research Experiences in Parallel Numerical Linear Algebra 

Daniel Okunbor

## 1. Introduction

Our goal in this project is to take a group of undergraduate students each year, who have had limited or no opportunities for research, and provide them with a significant research experience that will stimulate their interest in mathematical and computational sciences to the point that they will react positively to the possibility of attending graduate school for further education and research in mathematics related areas. The project is selected to illustrate methods of research that will take the students progressively from the definition of the problem, through the statement of objectives, literature search, and the actual research, to the point of writing up their results and conclusions. This will give them an introduction to the job of a researcher and to basic research techniques.

In this project, students will be given an opportunity to do research in the area of numerical algorithms for problems arising from different areas in science and engineering. In particular, students will be involved in the design, analysis, and implementation of parallel numerical algorithms for the solution of large sparse linear systems, eigenvalue, least squares, optimization problems. This research will necessarily involve investigating the current literature in these areas as well as the parallel architectures that can be utilized for this project. In the instructional phase of the project, students will also learn mathematical concepts in numerical linear algebra, message passing interface (MPI) and C programming language with appropriate language extensions to provide

[^9]a means of implementing the parallel numerical algorithms on parallel processors.

## 2. 1997 Summer

The REU program started and ended as planned. All participants except Tracy Harris were available throughout the entire eight weeks. Dr. Haddock, Dean of College of Arts and Sciences congratulated the participants for their selection and assured them of a successful summer program on the first day. The first half of the program was devoted to lectures and assisting students in preparation for research work in parallel numerical linear algebra. The theme and the emphasis of this year program was "parallel sparse systems." Participants were taught sparse matrix concepts, parallel computing approaches and programming methodologies in C language. Parallel constructs for parallelization of matrix manipulations and numerical direct and iterative algorithms for the solution of large systems of linear equations on different platforms were described in depth. Laboratory sessions were conducted (with the help of the graduate assistant, Brian Lightfoot) to assist participants in tutorials on C programming language and in programming parallel systems such as Intel iPSC/860 and network of Sun Ultra sparcstations. The presentations of the invited guest speaker, Dr. Vipin Kumar of the University of Minnesota, provided useful research insights and state-of-the-art algorithms for parallel sparse matrix technology. Dr. Kumar's presentations (which took approximately 4 hours) and with several research lectures delivered by Drs. Okunbor and Ercal helped to prepare participants for research projects in the second half of the program. Several extra- curricular activities were also included in the program during this first half. Participants engaged in research activities during the second half. Every participant was assigned to one of the three research groups. Daily meetings with all groups helped to answer research questions that they have while monitoring the student progress. Every participant was involved in literature survey, design of algorithm for the assigned project, actual numerical experimentations, oral presentations (at the daily meetings) and writing a technical report. Participants individually had several discussion sessions with the PI and co- PI to address concerns that occurred during the research phase of the program. One of the participants, Mr. Assad Ebrahim stayed one month longer to draw on the expertise of the PI in the preparation of a research paper on parallel adaptive block nest dissection that was presented at the August 1997 MAA Fest held in Atlanta, Georgia. Every student within each research group had the opportunity of presenting his or her portion of the group research paper at the

November 1997 Argonne Symposium for Undergraduates in Science, Engineering and Mathematics held at Argonne National Laboratory. Evaluation forms were completed by participants at the end of the program. Six students have indicated that they would pursue graduate study as their future career goals quoting the research knowledge that they acquired during this short period as the motivating factor.

### 2.1. List of Participants.

1. Linda J. Alexander, Junior, Mathematics, South Dakota School of Mines and Technology, South Dakota (plans to go to graduate school in actuarial sciences)
2. Assad El-Karim Ebrahim, Junior, Mathematics, Swarthmore College, Pennsylvania (plans to go to graduate school in scientific computing)
3. Martin Eckles, Junior, Computer Science, Hendrix College, Arizona (plans to go to graduate science school in computer science)
4. Kimthu Tran Pham, Junior, Mathematics, Diablo Valley College, California plans to go to graduate school in applied mathematics or computer science)
5. Paul Michael Ruth, Junior, Computer Science and Mathematics, Baker University, Kansas (wants to go to graduate school to obtain doctoral degree in computer science)
6. Samuel J. Tratechaud, Junior, Computer Science, Southeast Missouri State University, Missouri (not decided)
7. Michael Wolfrom, Junior, Mathematics and Computer Science, Rutgers University Camden, New Jersey (plans to pursue a master's degree in computer science specializing in scientific computing)

### 2.2. List of Publications.

1. L. Alexander and K. Pham, "Parallel Implementations of Cholesky Decomposition", Argonne Symposium for Undergraduates, 1997.
2. A. Ebrahim, P. Ruth and M. Wolfrom, "Adaptive Blocking for a Parallel Nested Dissection", Argonne Symposium for Undergraduates, 1997.
3. M. Eckles and S. Tratechaud, "Parallel Preconditioned Conjugate Gradient Algorithm", Argonne Symposium for Undergraduates, 1997.

## 3. 1998 Summer

The first half of the program was devoted to lectures and assisting students in preparation for research work in parallel numerical linear algebra. The theme and the emphasis of this year program was "fast matrix multiplications." Participants were taught different basic linear algebra subroutines at different levels (1,2 and 3), eigenvalue/eigenvector problems, sparse matrix concepts, parallel computing approaches and programming methodologies in C language. Parallel constructs for the parallelization of matrix manipulations and numerical direct and iterative algorithms for the solution of large systems of linear equations on different platforms were described in depth. Laboratory sessions were conducted (with the help of the graduate assistant, Mark Allen) to assist participants in tutorials on C programming language and in programming parallel systems such as the Intel iPSC/860 and parallel distributed computer system based on Sun Ultras.

Several research lectures delivered by Drs. Okunbor and Ercal helped to prepare participants for research projects in the second half of the program. Several extra- curricular activities were also included in the program during this first half. Participants engaged in research activities during the second half. Every participant was assigned to one of the four research groups (see titles and description below). Daily meetings with all groups helped to answer research questions that they have while monitoring the their progress. Every participant was involved in literature survey, design of algorithm for the assigned project, actual numerical experimentations, oral presentations (at the daily meetings and two formal presentations) and writing a technical report. Participants individually had several discussion sessions with the PI and coPI to address concerns that occurred during the research phase of the program. Every student within each research group would have the opportunity of presenting his or her portion of the group research paper at the November 1998 Argonne Symposium for Undergraduates in Science, Engineering and Mathematics held at Argonne National Laboratory. Evaluation forms were completed by participants at the end of the program. All students have indicated that they would pursue graduate study as their future career goals quoting the research knowledge that they acquired during this short period as the motivating factor.

### 3.1. List of Participants.

1. Alphan Altinok, Senior, Computer Science, Southeast Missouri State University, Cape Girardeau, Missouri (plans to pursue a graduate study program in Computer Science at the same university with the hope of staying in academia)
2. Kirby C. Bohling, Junior, Computer Science and Mathematics, University of Nebraska-Omaha, Omaha, Nebraska (plans to work and pursue a graduate degree)
3. Michael J. Carlisle, Junior, Mathematics and Computer Science, New College of the University of South Florida, Clearwater, Florida (plans for an academic career in mathematical science or computer science)
4. Neil A. Chilson , Junior, Computer Science, Harding University, Searcy, Arkansas (plans for graduate school and ultimately engaging in ground breaking research in Computational Mathematics)
5. Lisa Grignon, Junior, Mathematics, Elon College , Charlotte, North Carolina (plans for graduate school and would like to engage in Mathematics or Computer Science research/instruction at the university level)
6. Adam B. Johnson, Senior, Computer Science and Mathematics, University of Arkansas at Little Rock, Little Rock, Arkansas (plan to attend graduate school to obtain at least a masters level degree in Computer Science)
7. Marvin Otto, Senior, Computer Science and Physics, Mid America Nazarene, Sierra Vista, Arizona (plans to pursue a doctoral degree in Computer Engineering and engage in research in robotics)
8. Anson Tripp, Junior, Mathematics and Computer Science, Rensselaer Polytechnic Institute, Troy, New York (plans to pursue a masters degree in Statistics)

### 3.2. Research Publications.

1. Alphan Altinok and Kirby C. Bohling, "Parallel Implementations of Fast Matrix Multiplication Algorithms",Argonne Symposium for Undergraduates, 1998.
2. Michael J. Carlisle and Neil A. Chilson, "A Direct Parallel Solver for Sparse Positive Definite Matrix Systems", Argonne Symposium for Undergraduates, 1998.
3. Lisa Grignon and Adam B. Johnson, "A Parallel Implementation of the All Pairs Shortest Path Algorithm", Argonne Symposium for Undergraduates, 1998.
4. Marvin Otto and Anson Tripp, "A Parallel Implementation of an Accelerated Bisection-Type Algorithm for Finding Eigenvalues", Argonne Symposium for Undergraduates, 1998.

## 4. 1999 Summer

This year REU summer program was held at the University of Maryland Eastern Shore. The university is located in the small, historic town of Princess Anne and it offers students an atmosphere conducive for study and young adults. The REU summer program started and ended successfully. Seven undergraduate students participated in the program. As usual, the summer program was for a duration of eight weeks. The first three weeks was devoted for teaching participants different concepts in numerical linear algebra and parallel processing. Participants were instructed on basic linear algebra subroutines, numerical methods for solving systems of linear equations and eigenvalue problems, parallel basic linear algebra subroutines, and parallel direct and iterative algorithms for sparse systems of linear equations. Different technical papers were reviewed to introduce participants to research in numerical algebra for the purposes of preparing them for the next phase of the program. Since C programming language was the computer language that the parallel distributed system at the University of Missouri-Rolla uses, tutorials were provided to assist students to learn C programming language. The Message-Passing Interface (MPI) was the inter-processor communication software used. Participants have to learn MPI with the assistance of responsible professors and graduate assistant.

During the second phase, participants were assigned to research groups of two and three. Each research team was provided with a research topic and was requested to develop a research proposal for the first week. With visits to the local library and the University of Maryland at College Park library, each team completed several technical research papers on the basis on which they wrote a research plan that was used during the remaining period of the summer program. Each research team conducted a very qualitative research work during the last four weeks of the program. The participants read journal papers and developed useful parallel algorithms. They wrote C programs for algorithms using MPI for inter-processor communication. All the C programs were implemented on the parallel distributed system located at the University of Missouri-Rolla. Each team wrote a technical research paper describing all the implemented algorithms and the numerical results. The research papers will be presented at the Argonne Symposium to be held in November.

All the participants expressed interest in graduate study in Mathematics or Computer Science at the completion of their undergraduate
study. They were all impressed with the way the program was conducted and some have indicated that they would like to serve as graduate assistants for the REU at the University of Maryland Eastern Shore in the future. The extra-curriculum activities including visits to Washington D.C., Baltimore and Ocean City were very enjoyable and participants were very pleased with it, especially as all expenses were paid for by the support from the National Science Foundation.

### 4.1. List of Participants.

1. Christina Colgan, Junior, Mathematics, University of Wyoming.
2. Anne Lippert, Junior, Mathematics, University of Chicago.
3. Matthew Blair, Junior, Mathematics, University of Michigan at Lansing.
4. Johnnel Parrish, Junior, Computer Science, University of Maryland Eastern Shore.
5. Andrea Height, Junior, Computer Science, University of Maryland Eastern Shore.
6. Ryan Gantner, Junior, Mathematics, University of Wisconsin at Madison.

### 4.2. Research Publications.

1. J. Matulja, R. Gantner, and A. Height, "Parallel Lanczos Algorithm for Symmetric Matrices", to be presented at Argonne Symposium for Undergraduates, 1999.
2. C. Colgan and A. Lippert, "Parallel Orthogonal Methods for Large Least Squares Problems", to be presented at Argonne Symposium for Undergraduates, 1998.
3. M. Blair and J. Parrish, "Parallel Polar Matrix Decomposition", to be presented at Argonne Symposium for Undergraduates, 1998.

Department of Mathematics and Computer Science, University of Maryland Eastern Shore, Princess Anne, MD 21853

E-mail address: dokunbor@mcs.umes.umd.edu

# The Duluth Undergraduate Research Program 

Joseph A. Gallian

## Introduction

With one exception, every year since 1977 I have run a ten-week undergraduate research program in mathematics at the University of Minnesota, Duluth. In this article I describe various aspects of my program such as funding, problem selection, recruitment, structure, followthrough, and results.

## Funding

Over the years primary support for my programs has come from the National Science Foundation and the National Security Agency. My department, college and the graduate school have also contributed substantially. In 2000, participants will receive a stipend of $\$ 1500$, a travel allowance of up to $\$ 500$ and a subsistence allowance of $\$ 1250$. The subsistence allowance covers the cost of living expenses and field trips for the summer.

## Problems

Obviously, the selection of appropriate problems is of fundamental importance to a successful research program. I search for problems that meet the following criteria:

- not much background reading is required;
- partial results are probable;
- recently posed;
- new results will likely be publishable.

[^10]Graph theory and combinatorics provide the source of most of my problems. Many of my problems involve group theory. I find problems by perusing journals, attending conferences and writing people. As a rule of thumb, I begin the summer with twice the number of problems as I have students. I occassionally have a student continue work begun in a previous program by someone else. Each student has his or her own problem but I encourage students to discuss their problem with fellow program participants. Matching students with problems is a critical task. The skill with which this is done is a major factor in the success of a program. Undergraduate students, even the most talented ones, have a tendency to become frustrated and want to give up too soon. Here I serve as a counselor and cheerleader, offering an idea, a reference or a pep talk. Of course, it sometimes happens - about half the time in fact - that a problem is inappropriate. Sometimes a problem is too easy or too hard; sometimes we discover it is already solved by someone else. In these cases I simply assign a new one. Often one student can get over a hump with the help of another student.

It is my practice to invite one or two former participants to return to act as research advisors to new participants. I choose someone with the personality and talent to interact well with others. David Witte first participated in my program as a student in 1979 and he returned as a research advisor for the next eight years. In several other years he was here as a long-term visitor. From 1985 through 1988, Douglas Jungreis was a student in two programs and a research advisor in two. David Moulton was a student in one program, a research advisor in eight and a long term visitor once. Dan Isaksen was a student in one program, a research advisor in three and a long-term visitor in 1997. Daniel Biss was a student in one program and an advisor in three. Samit Dasgupta was a student one summer and advisor in two. Tim Chow was a student in one program and a long-term visitor in three others. Much of the success of my programs is due to these people.

## Recruitment

Recruiting well-qualified students has not been difficult. Indeed, each year I am embarrassed by the fact that I must reject extraordinarily talented students. I'm sure that faculty writing letters of recommendation for some of these students must suspect my judgment.

Applicants are generated by a mailing of announcements to a large number of mathematics departments nationwide and a mailing of an
announcement and descriptive letter to many students who place in the top 100 of the Putnam Competition. Word of mouth advertising by former participants has resulted many outstanding applicants. Detailed information and an application form is available from my web site: www.d.umn.edu/~jgallian. My programs are quite small. The majority of them have involved only three or four students. Recent programs have had 6-8 undergraduates along with one or two research advisors and a several visitors who also contributed as advisors.

Selection of participants is based on letters of recommendation, response to questions on the application form, performance in high school mathematical competitions and the Putnam Competition, reputation of home school and coursework. The Putnam Competition has proved to be a good predictor but it is not foolproof. I have had some students who did not place in the top 100 outperform some who finished in the top 5. Several students who did outstanding work in my program did not finish in the top 500 of the Putnam Competition. I have had several excellent students who have not participated in the Putnam Competition. Desire to succeed, enthusiasm and willingness to work are as important as raw talent. Personality is an important consideration as well. I try to select people who would be fun to spend the summer with.

## Structure

My programs are loosely structured. Each student is given his or her own problem together with an article or two as resource material. Each week the participants give a talk on their progress during the previous week to the group. Occassionally we have a guest speaker present a colloquium. We have had speakers from MIT, Stanford, Chicago, UCLA, Pennsylvania, AT\&T, Harvard, Minnesota, Oklahoma State, Michigan, Washington, Berkeley, Macalester, Simon Fraser, Washington, SUNY Stony Brook, Cornell, Oberlin and the National Security Agency. Typically, we have lunch as a group a few times a week. This gives me an opportunity to inquire about progress or difficulties. I meet with students individually upon request and when I feel such a meeting might be beneficial.

An important structural feature of my programs is the housing arrangements. Participants share adjacent three-bedroom apartments on campus. Rather than meet as a group to work in a formal setting, they simply work at home and interact with each other as a natural consequence of living together. It is quite common for participants to
receive ideas from each other. The research advisors and visitors share the apartments with participants and are available as a resource and sounding board on a continual basis. A few times a week I drop by the apartments to see how things are going.
"Field trips" are a component of the program. It is important that the students enjoy their summer. Together, we go kayaking and whitewater rafting, we visit the beautiful parks in the area, we bike, we play softball, and we picnic and walk along the shore of Lake Superior. The field trips are subsidized by the grant. Watching the morning sunrise over Lake Superior has become a program tradition. On weekends, the participants have access to two university vehicles at no cost. This makes it convenient for them to see movies, shop and eat out. Once a week we have lunch at a different restaurant.

In each of the past four years students from my REU program and the Michigan Tech REU program have held a two-day conference in which students from both programs present talks on their research. The site of the conference has alternated between Duluth and Houghton. The conferences have climaxed with a field trip or picnic.

By the eighth week of the program the students usually are writing up their work. Papers are written in a style suitable for submission to a research journal. The research advisors and I read the manuscripts for content and style and I decide whether they should be submitted for publication and where.

## Followthrough

Except for finding a sufficient number of appropriate research problems, the followthrough on manuscript preparation is my most difficult job. When the students leave Duluth I often have, at best, a first draft of their work. It typically takes many letters and phone conversations before the manuscripts are ready to be submitted to journals. Then, many months later, there are the inevitable referees' reports recommending revisions, necessitating another round of letters, phone calls and typing. By that time, the students are busy with other things and are not always eager to finish up. I have had a few students who did publishable work but did not have it published because of their unwillingness to carry out revisions demanded by editors. In some cases I have done the revisions myself but I generally refrain from this.

Ideally, publication of the work done in an undergraduate research program is a beginning rather than an end in itself. Many participants from my program have continued to publish as an undergraduate or graduate student.

In most instances I have been able to persuade the students' home school to provide support for the student to present his or her work at the annual meeting of the American Mathematical Society. This has proved to be a valuable experience for the students. They attend talks, meet people, have people ask about their work.

Followthrough also includes writing letters of recommendation for fellowships and admission to graduate school. In some cases I have written letters for former participants seeking employment after finishing the Ph. D. degree.

## Results

Through 1999, my program has had a total of 89 students with several participating more than once. To date the program has produced 81 papers that have been accepted for publication and another 20 or so are currently under review. These papers have appeared in mainstream journals such as: J. fur die reine und angewandte Mathematik (Crelle's Journal), J. Algebra, J. Combinatorial Theory Series A, J. Combinatorial Theory Series B, Discrete Math., Pacific J. Math., J. Number Theory, European J. Combinatorics, J. Graph Theory, and Graphs and Combinatorics. They include papers on graph theory, combinatorics, group theory, ring theory, field theory, and number theory. A complete program bibliography is available at www.d.umn.edu/~jgallian.

In many cases participation in my program has proved to be a benefit to those students who desire to enter graduate school. Sixty-nine of the 77 participants who have received their Bachelor's degrees have gone on to graduate school. The graduate schools chosen are: Harvard (14), MIT (12), Chicago (10), Berkeley (9), Princeton (4), Cornell (4), Michigan (3), Cambridge (3), Rutgers (2), UCLA, Texas, Vanderbilt, Brown, Stanford, UC San Diego, Penn, and Washington University. Of these sixty-nine, 55 have received a Graduate Fellowship (NSF, Hertz, NDSEG, AT\&T, Marshall, DoD, ONR). Thirty-five participants now have the Ph. D. degree.

## Conclusion

It has been my experience that carefully selected undergraduates are capable of doing professional level research in mathematics. These students have the ability, the desire and the time. They don't know how to start or how to finish. To begin, they need problems and guidance. To end, they need assistance with manuscript preparation and the publication process. In between, they need encouragement and reassurance. It has been my privilege to fulfill this role and to make a contribution to their development as research mathematicians.

University of Minnesota, Duluth, Duluth, MN 55812<br>E-mail address: jgallian@d.umn.edu

# The Summer Institute in Mathematics for Undergraduates (SIMU) at the <br> University of Puerto Rico - Humacao 

Herbert A. Medina and Ivelisse Rubio

## 1. Introduction

The Summer Institute in Mathematics for Undergraduates (SIMU) at the University of Puerto Rico - Humacao is a six-week research program in the mathematical sciences. Professors Ivelisse Rubio and Pablo V. Negrón from the University of Puerto Rico - Humacao, and Herbert A. Medina from Loyola Marymount University served as CoDirectors of the program in 1998 and 1999. Twenty-seven students participated in SIMU in 1998 and twenty-four participated in 1999.

The program is designed for talented Chicano/Latino and Native Americans ${ }^{1}$ undergraduates who want to engage in undergraduate mathematics research. The objectives of SIMU are:

1. to provide SIMU participants a rich and intensive research experience in the mathematical sciences-an experience that has proven successful in getting Chicano/Latino and Native American undergraduates to pursue graduate studies in the mathematical sciences;
2. to familiarize SIMU students with research protocols and techniques, with collaboration between peers, and with utilizing faculty as effective resources while conducting research-skills that

[^11]will help them succeed in their undergraduate and graduate education and their research careers;
3. to create an undergraduate research community that stimulates students to reach their fullest potential.
4. to offer SIMU participants workshops that will help them to develop skills and techniques needed in research careers in the mathematical sciences;
5. to build a network of researchers and peers that, through mentoring and collaboration, will help SIMU participants to continue their research activity and excel in their graduate education;
6. to introduce SIMU students to successful Chicano/Latino and Native American researchers and graduate students so as to encourage and motivate them to pursue careers in mathematics;
7. to provide SIMU participants the opportunity to attend undergraduate research forums where they will present the research they completed during SIMU;
8. to offer SIMU participants workshops that will teach them skills and techniques that will maximize their likelihood of admission to graduate programs best suited to their needs as well as their likelihood of securing financial support for such programs;
9. to enhance the overall academic portfolio of SIMU students by providing them the opportunity to work with and meet leading mathematicians so that their future applications to graduate school and fellowships are strengthened;
10. to monitor the educational progress and research activity of SIMU participants for at least five years after their participation in the institute, such monitoring being a measure of the program's success.
These objectives will contribute significantly towards the goal of SIMU: to increase the number of Chicanos/Latinos and Native Americans earning graduate degrees and pursuing careers in the mathematical sciences.

The reason for working to increase the number of Chicanos/Latinos and Native Americans pursuing research careers in the mathematical sciences can be summarized as follows. Historically, Chicanos/Latinos and Native Americans have been severely underrepresented in mathematics and science. This pattern continues today. For example, between 1989 and 1996, only $3.35 \%$ of all bachelor's degrees in the mathematical sciences were awarded to Hispanics and Native Americans in the U.S. and U.S. territories. The percentages for master's degrees during the same period is $1.78 \%$; and only $1.15 \%$ of all doctorate degrees in the mathematical sciences between 1988 and 1997 were awarded
to Hispanics/Latinos and Native Americans [3, 4]. These numbers are alarmingly low when one considers that these two ethnic groups account for $12.2 \%$ of the current U.S. population and that the U.S. Census Bureau projects that they will account for $18.6 \%$ of the U.S. population in 2025 [5, 6]. The leading position in mathematics and science that the U.S. occupies will be maintained only if all groups in our society actively participate in these fields. Indeed, if, as predicted by the U.S. Census Bureau, Latinos and Native Americans make up $18.6 \%$ of the U.S. population in the year 2025, and continue to account for only $1.15 \%$ of all doctorates in mathematics, then the U.S. will be leaving untapped a resource of millions of people from which to produce professional mathematicians; this puts the U.S. at a huge disadvantage to countries who tap their population effectively for mathematicians and scientists. One can ask the question, can a country that essentially does not use one fifth of its population to produce mathematicians continue to be a world leader in this field? We believe it is not possible, and SIMU's goal is to address this important issue.

## 2. Funding Information

The National Security Agency (NSA) provided the largest grants for the 1998 and 1999 SIMU's. The University of Puerto Rico has provided funds for SIMU in 1998, 1999 and will do so again in 2000. The National Science Foundation (NSF) provided significant funding for SIMU in 1999. The Alliance for Minority Participation (AMP) has funded the stipends and some travel to conferences for Puerto Rican students in both the 1998 and 1999 SIMU's. The National Aeronautics and Space Administration (NASA), through the Puerto Rico Space Grant program also provided funding for the 1998 and 1999 SIMU's. Finally, the Society for Advancement of Chicanos and Native Americans in Science (SACNAS) has paid expenses for three colloquium speakers during the past two years, and, after selecting them through a competitive process, has covered travel expenses of all SIMU students attending the 1998 and 1999 SACNAS Conferences.

## 3. Program Philosophy and Structure

The Co-Director's undergraduate research philosophy can be summarized as follows.

1. Create an intense and collaborative academic and intellectual environment by ensuring that students understand the high quality
and quantity of work that is expected of them and become selfmotivated to try to meet these standards and perhaps set higher ones for themselves;
2. Create a structure that maximizes faculty-to-student mentoring and support in the early stages of the program and that makes student-to-student interaction the primary dynamic while students are conducting mathematical research under the direction of their faculty mentor;
3. Immerse students in an intensive training that will give them all the necessary mathematical and computational knowledge and tools so that they successfully can engage in quality research; and
4. Engage students in challenging, interesting and accessible mathematical topics and research projects that results in a rich research experience.

SIMU is structured so that in addition to the Co-Directors, each summer two faculty direct the research of students participating in the program. The selection of these faculty, called "seminar leaders," is done by the Co-Directors on a year-to-year basis so as to allow a variety of mathematical fields to be the research areas of the program. The seminar leaders are mathematicians who are experts in their field of research, who have demonstrated excellence in and are committed to undergraduate education, and who share the vision and philosophy of SIMU. Each year, the seminar leaders are assisted by four seminar associates, two for each seminar. The organizers, in consultation with the seminar leaders, recruit the seminar associates. In the future, if possible, the seminar associates will come from the pool of former SIMU students who are in graduate programs in the mathematical sciences.

SIMU is designed so that the students are immersed in their research topic from the first day. Each day during the first three weeks, there are four different organized day activities and study groups at night which create an intense working environment that sets the pace and creates the conditions for the research that the students conduct during the institute. The pre-research seminar, working sessions, computational laboratory, and intense academic environment give students sufficient background so that after three weeks they can conduct an undergraduate-research project. That is, by the end of the third week, students are "undergraduate experts" in the mathematical field in which they will conduct research.
3.1. Pre-Research Seminars and Working Sessions. The preresearch seminar is a three-week intense training that provides the necessary background and prepares students to do quality research in a field of the mathematical sciences that is accessible to undergraduates. Each student participates in one of two pre-research seminars.

The pre-research seminars meet daily Monday - Thursday during the first three weeks of the institute for two seventy-five minute, morning sessions. The first daily session is devoted to the presentation of new material in an interactive lecture setting. The content of the seminars is designed to familiarize students with the fundamental concepts of the mathematical topic. During the second daily session, students work in groups on problems designed by the seminar leaders. The problems are designed to promote and enhance the material covered in the interactive lecture and to familiarize students with the different research projects. During these sessions, the seminar leader and associates float and help students to tackle the problems.

During the afternoons of this pre-research phase, students have a ninety-minute study session, where they are assisted by the seminar leader and associates. Also, seminar leaders and associates hold consultation sessions in the evening in the student residences. These sessions allow students to engage in at least three more hours of daily structured work. This proved in the 1998 and 1999 SIMU's to be a key in creating and sustaining a supportive and intensive academic and research environment. During all of these structured study sessions, students are encouraged to use their peers as the primary resource for answering questions.
3.2. Computational Laboratory. A computational laboratory meets Monday - Thursday in the afternoon for ninety minutes. It is dedicated to activities that supplement the mathematics from the preresearch seminar and prepare students to tackle the research project. The seminar leaders and associates design laboratories that give students the opportunity to use packages like Maple, Mathematica, Matlab, and Splus to solve problems, develop and test conjectures, and learn the value of the computer as a tool in the mathematical sciences. By the end of the third week of the institute, students are familiar with software packages and computational techniques that expose them further to the tools in use by mathematicians who work in the field.

After the first three weeks, students are ready to devote all their time to work in groups on their research projects.
3.3. Research Projects. By the end of the second week, the seminar leader has distributed a description of research projects. Students
begin to do preliminary reading and literature searches on the projects immediately. By the end of the third week, with the aid of their seminar leader, students have selected a research project, organized themselves and designed a plan of attack to tackle the project. At this juncture, each team of students makes a presentation to SIMU participants in their seminar in which they give an overview of their research project and the methods and techniques that they hope to use to tackle it. These presentations mark the end of the pre-research phase of SIMU.

For the remainder of the program, the seminar leader and associates are available to meet with students during the day and night if necessary. Each research team gives a daily progress report to the seminar leader or one of the associates. In addition, the Co-Directors are available during the day and night if students want to discuss their project with someone other than the seminar leader and research associates.

At the end of the program, each team of students makes an oral presentation to all SIMU students and faculty at an end-of-program symposium. During the 1998 and 1999 SIMU's, faculty from the UPR - Humacao, other UPR campuses, and mathematicians from the NSA and NSF have been present for the end-of-program symposium. Each research team also writes a technical report describing the results of their research. These technical reports are published each year by SIMU [1, 2].

The research topics during the first two years of SIMU were Gröbner bases, computational number theory, and probability and statistics. Computational algebra, particle methods in fluid dynamics, Gröbner bases, coding theory, and wavelets will be among the research topics during the next three years.

### 3.4. Other SIMU Activities: Colloquia, Workshops and

 Recreational Activities. Speakers from across the U. S. and Puerto Rico are invited to SIMU to give a colloquium talk. The aim is not only to give students the opportunity to hear a talk on current research, but also to provide students with another role model and future mentor in the mathematical sciences. Female and male colloquium speakers are chosen to represent a broad spectrum of mathematical disciplines as well as ethnic and cultural backgrounds, including those similar to the students' backgrounds.There are five colloquium speakers, one each during the first five weeks of the institute. Colloquium speakers arrive in Humacao on Thursday and stay until Sunday. Each speaker gives a one-hour colloquium talk on Friday afternoon. In addition to giving their talk, colloquium speakers are asked to attend the institute's seminars and
laboratories, to interact with students on an informal basis, to attend a recreational/cultural outing on Saturday, and to discuss graduate programs at their institution or career options at their laboratory, government agency or corporation.

SIMU has a collection of workshops designed to assist students in the pursuit of a graduate education and the development of skills that are important to mathematicians.

Dr. Colette Patt, Director of Diversity Programs in the Physical Sciences at the University of California, Berkeley, has given workshops to inform students about the graduate education options available to them, and the funding possibilities available for attending graduate school. Dr. Patt addresses questions/issues such as the significant differences between a master's and a doctoral program, the funding opportunities available for most graduate programs, and the benefits of obtaining a graduate degree. In addition to this basic information, Dr. Patt also presents successful techniques for applying to graduate school. She discusses the elements that constitute a good statement of purpose, the types of professors from whom one should seek letters of recommendation, and successful techniques for addressing not-sostellar semesters. In addition to the workshop, Dr. Patt meets individually with students to develop a blueprint for applying to graduate school. For example, each student receives individual academic counseling to help him/her develop a list of universities to which $\mathrm{s} / \mathrm{he}$ hopes to apply, a list of fellowships to which s/he should apply, a list of faculty whom s/he will ask for letters of recommendation, and a cover letter to accompany requests for letters of recommendation.

The other three workshops are led by the Co-Directors and are devoted to the development of skills that are important to every mathematician. The first is devoted to learning LaTeX, the typesetting program most widely used by mathematicians. The second workshop familiarizes students with good practices in preparing and delivering a mathematics oral presentation. The third instructs students on successful techniques in preparing a mathematics poster for a conference.

The working environment in SIMU is very intense. During the first three weeks from Monday to Thursday students work on a mathematical topic in a structured setting for more than eight hours per day and many more hours on their own. During the second three weeks, students work almost continuously on their project. Creating and sustaining an academic and research environment in which students are challenged, mentored, and supported by faculty and peers is one of the biggest challenges and one of the key components of undergraduate
summer institutes. To sustain this environment, it is important that students also engage in non-academic activities.

In the afternoons, students have time to go to the university gym and to the swimming pool. SIMU organizes a couple of athletic competitions during the first and fifth weeks of the institute. Also, on Saturdays, SIMU organizes outings to places like historic Old San Juan, the capital of Puerto Rico; El Yunque, the only tropical rain forest among the National Parks of the U.S.; and the Arecibo Observatory.

## 4. Results of 1998 and 1999 SIMU's

Almost all SIMU students have communicated orally and in writing in post-program questionnaires that the program was an excellent experience. Here are some quotes from SIMU students.

- SIMU helped to convince me that one of the things to which I want to devote my life is to mathematical research. It was an excellent experience.
Omar Colón-Reyes, University of Puerto Rico - Humacao
- SIMU's main purpose was to encourage us to work hard and to our fullest potential, however I feel that it did a lot more than that. It taught me to motivate myself. I encourage others to participate in the program.
Carina Nieves, Kean University
- What I love the most about SIMU is the attention that we get from the faculty. The level of interaction between students, TA's and professors is the best! SIMU has made me come to the conclusion that I can, and will, succeed in graduate studies in mathematics.
Omar Zuñiga, University of California, Riverside.
Most SIMU students continue to be members of the academic and research communities built during their time in Humacao. Indeed, several 1998 SIMU students were in contact with each other during their graduate school application process. A student from each of the past two SIMU's has created an Internet club for the SIMU students from that year.

To sum, the Co-Directors witnessed the transformations that students experienced in the 1998 and 1999 SIMU when they realized their fascination with and passion for the mathematics that they were learning and the research they were conducting. These types of transformations are often the motivating factor in inspiring a student to pursue a graduate degree, and the 1998 and 1999 SIMU were successful in achieving them.
4.1. Quantitative Results from the 1998 SIMU. Twentyseven students, sixteen men and eleven women, from sixteen universities participated in the 1998 SIMU. The fifteen students who participated in the Number Theory Seminar under the direction of Professor Carlos Moreno, City University of New York, worked on individual research projects; the twelve students who participated in the Gröbner basis seminar under the direction of Professor John B. Little, College of the Holy Cross, worked on three group research projects. Each student or group of students gave a talk at the end-of-program colloquium series and produced a technical report published by SIMU [1]. Twenty-five of the twenty-seven students presented posters on their research at the 1998 SACNAS Conference in Washington, D.C. in October, 1998. Thirteen SIMU students presented nine posters at a session sponsored by the Mathematics Association of America (MAA) at the Joint Mathematics Meetings in San Antonio in January 1999. At both conferences, one of the SIMU posters won an award. Many other students presented posters and gave talks based on their SIMU research at other conferences (e.g., Southern California Conference on Undergraduate Research, Nebraska Conference for Undergraduate Women in Mathematics, Junior Technical Meeting in Puerto Rico) and their universities. Over seventy percent of these students had not participated in undergraduate research prior to SIMU.

The success of these students in continuing their graduate education is also impressive. The Co-Directors gathered data on these students in a post-SIMU May 1999 questionnaire. A summary of the information gathered therein is as follows.

1. The Co-Directors received twenty-three questionnaires from a possible twenty-seven. (The Co-Directors are trying to get the other four students to return their questionnaires.)
2. Of the fourteen respondents who graduated on or before Spring 1999, ten ( $71 \%$ ) applied and were accepted to graduate programs. One of these students has postponed graduate school for a year, and the rest will be enrolled in a graduate program in Fall 1999. ${ }^{2}$
3. One of these students won a Ford Foundation Fellowship.
4. All respondents who graduated in Spring 1999, have applied or said that they would apply to graduate school in the future.
5. All of the students who began graduate school in Fall 1999 stated that SIMU helped them, by motivating and providing them with

[^12]important information, in their application process to graduate school.
4.2. Quantitative Results from the 1999 SIMU. Twentyfour students, twelve men and twelve women, from eighteen universities participated in the 1999 SIMU. They worked on eight research projects in groups of three. Four projects in Gröbner basis were directed by Professor John B. Little, and four projects in probability and statistics were directed by Professor Rudy Guerra, Southern Methodist University. All eight groups gave talks at the end-of-program symposium and wrote technical reports which have be published by SIMU [2]. Twenty-two students will present eight posters on their SIMU research at the SACNAS Conference in Portland, OR in October, 1999. The Co-Directors expect that at least a dozen students will present a poster in the MAA poster session at the Joint Mathematics Meetings in Washington, D.C. in January 2000.

Information gathered by the Co-Directors in the end-of-program student questionnaire shows that seventeen of the twenty-four 1999 SIMU students had not worked on mathematical research prior to SIMU; that all students plan to continue to engage in research after SIMU; and that SIMU has either "increased" or "increased significantly" the desire of nineteen students to pursue a graduate degree in mathematics or science.

## 5. The Future of SIMU

Professors Rubio and Medina will serve as SIMU Co-Directors in the future. They envision running SIMU for several years with twentyfour students each year. They will continue to apply yearly for funding to the NSA, and have applied for funding to NSF to establish SIMU as a Research Experience for Undergraduates (REU) site for 2000-2002.

The seminar leaders and research topics during the next few years include Reinhard Laubenbacher, New Mexico State University, Computaional Algebra; Ricardo Cortez, Tulane University, Particle Methods in Fluid Dynamics; John B. Little, College of the Holy Cross, Gröbner bases and Coding theory; and Herbert A. Medina, Orthogonal expansions and wavelets.

In addition to planning the next few SIMU's, the Co-Directors are building partnerships with faculty at schools that have graduate programs to facilitate admissions of SIMU students to these programs and to strengthen SIMU.

The Mathematics Department of Cornell University has included SIMU as part of its NSF Vertical Integration of Research and Education
in the Mathematical Sciences (VIGRE) grant proposal. If the proposal is funded, it would provide funding for 1. a Cornell mathematics faculty to travel to SIMU to give a colloquium talk every year; 2. a Cornell graduate student to serve as a SIMU seminar associate every year; 3. one year of Cornell support and two years of VIGRE support for SIMU students admitted to that department's graduate program. Cornell also would waive the application fee for SIMU students applying to its graduate mathematics program.

Ricardo Cortez has included SIMU as part of his NSF CAREER grant proposal. Dr. Cortez will serve as a SIMU Seminar Leader in 2002, and if his CAREER proposal is funded, SIMU students would be able to participate in research at Tulane; and Ricardo Cortez's salary during SIMU 2002 would come from his CAREER grant. Both of these activities would facilitate the application and admission process of students to the graduate mathematics program at Tulane.

Professor Laubenbacher is writing a proposal to the Alfred P. Sloan Foundation requesting graduate fellowships for SIMU students. If this proposal is funded, it will provide fellowships for SIMU students accepted to the graduate program in mathematics at NMSU. Two 1998 SIMU students and a 1999 SIMU seminar associate are currently enrolled in the mathematics graduate program at NMSU and the availability of fellowships would strengthen further the SIMU-to-NMSU pipeline.

## 6. Conclusion

The goal of SIMU is to increase the number of Latinos and Native Americans earning graduate degrees and pursuing careers in the mathematical sciences. The 1998 and 1999 SIMU's have demonstrated that the model of engaging students in quality undergraduate mathematics research and continuing to mentor them after the program is a successful one for a program with this goal. The Co-Directors plan to continue to fine tune the model so that within a few years, SIMU will have made an impact on the number of Latino and Native Americans earning graduate degrees in the mathematical sciences.

## References

[1] Summer Institute in Mathematics for Undergraduates (SIMU), 1998 Student Technical Reports, University of Puerto Rico - Humacao, 1998.
[2] Summer Institute in Mathematics for Undergraduates (SIMU), 1999 Student Technical Reports, University of Puerto Rico - Humacao, 1999.
[3] Susan T. Hill, Science and Engineering Degrees, by Race/Ethnicity of Recipients: 1989-1996, National Science Foundation, Division of Science Resources Studies, , NSF 99-332, Arlington, VA, 1999.
[4] Susan T. Hill, Science and Engineering Doctorate Awards 1997, National Science Foundation, Division of Science Resources Studies, NSF 99-323, Arlington, VA, 1999.
[5] United States Census Bureau, http://www.census.gov/population/estimates/nation/intfile3-1.txt.
[6] United States Census Bureau, http://www.census.gov/population/projections/nation/nsrh/nprh1530.txt.

Herbert A. Medina, Loyola Marymount University, Iveslisse Rubio, University of Puerto Rico - Humacao

E-mail address: hmedina@popmail.lmu.edu; ive@cuhwww.upr.clu.edu

# REU at Tennessee: ONE-on-ONE Style 

K. Renee Fister and Suzanne Lenhart

The Research Experience for Undergraduates (REU) program funded by the National Science Foundation has provided many opportunities for dedicated young men and women of various collegiate backgrounds to expand their mathematical horizons. From the standpoint of a student in the program, the giant step into research can be overwhelming. Through the REU program at the University of Tennessee in Knoxville, the students learn that the professors and scientists are also delving into new material for possibly the first time. A participant's confidence is enhanced when he/she envisions a technique to begin a problem that another distinguished researcher may have not visualized as a possibility. Obviously, it is a joint learning procedure with peaks and valleys as with any other research.

Our REU program started in 1987 and continues to the present. Larry Husch started the program and Suzanne Lenhart has been the director since 1990. Renee Fister, a former alumni of our REU program, was the graduate student assistant for the program fron 1991-1996.

## OUR APPROACH

Our program is based on the theme - "one advisor with one student." Each student in our program is working with an advisor on a research problem. We believe that this "one-on-one" approach simulates most of the research experiences in mathematics and it definitely mimics the experience of working on a PhD dissertation. Although some mathematics is investigated in groups of four or five researchers, usually tasks are broken down into subtasks that one or two researchers work on. Our program usually has 10 to 12 students and 10 to 12 advisors.

Received by the editor September 3, 1999.

This "one-on-one" approach brings up another question - How can the program pay for so many research advisors to be involved? Our funding from the National Science Foundation is supplemented by funding from the university's Science Alliance program. This additional funding enables us to pay for all these advisors. Consistently, we have a strong group of faculty members who want to be involved in our program.

## STRUCTURE

Each summer, two of our student participants work with scientists at Oak Ridge National Laboratories. This laboratory is nearby Knoxville and those students go to the laboratory to work on Tuesday and Thursdays. Lenhart is a part-time employee of the laboratory and is able to coordinate this research situation for the students. This interaction with Oak Ridge National Laboratory gives two students an indepth experience in this laboratory setting, but the others also learn about the laboratory. As a group, we visit the laboratory once each summer and meet some of the researchers there. At this visit, we discuss the differences between working at a government laboratory and working at a college or university. This unique lab connection feature is a draw for our program because the students have the opportunity to work with researchers not directly associated with academia and to work on sophisticated computer equipment.

Another unique feature of our program is the strong participation of our Math Ecology group. We usually have two projects each summer in the area of math ecology/biology.

The student group meets together on Monday, Wednesday, and Friday mornings. There are two short courses each summer - one on a "pure" math topic and one on an "applied" math topic. Each short course consists of ten lectures. These courses give the students a chance to interact with each other and to learn something new and sometimes unusual. Some of the more unusual course topics were "Circle Packing" and "Cryptography." Courses like "Boundary Elements Methods" and "Chaos" have been taught by researchers from Oak Ridge National Laboratory. There are no tests or grades in these courses, but the students are encouraged to work together on some problems and present solutions to the group. Each faculty member involved as an advisor presents a seminar to the group as an introduction to his/her research
area. One group meeting is devoted to discussing graduate school opportunities, jobs, the structure of academic departments and professorial ranks, and mathematics organizations. We want to educate about the spectrum of the mathematics community.

However, the main emphasis is working on their research projects. Each student meets with his/her advisor at least three times a week. The researchers involved have created excellent opportunities for students to work on diverse applications varying from math ecology to differential equations to algebraic structures.

## SUMMARIES OF SAMPLE PROJECTS

The following summaries were written by the students involved.

1. The central question of the project of Trevis Litherland [1] under the direction of Professor Philip Schaefer concerned the sign of the classical solution u of the boundary value problem

$$
\Delta^{2} u+4 k^{4} u=f(x) \text { in } D \quad u=\Delta u=0 \text { on } \partial D, k>0
$$

where $D$ is a bounded domain in $\Re^{n}$. Given $f(x)<0$, it was asked whether u must also be nonpositive on $D$. For the $n=1$ case, the ordinary differential equation case, it was shown via a Green's function that for $0<k \leq \frac{\pi}{2}$, u was indeed nonpositive. However, while it was known that $u$ could not always be positive in $D$, a counterexample solution was produced for $k=\frac{3 \pi}{2}$. Finally, it was noted that the Green's function for the given problem, exhibited the unexpected symmetry $g(x, s)=g(1-s, 1-x)$.
2. Lora Ballinger worked with Professor Louis Gross to develop a simple spatially explicit model for fish movement in the Everglades. In the Everglades, fish movement (over relatively large distances) is governed by the hydrology of the region throughout the year, and is density-driven during the rewetting season. Under that assumption, a grid-based model was developed incorporating constant fraction birth and death terms, periodic drydown and rewetting and rules for movement. A major focus in the development was determining appropriate functions for the water-dependency and the density -dependency fo the motion. Reasonable values for the parameters were found, and some sensitivity analysis was done. The model was constructed, in part, to aid in the development of a more complex model for fish movement which will be a part of the Across-Trophic-Level System Simulation project.
3. Vrej Zarakian [2], under the direction of Dr. Leonard Gray of Oak Ridge National Laboratory, examined error indicators for the boundary element method. He implemented (in FORTRAN code) and
tested three such indicators; $\frac{d E}{d T}$ (the tangential derivative of the boundary integral equation), $\frac{d E}{d N}$ (the normal derivative of the boundary integral equation or hypersingular equation), and $\frac{d E}{d T(n)}$ ( a variation on $\frac{d E}{d t}$ in which the required boundary integrals are replaced by contour integrals over a subsection of the boundary). He found that both $\frac{d E}{d T}$ and $\frac{d E}{d N}$ perform adequately - in a qualitative sense (with $\frac{d E}{d T}$ showing superior performance in Neumann problems, $\frac{d E}{d N}$ in Dirichlet problems, and the two demonstrating equally good performance in problems with mixed boundary conditions). He discovered that $\frac{d E}{d T(n)}$ is an unsatisfactory error indicator. He was unsuccessful in establishing a quantitative relationship between the two reasonable error indicators and the actual error.
4. Becky Cantonwine and Dr. Conrad Plaut studied semigroups of subsets of finite groups. They began the project by looking at all such semigroups, but quickly narrowed their area of study to those semigroups that were totally ordered by inclusion. Eventually they imposed even more restrictions on the structures of the semigroups in question. The final set of requirements was that the semigroup was totally ordered by inclusion, each subset in the semigroup was symmetric, the intersection of the subsets was exactly the set containing the identity of the group from which the subsets were formed, and that the union of the subsets was exactly that group. After naming the semigroups $\Sigma(G)$, they proved more than a dozen theorems and corollaries about the semigroups in this set. For example, an important discovery was that every semigroup in $\Sigma(G)$ is abelian, even if the group $G$ is not.

## RECRUITMENT AND ORGANIZATION

We send advertisement flyers to about 500 colleges, hand out flyers at the annual math meetings and have information on the department home page. Our application process requires an information form, a letter of interest, two letters of recommendation, and a transcript. Students are asked to express preferences among a list of possible research areas. We usually receive about 90 applications. Each advisor chooses the students with whom he/she wants to work. We have also visited some nearby campuses to recruit students. Over the years, our student participants have been about 50 percent female and 50 percent males. We try to recruit from a variety of types of undergraduate institutions, including small colleges to large universities

Besides our summer math program, there are usually several other science summer student research programs funded each summer by the

UT Science Alliance. There are usually several activities scheduled that involve all of the science students together.

Travel expenses to the program are reimbursed and we arrange for the housing. Upon arrival, the students are housed within an attractive, furnished, two-bedroom apartment that has a small kitchen and living accomodations for four students. To eleviate undue housing costs, participants in other science summer programs room with each other. It provides a conducive atmosphere to talk about mathematics as well as its relationship to other disciplines. The first evening the students meet their designated research partner/professor at an informal dinner party. Then the following day they begin the "real thing".

Each student is contacted during the following year and a newsletter is sent around with details about student plans and activities.

## FINALE TALKS

On a more formal note, the students are required to present their work in a thirty minute talk on the last two days of the program. In the fifth week of the program, the students give shorter practice talks, reporting on their progress. The director gives an informational session a few days before on "how to give talks." The director and the graduate student assistant attend the practice talks and give advice and constructive criticism. In the practice, the students can discuss what they have learned and explain the expected course of their project. The longer talks are intended to be a "finale" to our program. Their advisors and the entire math faculty are invited to attend these talks. This process gives them a chance to show what they have accomplished and to give them an opportunity to learn to present their work in a professional manner. Indeed, the intent is to encourage them to present talks at local and national meetings and to organize their work in a possibly publishable form. Rather than focusing on the publication of results, we chose to make the final talks as the high point of the program. But we acknowledge that many publications have resulted from this research program. About 20 percent of the projects have resulted in publications.

## SOCIAL ACTIVITIES

At UT, the social activities are limitless. The students have many opportunities to visit the Great Smoky Mountains. Usually, at least one trip is planned with the other science summer groups to hike certain trails of these majestic hills. In addition, some of the professors
and graduate students help lead the groups through the trails. This is a great time for all from a couple of angles. The students and others have an adventure climbing the trail, and each party has the opportunity to talk to each other in an unhurried setting. Furthermore, the group has the option of playing on an intramural softball team consisting of faculty, staff, graduate students, and them. The gloves and bats are shared, and the excitement of winning and losing as a team creates a nice atmosphere. Other excursions include trips to Chattanooga to see the Aquarium and the ever-popular Rock City, white-watering rafting on the Ocoee River, a lake-front picnic and boating party, and trips across town to see the minor league Toronto Bluejays squad play the Chattanooga Lookouts. As if this is not enough, a volleyball competition is held among all the science summer programs on a midweek evening. The math REU provides drinks and snacks to entice the other groups to come meet the challenge.

In addition, lunches are scheduled with the mathematics faculty. These lunches give both the students and faculty members the chance to learn about each other.

## CONCLUSION

Definitely, the students must do significant background and original work to obtain reasonable and accurate results. Also, they do have the opportunity to share their mathematical viewpoints with other distinguished colleagues, whether they be professors, graduate students, or other participants. They have several chances to explore different areas of mathematical research.

We feel that this "one-on-one" style reflects the way that much of mathematical research is accomplished. Working with research mathematicians at the frontiers of exciting mathematics, our participants have experiences which have a positive impact on their career decisions and aspirations. Participation in our program helps these students make a well-informed choice about their mathematical future.

## REFERENCES

1. Trevis Litherland and Phil Schaefer, On a Maximum Principle for fourth order equations, Int. J. Math. Educ. Sci. Technol. 26 (1995), 619-626.
2. Glaucio Paulino, L. J. Gray and Vrej Zarikian, Hypersingular Residuals -a new approach in error estimation in the boundary element method, Int. J. for Numerical Methods in Eng. 39 (1996), 2005-2029.

Murray State University Murray, KY 42071, University of Tennessee, Knoxville, TN 37996

E-mail address: kfister@math.mursuky.edu, lenhart@math.utk.edu

# The Summer Program for Undergraduate Research at the University of Virginia 

James Rovnyak

The involvement of our department with summer undergraduate research began in the early 1990's, when several faculty members directed REU students in summer projects using both NSF and departmental funds. A more formal Summer Program for Undergraduate Research (SPUR) was created in 1996. The idea of the program is spelled out in our flier:
"The purpose of the Summer Program for Undergraduate Research is to give mathematically talented undergraduate students an early experience with research. ... Projects are expected to be carried out in about eight weeks. Students will be paid a fellowship stipend of [current rate] to be in residence at
the University for the project period. A full-time effort is expected, and participating students should not at the same time be working or taking courses in the summer session. All University students are eligible. Each project must have a faculty supervisor who is a member of the Department of Mathematics including the Division of Statistics. Upon completion, the student will write a report describing the activities of the project." Participating students have usually completed two or three years, but we have had success with an occasional first-year student as well. The topics have included algebraic geometry, cobordism classifications, combinitorics, function theory, Jordan algebras, mathematical physics, operator theory, probability theory, programming with mathematical software, representation theory, and stochastic differential equations with applications in statistics.

Received by the editor September 8, 1999.

The program does not have any explicit or implied condition that projects should result in a product such as a report or a paper. Students often express reservations that they will not be able to do what is expected of them. All we ask is that at the end of the project the student should write a letter which describes the activity and their experiences. The point is to encourage both students and faculty to take a chance. This yields an occasional bust, but it also produces some projects that exceed expectations. Many students do indeed produce reports and papers from their projects.

In all, 30 projects have been undertaken by 24 students ( 4 of the 24 students did more than one project). Our steady-state level of operation is $5-7$ projects each summer. A strength of the program is that many faculty members participate: to date, 13 tenured faculty members, 1 untenured faculty member, and 2 postdocs have served as supervisors.

The undergraduate student stipend is tied to what we pay graduate students for summer support. In 1999, this was $\$ 2900$. The point of the stipend is to allow students to be in residence for the main part of the summer and to devote their full attention to the project. The program is funded from two sources. NSF grants with REU attachments are one important source. The bulk of the money, however, comes from private funds which are available to our department. These private funds are restricted and can only be spent on full-time students at the University of Virginia (we do not advertise nationally). In the last four summers (1996-99), NSF funded 5 of the 25 projects for $\$ 14,100 ; 20$ were supported with our private funds for $\$ 56,450$.

Student interest is high. Assuring an adequate supply of projects and faculty advisors is the main problem. We solve this problem by having students initiate contact with a faculty member (it is much harder to turn down a student than to make excuses to a department head or director of the REU committee). Students learn of the summer research program through advertisements, which direct them to me. I interview each personally. Usually undergraduate students do not know much about research in mathematics, and the idea fascinates them or they are simply curious about it. I try to explain the process a bit, but mainly I prompt students how to approach a faculty member. I suggest that they say something like this:
"I have heard about the summer research program, and I am very interested. Do you have any ideas for projects that might be suitable for somebody with my background?'
I caution students that what they are asking is difficult, and the faculty member might be discouraging and say that he or she cannot think of
anything at the moment, and it is a problem because of a host of reasons, many of which are legitimate. Then the student should then say:
"Well, would you keep me in mind if something comes up. I am really interested in doing this."
I do not imagine that such a conversation takes place literally, but our experience is that students do follow up on these tips and initiate contact with faculty members, and some two-way conversations take place. If the faculty member thinks that summer research is unrealistic for the particular student, the matter ends. Often the faculty member sees some possibility, perhaps weeks later. There might be some further discussions between the faculty member and student, and at some point I get stopped in the hall, or I receive email from the faculty member saying, "Jim, such-and-such a student has been talking to me about a summer project, and I think I have something. If we still have money for this, I'm game." Faculty response to such requests by students has been outstanding. Projects tend to be custom designed for particular students. Faculty feel more comfortable working with students who are known to them. The project is undertaken by mutual agreement.

In a small number of cases, undergraduate students aquire a faculty mentor on their own, and a project is already informally underway when the student comes to see me about our program. In such cases, it is a simple matter to sign up the student for summer support and put the project into high gear.

Department of Mathematics, University of Virginia, Charlottesville, Virginia 22903

E-mail address: rovnyak@Virginia.EDU

# INVERSE PROBLEMS FOR ELECTRICAL NETWORKS 

Edward B. Curtis and James A. Morrow

## 1. Introduction

We typically bring eight to ten undergraduates to the University of Washington during the summer to work on inverse problems concerning discrete electrical networks and continuous electrical bodies. We have directed an REU site focused on such problems over a period of eleven years and have made improvements in our program each year.

During the first week, the students learn about the conductivity equation, the Dirichlet problem, and related problems for electrical networks. They learn about circular planar graphs, critical graphs, and medial graphs. They also learn about inverse problems in these areas. After this introduction, we help them formulate open problems. They then proceed to investigate these problems and try to solve one or more of them. Finally, we have them write up the results of their investigations. The students have a lot of freedom in choosing their own research projects. They are not given fixed assignments nor, after the first week, are they expected to study the same material. We encourage them to be bold with conjectures and not to worry about success or failure. In our recent REU programs this approach has led to a wide variety of ideas for projects. Some worked out quickly to produce good results. Others seemed promising, but could not be completed in the short time of program. Several students have continued their work during the following year and have produced more results. Three of the students have written joint papers with the principal investigators and the results of others will appear either as separate publications or in a monograph devoted to the subject of discrete inverse problems.

Received by the editor September 10, 1999.

The quality of the students in the program has been high. Two of the students have been given honorable mention in the competition for the Alice T. Schafer Award. Several of the students were accepted into the special mathematics program at the University of Budapest. A recent student spent the year at the University of Pennsylvania in their special program. Five students have been awarded NSF fellowships. Many of the students have gone on to graduate school in mathematics or a related discipline and many of them have been awarded fellowships. Several of them are now faculty members at highly regarded universities. One was awarded a Sloan Dissertation Fellowship, is a recent recipient of an NSF grant, and is currently a faculty member in the Mathematics Department at MIT. In addition he worked for two summers in the Mathematics Group at Microsoft. A participant in our program has recently won an award for his solution of an old problem in geometry. Many of these students have said they would not have considered graduate school if it had not been for the REU program and some have switched from another discipline to mathematics because of their experience in the program.

The projects we propose involve original work in an active area of mathematical research. The AMS meetings at Arcata in 1989 and Seattle in 1995 on inverse problems and the various special meetings of SIAM and IEEE on inverse problems attest to the current interest in inverse problems. The proposers are active in this area, are in constant contact with other workers and know the current state of research. The virtue of problems in inverse conductivity is that they can be approached by undergraduate students with a good understanding of calculus, differential equations, linear algebra and elementary physics. In the first week we give some lectures, and before the week is over some students begin to formulate ideas. Soon we meet in a seminarlike setting where the instructors and the students bring up ideas which we discuss. There are occasional guest lectures and there are faculty members available to consult on questions about graph theory, inverse problems, partial differential equations, numerical analysis and optimization. At the end of the term students submit papers describing their work. Some continue to work on their papers after the formal termination of the program.

## 2. Inverse Problems

The inverse conductivity problem for an electrical network is that of determining the conductivity in the interior of the network from measurements made at the boundary. This problem will now be described in some detail.

A graph $\Gamma$ consists of edges and vertices. An electrical resistor network is a graph in which the edges are conductors and the vertices are nodes. A conductivity on a graph $\Gamma$ is a function $\gamma$ which assigns to each edge $e$ a positive real number, $\gamma(e)$, which is the conductance of $e$. If voltages are imposed at a specified set of boundary nodes, there will be a unique set of voltages satisfying Kirchhoff's current law at each interior node The problem of finding these interior voltages is called the Dirichlet Problem and the solution is called the potential. This potential determines the current flow in the network and thus the current flow into each of the boundary nodes. If the boundary voltages are represented by the vector $\phi$ and the current into the boundary nodes is represented by the vector $\psi$ then there is a linear relationship $\Lambda \phi=\psi$. The map $\Lambda=\Lambda(\Gamma, \gamma)$ depends on the graph $\Gamma$ and the conductivity $\gamma$ and will be referred to as the voltage to current map or the network response.

The simplest version of an inverse problem on an electrical network is the problem of finding the conductivity $\gamma$ when $\Gamma$ and $\Lambda$ are given. In joint work, $[\mathbf{2}, \mathbf{3}]$, with some of the students in the REU program, we have shown that for a large class of networks (circular planar networks) $\Lambda$ uniquely determines $\gamma$ and we have given a characterization of the linear maps $\Lambda$ that are responses of such networks. We have also studied algorithms for finding the conductors from $\Lambda$ and have made extensive numerical calculations. In the next few paragraphs we make some definitions and describe some of these results.

A graph $\Gamma$ with designated boundary vertices is called circular planar if $\Gamma$ can be embedded in the two dimensional disk, $D$, so that the boundary vertices lie on the boundary of $D$ and the rest of $\Gamma$ is in the interior of $D$. The embedding in $D$ determines a (circular) ordering of the boundary vertices and this determines an ordering for a basis of the boundary potentials. This in turn determines a response matrix which will also be denoted by $\Lambda$. The notation $p_{1}<p_{2} \cdots<p_{k}$ means that the sequence of boundary vertices $P=\left(p_{1}, p_{2}, \cdots, p_{k}\right)$ is in circular order. A pair of sequences of boundary vertices $(P ; Q)=$ ( $p_{1}, p_{2}, \cdots, p_{k} ; q_{1}, q_{2}, \cdots, q_{k}$ ) such that the entire sequence
$\left(p_{1}, p_{2}, \cdots, p_{k}, q_{k}, \cdots, q_{1}\right)$ is in circular order is called a circular pair.
A circular pair $(P ; Q)=\left(p_{1}, \ldots, p_{k} ; q_{1}, \ldots, q_{k}\right)$ of boundary vertices is said to be connected through $\Gamma$ if there are $k$ disjoint paths $\alpha_{1}, \ldots, \alpha_{k}$ in $\Gamma$, such that $\alpha_{i}$ starts at $p_{i}$, ends at $q_{i}$ and passes through no other boundary vertices. We say that $\alpha$ is a connection from $P$ to $Q$. We consider two ways to remove an edge from $\Gamma$ :
(1) By deleting an edge.
(2) By contracting an edge to a single vertex and thereby also eliminating a vertex. (An edge joining two boundary vertices is not allowed to be contracted to a single vertex).

We say that removing an edge breaks the connection from $P$ to $Q$ if there is a connection from $P$ to $Q$ through $\Gamma$, but there is not a connection from $P$ to $Q$ after the edge is removed. A graph $\Gamma$ is called critical if the removal of any edge breaks some connection. In [2] the following theorem is proved:

Theorem 1. Suppose $(\Gamma, \gamma)$ is a circular planar resistor network which is critical as a graph. Then the values of the conductors are uniquely determined by, and can be calculated from $\Lambda_{\gamma}$.

Notation: Suppose $A=\left(a_{s, t}\right)$ is a matrix, $P=\left(p_{1}, \ldots, p_{k}\right)$ is an ordered subset of the rows, and $Q=\left(q_{1}, \ldots, q_{m}\right)$ is an ordered subset of the columns. Then $A(P ; Q)$ will denote the $k \times m$ matrix obtained by taking the entries of $A$ which are in rows $p_{1}, \ldots, p_{k}$ and columns $q_{1}, \ldots, q_{m}$. Specifically, for each $1 \leq i \leq k$ and $1 \leq j \leq m$,

$$
A(P ; Q)_{i, j}=a_{p_{i}, q_{j}}
$$

In [2] the following theorem is proved:

Theorem 2. The set of response matrices of circular planar resistor networks graphs with $n$ boundary nodes is the set of symmetric matrices $M$ with row sum zero such that

$$
(-1)^{k} \operatorname{det} M(P ; Q) \geq 0
$$

for all circular pairs $(P ; Q)=\left(p_{1}, p_{2}, \cdots, p_{k} ; q_{1}, q_{2}, \cdots, q_{k}\right)$.

The students and the proposers worked closely together in the development of these results. It was Edith Mooers who first developed an interest in critical networks and David Ingerman who saw clearly the relationship between determinants and connections in a network. Thad Edens, one of our first students, suggested some ideas which led to our characterization of response matrices for rectangular networks. The work with Mooers and Ingerman has attracted the interest of such prominent research mathematicians as Colin de Verdiere at the Universite de Grenoble and Gilbert Strang at MIT.

A good feature of inverse problems for electrical networks is that they naturally lead to new questions. Students have been adept at formulating their own versions of network problems. These problems
have led to interesting projects and stimulated further questions. For example in 1990 an REU student discovered a method for creating an artificial probe inside a rectangular network by a suitable choice of boundary conditions. His ideas have interested many subsequent students. In a subsequent summer, a student found an analog of this method that allowed him to recover conductors inside a three dimensional rectangular network with an accuracy far greater than was previously possible. We have had students working on versions of this problem for several summers. In a later summer a student clarified what is necessary for the original method to work. He used a tool (the medial graph) that we had only recently introduced and he became an expert on the medial graphs of some rather complicated networks. By using medial graphs he found examples where the original method works and examples where it fails.

The medial graph $M(\Gamma)$, of a circular planar graph, $\Gamma$, is a convenient tool for studying inverse problems. It is obtained by placing a vertex on each edge of $\Gamma$ and of the bounding circle, and joining these edges in a counterclockwise fashion around each cell of $\Gamma . M(\Gamma)$ is a union of connected paths with endpoints on the bounding circle. These paths are referred to as geodesics or pseudo-lines. The cells of the medial graph can be two colored, with the black cells corresponding to vertices in the original graph, $\Gamma$. Medial graphs have been extensively studied by Branko Grunbaum in $[\mathbf{6}]$ and were indispensable for proving the results of [3]. We have become more and more convinced that, at least in the circular planar case, the medial graph is the fundamental object.

In the summer of 1996, an REU student found an algorithm for recovering the $Y-\Delta$ equivalence class of $\Gamma$ from $\Lambda(\gamma)$. In his paper he conjectured a relation between the black cells on the boundary of the medial graph and certain connection relations in the medial graph. Although he was unable to prove his conjecture, David Ingerman succeeded in giving a proof in his thesis, [7]. Using this, "key lemma", an REU student later succeeded in giving an elegant algorithm to find the $Y-\Delta$ equivalence class of $\Gamma$ from $\Lambda(\gamma)$. At the same time, a group of REU students reconsidered the problem of locating a faulty resistor in a network from boundary information. This problem was first studied by an REU student in the summer of 1989. By using our knowledge of the medial graph REU students in the summer of 1997 were able to give an algorithm for finding a broken resistor in a rectangular network. Building on their ideas, in the summer of 1998 another group of students were able to find a broken resistor in certain critical circular
planar networks. With certain additional information they can find a broken resistor in a general circular planar network.

The original motivation for the electrical network problem was the continuum inverse conductivity problem, which we now describe. Suppose $\Omega$ is a region in $\mathbf{R}^{n}, n \geq 2$, which represents an electrical conductor (e.g. $\Omega$ could represent the human body). Let $\gamma$ be the electrical conductivity of $\Omega$, that is $\gamma(x)$ is the conductivity at point $x$ in $\Omega$. Let $\phi$ be an electrical potential applied to the boundary $\partial \Omega$. This boundary potential will determine a unique potential $u$ throughout the interior which satisfies the conductivity equation:

$$
\operatorname{Div}(\gamma \operatorname{Grad}(u))=0
$$

The electrical current $J$ in $\Omega$ is determined by Ohm's law: $J=\gamma$ $\operatorname{Grad}(u)$. The conductivity equation is a continuous version of Kirchhoff's law (conservation of current). To each boundary potential $\phi$ there is associated a boundary current $\gamma \frac{\partial u}{\partial n}$. The map from boundary potentials to boundary currents is called $\Lambda$. The inverse conductivity problem for a continuum is to determine $\gamma$ from $\Lambda$. The work of Nachman, Sylvester, Uhlmann, Vogelius, et al. shows that $\gamma$ is uniquely determined by $\Lambda$. However questions of actually computing the conductivity are still open (as well as characterizing the map $\Lambda$, etc.). In the summer of 1994, two students worked on the problem of computing the conductivity and location of a disk of constant conductivity which is contained inside a larger disk of conductivity $\gamma=1$. They worked closely with John Sylvester on their project and devised an algorithm to compute the conductivity and radius of the smaller disk.

The projects just cited are typical examples of ideas that came from the students. In most cases the students have formulated their own problems and learned or devised techniques to attack their problems. The area of research that we have chosen lends itself to the development of new ideas and problems.

## 3. Selection of the students

We have a website with information about the program and selected papers for interested parties to read. This year's web address was http://www.math.washington.edu/~morrow/reu99/reu.html. We have made a selected list of universities and colleges in the United States and either by mail or by personal contacts we will make the local undergraduate advisers aware of the program. This list includes institutions at which research opportunities are limited and which are
located throughout the United States. We will distribute flyers and application forms (we enclose this year's versions) to faculty and students and follow up to attempt to get the best applicants.

In addition to the usual applications from students on the West Coast, we have many requests from Eastern, Southern and Midwestern students, who probably learned about our program from a faculty member. We have had students from University of South Alabama, Bowling Green University, University of California at Berkeley, University of California at Davis, Bryn Mawr, Colorado School of Mines, Columbia University, University of Dayton, Duke University, Eckerd College, Harvard University, Harvey-Mudd, Humboldt State, University of Mississippi, MIT, Northwestern, University of Portland, Pomona College, Rice, Seattle University, Seattle Central Community College, Texas A\&M University, Tufts, UCLA, University of Notre Dame, University of Utah, Eastern Washington, University of Washington, Westfield College, Whitworth, and Wittenburg University. As can be seen from this list we have had students from a wide range of institutions located across the United States.

Our program is suitable for students with the following background:

- Differential equations at the level of Boyce and DiPrima (typically a sophomore course).
- Linear algebra at the sophomore or junior level including some discussion of numerically solving linear equations
- Advanced calculus, especially Green's theorem
- First year physics (mechanics, electricity and magnetism)
- Fortran, C, Mathematica, Maple or Matlab

We do not expect that students will know numerical analysis or partial differential equations. By the end of the program many of them will have become quite familiar with certain aspects of these subjects.

We ask prospective students to describe their mathematical education, list any special awards and write a short essay describing their interest in the program. We include a copy of the form we used this year. We ask for one letter of recommendation. We plan to select the students in early April.

## 4. Activities and Projects

We send the students reading material and a list of references late in April and expect the students to do some reading before the program begins. We send them articles (for example $[4,5,2,3,8]$ ) and student papers from our previous REU programs.

The program will run for eight weeks. During the first week there will be lectures describing known results, open problems and numerical approaches. We will assist students in the formulation of projects as soon as they have understood the background material. Some afternoons there will be a session in the computer lab. The students will learn about the MSCC computing facilities. There will a short course on the operating system and its software. As an introduction to computation students may wish to compute the solution of the Dirichlet or Neumann problem on a square or a circle. Many students prefer to proceed to more complicated regions and make computations for graphs which are not planar. We find that some students master this in a few days, others take more than a week.

We don't present students with a list of problems to be solved in the manner of exercises in a textbook. We never assign problems to students. However, we do suggest problems that we think they will find interesting, and we suggest promising directions. Some students choose to extend work done by former students in the program. In many cases our former students did not completely solve the problems they worked on. Thus we have had students work on parts of the same problem over a period of several years. For example substantial work on non-planar networks has been done in six different years.

Some students prefer to work on problems that they formulate entirely themselves. We encourage this and help them as much as possible. We also direct such students to other faculty and graduate students when we think it is appropriate.

We encourage students to formulate and work on problems as soon as possible. When we first began to involve undergraduates in research, we expected that it would be a long time before they could do this. We have have been pleased to find that they actually need only a few days. After the first week, formal lecturing ceases and is replaced by one-on-one interaction between faculty and students. Students lecture on their work and we have guest lectures by other faculty. We make appointments with each student every day to discuss their progress and to make suggestions.

Students can choose to work on their own or in teams. If they desire, we meet with them in the afternoon. Most of them said they were very happy to be given the freedom to choose their own problems and work on their own time schedules. Several said that this was the most intense learning experience that they had ever had.

The students are told early on that they will be expected to write an exposition of their work and it is suggested that this writing should begin as soon as possible as this is frequently a lengthy and difficult
process. We have encouraged the students to show us written work as soon as the second or third week. We start to help them right away in the formulation and expression of their ideas. Because the students made a quick start on their projects and began writing in the first few weeks, this year's papers were better written than those in previous programs. We are continuing to discover what students are capable of doing.

We have learned a lot from the student responses to the evaluation forms that we distributed. The students have found that a solid understanding of linear algebra is essential. It is easy for us to overlook the fact that such an elementary subject is so important. We have become more aware of this requirement and have tried to assist the students in learning all the linear algebra that they need. The students said that the paper writing is one of the most difficult parts of the program. We agree with that assessment and we assist them in learning how to write by carefully reviewing their work with them. We ask them to be very critical of their own work and to strive to make it as clear and unambiguous as possible.

Finally we tell them that this program is intended to give them the freedom to experiment. We want them to find a problem, learn all they can about it, attempt to solve it and write up the results. We stress the importance of determination and that they must keep trying even in the face of repeated failures. We tell anecdotes about former students and their work. We point out that often what seems to be a crazy idea turns out in fact to be quite fruitful. We also tell them that sometimes the discoverer never sees the real importance of his or her ideas. What they should be getting from this program is an idea of what it is like to do mathematical research and whether it suits them or not. In that sense the program should be a success for everyone.

## References

[1] E.B. Curtis, T. Edens, and J.A. Morrow, Calculating the Resistors in a Network, Proc. Annual International Comference of the IEEE Engineering in Medicine and Biology Society, 11. (1989), 451-2.
[2] E. B. Curtis, D. Ingerman, and J. A. Morrow, Circular Planar Graphs and Resistor Networks, to appear in Linear Algebra and Applications.
[3] E. B. Curtis, E. Mooers, and J. A. Morrow, Finding the Conductors in a Circular Network from Boundary Measurements, Mathematical Modelling and Numerical Analysis 28, No. 7 (1994), 781-813.
[4] E. B. Curtis and J. A. Morrow, Determining the Resistors in a Network, Siam J. Appl. Math. 50 (1990), 918-930.
[5] E.B. Curtis and J.A. Morrow, The Dirichlet to Neumann Map for a Resistor Network, Siam J. Appl. Math. 51, No. 4, 1991, 1011-1029.
[6] B. Grunbaum, Convex Polytopes, Pure and Applied Mathematics, Vol. 16 Interscience, 1967.
[7] D. Ingerman, Discrete and Continuous Inverse Boundary Problems on a Disc, to appear in SIAM J. Math. Anal.
[8] D. Ingerman and J. A. Morrow, On a Characterization of the Kernel of the Dirichlet-to-Neumann Map for a Planar Region, SIAM J. Math. Anal. 29, No. 1, 106-115.

University of Washington, Seattle, WA 98195
University of Washington, Seattle, WA 98195
E-mail address: curtis@math.washington.edu
E-mail address: morrow@math.washington.edu

# Williams College SMALL Undergraduate Research Project Description 

Colin Adams, Thomas Garrity, Susan Loepp and Cesar Silva

Twelve years ago, the number of mathematics majors at Williams College averaged ten a year and this had been true for a period of at least ten years. For the years 1994-1998, an average of 28 Williams College graduating seniors majored in math, about $5.5 \%$ of each class (compared with the national average of $1-2 \%$ ). Since then, the numbers have increased further, so that for the 1999-2000 academic year, we have over 40 senior majors, which constitutes $7.5 \%$ of the Williams College senior class. Many factors have contributed to this large jump in the number of math majors. However, certainly a very visible change that occurred at Williams was the introduction of the SMALL Geometry Project eleven years ago. Williams College had undergraduate research in mathematics for many years, pre-dating the SMALL program. Before SMALL, this consisted of work done by one or two students working with individual faculty (including among others, Curtis McMullen, 1998 winner of the Fields Medal). In the summer of 1988, we began the SMALL Geometry Project. (SMALL is an acronym from the names of the founding faculty, Silva, Morgan, Adams, Lenhart, Levine). In the first year, a total of fifteen students worked with five faculty. Over the years since then, the size of the program has fluctuated, usually having between 15 and 24 students, normally with 6 to 8 students from outside Williams College. The goal of the program has been to introduce undergraduates to the excitement and challenge of doing original research in mathematics while still relatively early in their careers. Too often, we have seen talented students lose interest in mathematics because they perceive it as a lifeless subject within which one simply studies the work of long dead mathematicians. In SMALL, they get to work on current projects under the direction of

Received by the editor October 28, 1999.

204 COLIN ADAMS, THOMAS GARRITY, SUSAN LOEPP AND CESAR SILVA
faculty who are themselves actively engaged in cutting edge research. They see the vitality of mathematics and their own potential for making a contribution. They understand the ultimate goal of learning, namely to utilize that learning in order to make their own advances and further the pursuit of knowledge. The students are broken up into groups of two to four, each group working with an individual faculty member. The students learn to work as a group, assigning sub-projects to individuals, and helping each other over the hurdles. Each group, along with their Faculty advisor, decides on the structure of their daily routine. Some meet once or even twice a day at a scheduled time with the faculty member, while others prefer a more open door policy where the students come to see the faculty member whenever they have questions. When not talking to the faculty advisor, the groups are working together or individually in the Mathematics Library (which is surrounded by the faculty offices), in the Math Computer lab or in one of several classrooms reserved for their use. See the pictures at http://www.williams.edu/Mathematics/small.html for a sense of the facilities. There are a variety of weekly activities that all students attend. Every Tuesday at 10:00 am, convocation is held. Here, announcements are made. They are sometimes followed by short progress talks by the student groups. Tuesday at noon the students, together with at least another one hundred students working on research in the sciences over the summer, attend research talks by faculty from the sciences. Lunch is provided by the College. Every Wednesday at 1:00, there is a mathematics colloquium talk by a faculty member from inside or outside Williams. These talks are usually directed at the students, although the topic is typically current research. At 4:00 on Fridays, there is a tea, giving students a chance to talk to each other and to faculty about their progress over the week. In addition, there are a variety of social events organized by students and faculty which increases the opportunities for interaction. In past summers, the enthusiasm has been so great that students have often worked more than the standard forty-hour workweek, including working late into the night and over the weekends. Over the summer, students present their work to the rest of SMALL in presentations. In addition, we have had students present their work in numerous other venues, including MAA and AMS national and regional math conferences, as well as the Regional Geometry Institute that took place at the Five Colleges in the summers of 1991-93, the Hudson River Undergraduate Math Conference in 1994-1998 (for which three of the faculty in SMALL are founders and/or organizers) and at joint meetings with the Mt. Holyoke REU. Over the last three summers, a total of

45 students have presented talks at the MAA Mathfests in Atlanta, Toronto and Providence. Nine of these students won prizes for the best talks in their MAA paper sessions. Housing has been provided free of charge by Williams College. The last three summers, the College provided us with Agard House for the exclusive use of our program. (See pictures at http://www.williams.edu/Mathematics/small.html ). Having all of the students living together in a single building allows for the mathematical interaction to expand beyond the boundaries of the "work day". The College has also provided us with computing facilities and classroom space. The faculty who have participated in the program include: Colin Adams, Duane Bailey, Deborah Bergstrand, Gerald Bope, Edward Burger, Elizabeth Camp, Charles Chace, Richard De Veaux, Thomas Garrity, David Levine, William Lenhart, Susan Loepp, Robert Mizner, Frank Morgan, Cesar Silva, Alice Underwood, and David Witte. Group topics have included knot theory, hyperbolic manifolds, minimal surfaces, symmetry groups, combinatorics, graph theory, computational geometry, algebraic geometry, dynamics and ergodic theory, parallel processing, topology of robotics, CR structures, neural networks, and commutative algebra. Students from SMALL have gone on to numerous graduate schools, including Berkeley, Carnegie-Mellon, Duke, Harvard, Michigan, MIT, Northwestern, Stanford, UCLA, UCSD, University of Chicago, University of Illinois, UNC, University of Penn., University of Texas, University of Washington, Washington University, Wisconsin and Yale, among others.

The following list of references includes student work on mathematics, which was either generated by the SMALL Project from 1988 - 1999 or followed up on work begun in SMALL.

References, 1988-99, Externally submitted.

1. C. Adams and W. Sherman, "Minimum Ideal Triangulations of Hyperbolic 3-Manifolds", Journal of Discrete and Computational Geometry, 6:135-153 (1991).
2. D. Bergstrand, K. Jones, S. Seager, W. Sherman, "Posets with Isomorphic Upper and Lower Bound Graphs", to appear in SIAM Journal of Discrete Math.
3. D. Bergstrand, F. Harary, K. Hodges, G. Jennings, L. Kuklinski, J. Weiner, "The Sum Number of a Complete Graph", Bull. Malaysian Math. Soc. (2) 12 (1989), no. 1, 25-28.
4. D. Bergstrand, F. Harary, K. Hodges, G. Jennings, L. Kuklinski, J. Weiner, "Product Graphs are Sum Graphs", Math. Mag. 65 (1992), no. 4, 262-264 .
5. M. Alfaro, M. Conger, K. Hodges, A. Levy, R. Kochar, L. Kuklinski, Z. Mahmood, K. von Haam, "The Structure of Singularities in

Phi-Minimizing Networks in $R^{2 "}$, to appear in Pacific Journal of Math, 149 (1991), 201-210.
6. C. Adams, J. Brock, J. Bugbee, T. Comar, K. Faigin, A. Huston, A. Joseph, D. Pesikoff, "Almost Alternating Links", Topology and its Applications 46 (1992) 151-165.
7. M. Conger, Gordo, (a compute program to input knots), program library of Geometry Supercomputer Project, Minnesota.
8. Manuel Alfaro, "Existence of shortest directed networks in R2," Pacific J. Math. 167 (1995), 201-214.
9. Joel Foisy, Manuel Alfaro, Jeff Brock, Nick Hodges, Jason Zimba, "The standard double soap bubble in $R^{2}$ uniquely minimizes perimeter," Pacific J. Math. 159 (1993), 47-59. (Featured in 1994 "What's Happening in the Mathematical Sciences.")
10. M. Alfaro, M. Conger, K. Hodges, A. Levy, R. Kochar, L. Kuklinski, Z. Mahmood, K. von Haam, "Segments Can Meet in Fours in Energy-Minimizing Networks", J. Undergraduate Math 22, (1990), 9-20.
11. M. Alfaro, T. Campbell, J. Sher, A. Soto, "Length Minimizing Directed Networks can Meet in Fours", J. Und. Math 22 (1990), 53-59.
12. M. Conger, "Energy-Minimizing Networks in $R^{n}$ ", J. Und. Math 22 (1990), 53-59.
13. A. Levy, "Energy-Minimizing Networks Meet Only in Threes", J. Und. Math 22 (1990), 53-59.
14. Tom Colhurst, Chris Cox, Joel Foisy, Hugh Howards, Kathryn Kollett, Holly Lowy, Stephen Root, "Networks minimizing length plus the number of Steiner points," in Ding-Zhu Du and Panos M. Pardalos, ed., Network Optimization Problems: Algorithms, Complexity and Applications, World Scientific, 1993, pp. 23-36.
15. Christopher Cox, "Flow-dependent networks," undergraduate thesis, Williams College, Networks 31 (1998), 149-156.
16. Christopher Cox, Lisa Harrison, Michael Hutchings, Susan Kim, Janette Light, Andrew Mauer, Meg Tilton, "The shortest enclosure of three connected areas in $R^{2}$," Real Analysis Exchange 20, (1994/95), 313-335.
17. Frank Morgan, Christopher French, Scott Greenleaf, "Wulff clusters in $R^{2 " \prime}$, to appear in J. Geom. Anal. (1998).
18. G. Jennings, W. Lenhart, "An Art Gallery Theorem for Line Segments in the Plane", Pattern Recognition Letters, (1993), 727-732.
19. E. Burger, C. Kollet, "On the Structure of Quadratic Irrationals Associated with Generalized Fibonacci and Lucas Numbers", The Fibonacci Quarterly, 34, (1996) .
20. Michael Hutchings, "The structure of area-minimizing double bubbles, J. Geom. Anal., to appear.
21. Hugh Howards, Michael Hutchings and Frank Morgan, "The Isoperimetric Problem on Surfaces", Amer. Math. Monthly, to appear.
22. R.D. De Veaux, J. Schumi, J. Schweinsberg, and L.H. Unger, "Predictions Intervals for Neural Networks via Nonlinear Regression", Technometrics, to appear.
23. C. Adams, B. Brennan, D. Greilsheimer, A. Woo, "Stick numbers and composition of knots and links", J. Knot Theory and Ram., 6 (1997) 149-161.
24. Joseph Masters, "The perimeter-minimizing enclosure of two areas in $S^{2 "}$, Real Analysis Exchange 22 (1995/6), 1-10.
25. "Stick Knots", Eric Furstenberg, Jie Li, Jodi Schneider, Chaos, Solitons and Fractals special issue devoted to knots and their applications, Vol. 9, No.4/5, April/May 1998, 561-568.
26. "Exploring Knots", C. Adams, E. Furstenberg, J. Li, J. Schneider, Mathematics Teacher, Vol. 90, No. 8, Nov. 1997 640-646,652.
27. Frank Morgan, Ted Melnick, Ramona Nicholson, "The Soap Bubble Geometry Contest", The Mathematics Teacher 90 (December, 1997), 746-750.
28. "Prediction Intervals for Neural Networks via Nonlinear Regression", Richard D. De Veaux, Jennifer Schumi, Jason Schweinsberg, David Shellington and Lyle H. Ungar, Technometrics, Vol. 40, No. 4, in press.
29. "Alternating Graphs", Colin Adams, Kerryann Foley, Jonathan Kravis, Ryan Dorman, Sam Payne, submitted for publication.
30. "The soap bubble geometry contest," Frank Morgan, Ted Melnick and Ramona Nicholson. The Mathematics Teacher 90 (December, 1997), 746-750.
31. "Lightly mixing on dense algebras", C. Silva, E. Muehlegger, A. Raich, and W. Zhao, Real Analysis Exchange, Vol. 23, No. 1, to appear.
32. "Cost-minimizing networks among immiscible fluids in R2", David Futer, Andrei Gnepp, David McMath, Brian Munson, Ting Fai Ng, Sang Pahk, Cara Yoder, Pac. J. Math., to appear.
33. "Constructing Chains of Excellent Rings with Local Generic Formal Fibers," Mark Florenz, Davina Kunvipusilkul, Junghee Yang, Communications in Algebra, to appear.
34. "Characterization of Completions of Reduced Local Rings," Dan Lee, Leanne Leer, Shara Pilch, Yu Yasufuku, 1999, submitted for consideration for publication.

208 COLIN ADAMS, THOMAS GARRITY, SUSAN LOEPP AND CESAR SILVA
35. "Excellent Rings with Singleton Formal Fibers," Dan Lee, Leanne Leer, Shara Pilch, Yu Yasufuku, 1999, submitted for consideration for publication.
36. "Infinite ergodic index Zd actions in infinite measure", E. Muehlegger, B. Narasimhan, A. Raich, C. E. Silva, M.Touloumtzis, and W. Zhao Colloquium Math., to appear.
37. "Lightly mixing on dense algebras," E. Muehlegger, A. Raich, C. E. Silva and W. Zhao, Real Analysis Exchange, Vol. 23, No. 1, (1998), 259-266.
38. "A Power weakly mixing infinite transformations," S.L. Day, B.R. Grivna, E.P. McCartney and C. E. Silva, New York J. of Math., 5 (1999), 17-24.

Other Papers:
39. Frank Morgan, Michael Hutchings and Hugh Howards, "The Isoperimetric Problem on Surfaces of Revolution of Decreasing Gauss Curvature", preprint (1997).
40. M. Alfaro, J. Goodell, K. Hodges, K. Pootrakool, A. Soto, S. Taubs, Dynamics Group Report, Undergraduate Research, Williams College, 1989.
41. M. Alfaro, M. Conger, M. Gray, M. McDougall, W. Sherman, K.von Hamm, Topology Group Report, Undergraduate Research, Williams College, 1988.
42. J. Baraka, A. Bardine, J. Emerso, K. Hodges, K. Pootrakool, A. Soto, S. Taubs, "An Exposition of Macaulay's Resultant", Undergraduate Research, Williams College, 1989.
43. J. Baraka, L. Borowsky, B. Jarman, M. Knell, Computational Geometry Report, Undergraduate Research, Williams College, 1989.
44. D. Bergstrand, F. Hurary, K. Hodges, G. Jenning, L. Kuklinski, Z. Mahmood, W. Sherman, J. Weine, "Some results on Sum Graphs", Undergraduate Research, Williams College, 1988.
45. W. Sherman, "Hyperbolic Manifolds from Ideal Tetrahedra", Undergraduate Honors Thesis, Williams College, 1988.
46. M. Alfaro, J. Brock, J. Foisy, N. Hodges, J. Zimba, "Compound Soap Bubbles in the Plane", Undergraduate Research, Williams College, 1990.
47. J. Bope, D. Goodwin, J.Foisy, S. Friedland, M. Gallagher, J. Gyori, T. Tucker, "Ramsey Numbers and SCUBE", being rewritten for submission to undergraduate research journal, 1990.
48. J. Bugbee, S. Friedland, D. Robb, T. Tucker, J. Zimba, "Classifying Vector-Valued Hermitian Forms", Undergraduate Research, Williams College, 1990.
49. A. Huston, A. Beveridge, M. Donofrio, J. Emerson, K. Fiesler, D. Robb, "Topology of Robotics", Undergraduate Research, Williams College, 1990.
50. A. Beveridge, M. Gallagher, D. Goodwin, J. Gyori, A. Joseph, "Results on Sum Graphs", in preparation, 1990.
51. L. Kuklinski, "Niche Graphs: Families, Constructions, and Boundaries", Undergraduate Honors Thesis, 1990.
52. M. Alfaro, "Existence of Shortest Directed Networks in $R^{2}$ ", Undergraduate Honors Thesis, 1990.
53. Joel Foisy, "Soap bubble clusters in $R^{2}$ and $R^{3}$," undergraduate thesis, Williams College, 1991.
54. Graph Editor, a computer program developed by students over three summers for manipulating graphs.
55. Margaret H. Forbush, Elizabeth Hanson, Susan Kim, Andrew Mauer, Rhian Merris, Seth Oldham, Jennifer O. Sargent, Kate Sharkey, and Dave Witte "Hamiltonian Paths in Projective Checkerboards", preprint.
56. Marcus Christian, Dylan Cooper, James Goodell, "LengthMinimizing Networks in $R^{3}$ : The Tetrahedron and New Examples", Geometry Group Report, January, 1989.
57. Susan Kim, "Source and sink networks: no stochastic Steiner points in cost minimizers," undergraduate thesis, Williams College, 1993.
58. Christopher French, Kristen Albrethsen, Heather Curnutt, Scott Greenleaf, Christopher Kollett, "The planar double Wulff crystal," Geometry group report, 1993.
59. Margaret Forbush, Susan Kim, Andrew Mauer, Rhian Merris, Seth Oldham, Jennifer O. Sargent, and Kate Sharkey, "Hamiltonian Paths in Checkerboards on the Torus, Klein Bottle, and Projective Plane", Undergraduate Research, Williams College, 1992.
60. Cordelia Aitkin, Thomas Colthurst, Kathryn Kollett, Steven Levandosky, Andrew Perry, Eugene Stern, "On the Diophantine Equation $S x k=(S x) 2 "$, SMALL Combinatorics Group, 1991.
61. A. Beveridge, M. Donofrio, L. Klein, C. Macauley, Jonathan Mattingly, Jessica Polito, Stephen Root, "On Dynamics of the General Tent Map," SMALL Dynamics Group, 1991.
62. C. Aitkin, S. Fiedler, A. Huston, J. MacEachern, B. McLean, A. Perry, "Peaks of the Divisor Function", SMALL Number Theory Group, 1991.
63. Stephen Fiedler, H. Lowy, B. McLean, J. Mynntinen, E. Swanson, J. Terilla, "Invariants of Vector-Valued Forms", SMALL Differential Geometry Group, 1991.
64. H. Howards, L. Klein, J. Mynttinen, J. Maceachern, J. Polito, J. Terilla, "Universally Linked Graphs", Topology Group, 1991.
65. Eric Chiu, Meg Forbush, Lisa Harrison, Dianne Keller, Shawn McDougal, Rhian Merris, Seth Oldham, "Characterizing Peaks of the Divisor Function", SMALL 1992 Analytic Number Theory Group.
66. Douglas Briggs, Eric Chiu, Noah Graham, Charles Huse, Meg Tilton, "Invariants of Vector-Valued Forms", The SMALL CR Structures Group, 1992.
67. Hugh Howards, "Soap bubbles on surfaces," undergraduate theis, Williams College, 1992.
68. Kristen Albrethsen, Derek Bruneau, Matthew Carroll, Tonya McLean, Amy Tsang, SMALL Symmetry Group Report, 1993.
69. Froney Crawford, Jason Eglit, Craig Epifanio, Jamie Kerman, Kira Maginnis, "Testing the Density Matrix Perturbation Theory of the Doppler Effect", SMALL Mathematical Modeling Group, 1993.
70. Matthew Carroll, Heather Curnutt, Scott Greenleaf, Joshua Kucera, Tonya McLean, "Minimal Networks: Approximate Solutions, Triangle Metric", SMALL Minimal Surface Group 1993.
71. Michael Pelsmajer, Jeffrey VanderClute, Penny Vongsinsirikul, Ted Welsh, Aaron Williams, "On Gelfand's Powers Problem", SMALL Dynamics Group, 1993.
72. Megan Barber, Jennifer Tice, Brian Wecht, "Geodesics and Geodesic Nets on Regular Polyhedra", Geometry Group Report, 1995.
73. Megan Barber, Jennifer Tice, Brian Wecht, "Immiscible Fluids", Geometry Group Report, 1995.
74. Megan Barber, Jennifer Tice, Brian Wecht, "Double Salt Crystals", preprint.
75. Megan Barber, Jennifer Tice, Brian Wecht, "Double Salt Crystals", (research announcement), preprint.
76. "The Splittability of Almost Alternating Links", C. Arthur, D. Bruneau, T. Graber, J. Kucera, P. Vongsinsirikul, and T. Welsh, preprint.
77. "The Reduction of Almost Alternating Knots and Links", Charene Arthur, Derek Bruneau, Tom Graber, Josh Kucera, Penny Vongsinsirikul, and Ted Welsh, Knot Theory Group Report, 1993.
78. Jennifer Sargent, Janette Light, Douglas Briggs, Michael Hutching, Kate Sharkey, Timothy Pinto, "T-Colorings of Cycles and Other Graphs", SMALL Graph Coloring Group.
79. Christopher French, Christopher Kollett, Michael Pelsmajer, Jeffrey VanderClute, Aaron Williams, "Recurrence Relations and Simple Continued Fractions," SMALL Number Theory, 1993.
80. Ted Welsh, "Hyperbolic 3-orbifolds", Undergraduate thesis, Williams College, 1995.
80. "On the arithmetical structure of recurrent sequences modulo m and issues from Diophantine analysis", Kariane Calta, Alex Powell, Brian Wecht.
81. "Geodesic nets on regular polyhedra", Alexei Erchak, Ted Melnick, Ramona Nicholson.
82. "Double clusters of immiscible fluids", Alexei Erchak, Ted Melnick,Ramona Nicholson.
83. "Cost-Minimizing Networks Among Immiscible Fluids in R3", David Futer, David McMath, Brian Munson, Sang Pahk, Geometry Group Report, 1997.
84. "Infinite ergodic index Zd actions in infinite measure", E. Muehlegger, B. Narasimhan, A. Raich, C.E. Silva, M.Touloumtzis, and W. Zhao, to be submitted for publication.
85. "The Crossing Numbers of $P_{n} \times P_{2} \times P_{1}$ and $P_{n} \times P_{3} \times P_{1}$," D. Bergstrand, S. Foster, F. Winston, and A. Woo, in preparation.
86. "Cost-minimizing networks among immiscible fluids in $R^{3}$," David Futer, David McMath, Brian Munson, Sang Pahk, to be revised and submitted for publication.
87. "Outer-Crossing Numbers of Complete Tripartite and Hamiltonian Graphs," Christine Connor, Courtney O'Connor, Aaron Weinberg.
88. "A power weakly mixing infinite transformation," Sarah Day, Brian Grivna, and Earle McCartney, Cesar Silva, preprint.
89. "Polar Varieties in Higher Co-dimensional Geometry," Heath Dill, Edward Johnson, Tristan Smith, Craig Westerland.
90. "The Fibonacci sequence modulo $p$ ", H. Dill, Senior Honors Thesis, 1998.
91. "Cost-Minimizing Networks and Polyhedral Cones", Brian Munson, Undergraduate thesis, Univ. of Oregon, 1998.
92. "Cost-Minimizing Networks Among Immiscible Fluids in $R^{3}$ ", David Futer, Andrei Gnepp, David McMath, Brian Munson, Ting Ng, Sang Pahk, Cara Yoder, preprint.
93. M. Florenz, D. Kunvipusilkul, J. Yang, "Chains of Excellent Rings with Local Generic Formal Fibers" SMALL Commutative Algebra Group, 1998.
94. "Two Counterexamples on Immiscible Fluids", Andrei Gnepp, Ting Ng, Cara Yoder, to be submitted.
95. "Isoperimetric Domains on Polyhedra and Singular Surfaces", Andrei Gnepp, Ting Ng, Cara Yoder, to be submitted.
96. "Minimal Volume Maximal Cusps in Hyperbolic 3-Manifolds", Colin Adams, David Biddle, Carol Gwosdz, Katherine Paur, Scott Reynolds, in preparation.
97. "Isoperimetric domains on polyhedra and singular surfaces", Andrei Gnepp, Ting Fai Ng, Cara Yoder, Geometry Group Report, 1998.
98. "Two counterexamples on immiscible fluids", Andrei Gnepp, Ting Fai Ng, Cara Yoder, Geometry Group Report, 1998.
99. "Component bounds for area-minimizing double bubbles", Cory Heilmann, Yvonne Lai, Ben Reichardt, Anita Spielman, Geometry Group Report, 1999.
100. "Proof of the Double Bubble Conjecture in $R 4$ and certain higher dimensional cases ", Cory Heilmann, Yvonne Lai, Ben Reichardt, Anita Spielman, Geometry Group Report, under revision for submission for publication, 1999.
101. "Minimal Volume Maximal Cusps in Hyperbolic 3-manifolds", by C. Adams, D. Biddle, C. Gwosdz, K. Paur and S. Reynolds, to be submitted for publication, 1998.
102. "Crossing Number for Alternating Knots in $S x I$ ", by T. Fleming, M. Levin, A. Turner, to be submitted for publication, 1999.
103. "Some Results Concerning Uniqueness of Triangle Sequences", by Tegan Cheslack-Postava, Alex Diesl, Thomas Garrity, Matthew Lepinski and Adam Schuyler., 1999.
104. "A Bound on the Distance from Approximation Vectors to the Plane", by Tegan Cheslack-Postava, Alex Diesl, Thomas Garrity, Matthew Lepinski and Adam Schuyler, 1999.
105. "Some Results Concerning Terminating Triangle Sequences" by Tegan Cheslack-Postava, Alex Diesl, Thomas Garrity, Matthew Lepinski and Adam Schuyler, 1999.

Williams College, Williamstown, MA 01267
E-mail address: Thomas.A.Garrity@williams.edu

# Research Experiences for Undergraduates in Industrial Mathematics and Statistics at WPI 

Bogdan Vernescu and Arthur Heinricher

## Program Description

The REU in Industrial Mathematics and Statistics at WPI is an 8 week program that focuses on mathematics and statistics applied to industrial problems. The program, sponsored by NSF, started in 1998 and provides support for 10 undergraduates and 1 graduate student per year.

Our program benefits from an active mathematics faculty in the Mathematical Sciences Department that has a successful experience in conducting research work with the undergraduates within the WPI project-based undergraduate program. It also benefits from the experience gained by the Center for Industrial Mathematics and Statistics at WPI in running student research projects with local business and industrial partners.

Our goal is to provide a unique experience for students of mathematics by introducing them to mathematical research in an industrial environment. The summer research experience provides students with a glimpse of the ways that advanced mathematics is used in the real world to analyze and solve complex problems. It provides challenges not faced in standard undergraduate programs and thus develops skills not always developed in traditional mathematics education.

Received by the editor September 13, 1999.

One of the key features of the REU experience at WPI is that we put the students into a "professional situation." The students work in teams on problems provided by local business and industry. They work closely with a company representative, to develop solutions for problems of immediate importance to the company. The faculty advisor helps the students maintain a clear focus on the mathematical issues behind the project. The students are called on to produce more than a solution, they must communicate their solution to the company in a form that the company can understand and use.

Our program has an important impact which cannot be obtained in a standard course or REU experience. First of all, the problems are presented in their original language. The students will not receive the distilled mathematical essence of the problem. The process of taking the problem and identifying the key mathematical structure, of refining and redefining the problem, is a crucial part of the industrial mathematical experience. Students in the program have said that ${ }^{1}$

The hardest part is what to do, not how to do it. We were constantly reformulating all summer.

Other students said that the most valuable part was
... how to understand and decipher an industrial problem and make it into a mathematical problem.

In the last 2 -years, the 22 students recruited came from a wide variety of colleges in terms of size, location and prestige. For recruiting we send out a flyer to about 500 schools. At the same time we maintain a webpage linked to the NSF REU webpages. The webpage has an online application form, that was very much appreciated by our students.

## Project Logistics

- Week 1: Company representatives make presentations to define the problem. Student teams are formed to work on specific problems. By the end of the first week the teams will have searched the literature, gathered the relevant references and present a broad plan of attack for the problem and a timetable for the project.

[^13]- Week 2: Problem Statement presented to the company, along with a clear timetable of interim goals and a statement of the deliverable.
- Weeks 3-7: Interim Progress Reports provided to the company. Weekly presentations in the Department.
- Week 8: Students make their final presentations to the company and prepare the final report.

The students meet daily with the faculty and periodically with the industrial advisors. Visits to each of the sponsoring companies is organized, towards the beginning of the program. The purpose of the visits is to help the students see the dimension of the importance of their project by learning more about the range of problems facing the companies.

An important part of our industrial REU experience is teamwork. This is one of the skills required for a mathematician working in industry and one of the responsibility of the faculty advisor is to observe and guide the team-building process:

Communication and team work was important because often one person's idea was aided by another person's thoughts on implementing that idea.

In the industrial REU, as in the real-world industrial experience, the teams are not formed based on friendships. Students learn to adjust to work with others:

- One of the project partners was very helpful and we worked well together; the other one was very difficult to work with but we kept our patience and we made it through. I learned to be a more patient person.
- Team work can be a blessing and a curse.


## Presentations

At the end of each week, each team gives a presentation to the full group describing the progress they made during the week. In this way the students get involved in and can contribute to all the projects. This creates a challenging environment and gives the students the necessary feed-back for successfully continuing the project:

Preparing and making presentations is helpful in that if forced us to step back and look at the big picture of the project. They also made sure we thoroughly understood
what we were doing because if we didn't it would have been obvious when we tried to present it.

At the end of the eight weeks, a "Project Presentation Day" is organized for the students to present their final results in front of the Department faculty, industrial advisors and representatives of the WPI Administration and Corporate Relations Office.

Each team of participants prepares a final report based upon the research they have completed during the summer. The purpose of this report is to describe the problem considered, the background literature read, the approach(es) taken, the results that have been obtained, and the questions motivated by their research. Participants are asked to begin writing parts of this report as early as the first week of the program so that the faculty advisor will have an opportunity to assist the students in developing a proper style for writing mathematics. All reports are bound together in a volume and the abstracts are made available on the web.

## Other Activities

One other important way in which we give the students a glimpse of the the way advanced mathematics is used in the real world is by meeting mathematicians and statisticians working in industry. For this we organize site visits at companies (e.g. Applied Mathematics Inc., BOSE, IBM Research Center, Pratt \& Whitney, United Technologies Research Center) and presentations at WPI by mathematicians and statisticians who work in industry (e.g. Fidelity Investment, Microsoft, Computer Science Corporation). Students get first hand information about the mathematics and statistics used in developing a product and at the same time can see the product being developed.

In order that student-faculty interaction is not limited to the academic dimensions, group recreational activities are planned for most weeks.

In order to encourage and maintain contact at a more informal level among the student and faculty participants, the PI and Co-PIs post on the web a newsletter linked to the homepage of the Center for Industrial Mathematics and Statistics. (The URL for the Center is http://www.wpi.edu/~cims.) This newsletter contains news of the whereabouts and activities of former participants and announcements of future programs. It is hoped that this will facilitate networking among former participants.

INDUSTRIAL MATHEMATICS AND STATISTICS AT WPI

## Project Descriptions

The following is a list of the projects completed in the past two summers. In each case, the title is followed by the corporate sponsor and a brief listing of the mathematics involved in the project. (A more complete description of the project can be found on the CIMS web.)

- Optimal Shape Design in Metal Processing

Sponsor: Morgan Construction, Worcester
Solid mechanics, optimal control theory, numerical optimization

- Statistical Models Predicting Demand for Loans Against Permanent Life Insurance Policies

Sponsor: John Hancock Mutual Life Insurance, Boston
Statistical analysis, financial mathematics

- Pricing a Child Rider Insurance Policy

Sponsor: John Hancock Mutual Life Insurance, Boston
Probabilistic modeling, actuarial science

- Leak modeling in Pressurized Pipes

Sponsor: Veeder-Root, Simsbury, Connecticut
Differential equations, signal processing

- Risk Measures for Cession Strategies in Automobile Insurance

Sponsor: Premier Insurance, Worcester
Statistical analysis, optimization theory

- Design of a Bending Pipe

Sponsor: Morgan Construction, Worcester
Differential equations, solid mechanics, optimal control

- Estimating Success Probabilities of Variable Life InSurance via Simulation

Sponsor: John Hancock Mutual Life Insurance, Boston
Statistical analysis, simulation, actuarial science

- Optimal Control of HVAC Systems

Sponsor: United Technologies Research Center, Hartford, Connecticut

Differential equations, optimization,

## Conclusions

The REU program gives students valuable information for making career plans. In our 2-year experience, more than half of the students who were not considering going into a graduate program in mathematics or statistics have changed their mind.

I had not been planning to go to math graduate school but now I am. Before I had been leaning toward working with Russian/Linguistics but now I see myself working with statistics. Talk about a complete turn-around!

Before I had not been aware of math jobs besides actuarial work, professorships and accounting. Now I am interested in more industry-oriented mathematics.

I am now more interested in Industrial Mathematics and Statistics, now that I know how it is applied in the real world. I now better know my options for career choices.

I used to think that the only good job a person could get with a math degree was at a University and that the degree had to be a Ph.D. Now, I still want to get a Ph.D. but now I know that here are a lot of companies that hire mathematicians and they are not all insurance companies!

The REU in "Industrial mathematics and Statistics" gives the students an industrial experience when their career does not depend on it. It gives them a better feel for what options are available for careers in mathematics and what career is better suited for them:

I was ambivalent about grad school before the REU, but now I think I want to go... maybe for a Ph. D. in pure mathematics...

Students from our program have presented their work in MAA and AMS national and regional conferences. All the students who have already graduated from college went to a graduate program in mathematics (Ohio State University, University of Delaware, Boston College...)

The REU is one of the places very talented students find, maybe for the first time, how mathematics can be difficult and beautiful at the same time. One of the students concluded her survey:

Math is frustrating most of the time, always was, always will be.
and expressed her interest in going to graduate school for a Ph.D. in mathematics.

The REU in Industrial Mathematics and Statistics at WPI was very well appreciated by the participants:

This is a great program. I would have never guessed it is only in its 2nd year. Everything was extremely well organized - the
projects, the field trips, the housing. I would highly recommend this program.

Worcester Polytechnic Institute, Worcester, MA 01609
E-mail address: vernescu@WPI.EDU

## Part II

## DESCRIPTIONS OF SUMMER ENRICHMENT PROGRAMS

# Carleton and St. Olaf Colleges' Summer Mathematics Program 

Deanna Haunsperger and Stephen Kennedy

The Carleton and St. Olaf Colleges' Summer Mathematics Program ${ }^{1}$ began in the summer of 1995 . Each year we admit eighteen lower-division students contemplating a major in mathematics or who have just declared a major in mathematics and who demonstrate some of the skills and a desire to continue on to a graduate degree in the field. By selecting lower-division students, we hope to provide these women with support and direction as they make choices critical to their future and to make certain that they have enough time (two or three academic years) to structure their undergraduate experience in a way that will best prepare them for graduate study.

The National Science Foundation has been the primary source of funding for this program since its inception with important supplemental funding contributed by the National Security Agency. The program is directed by Deanna Haunsperger, Stephen Kennedy, and Gail Nelson of Carleton College, and Jill Dietz of St. Olaf College. A former director, Laura Chihara, formerly of St. Olaf College, played an instrumental role in the development of the program.

## The Problem

The numbers are familiar to everyone in mathematics: between July 1, 1994 and June 30, 1995 U.S. colleges and universities awarded 20154 Bachelor's degrees in mathematics. Of these, 9061 (45\%) went to women ${ }^{2}$. In that same period 1226 Ph.D.s were awarded (fall count), of these $280(23 \%)$ went to women ${ }^{3}$. In this decade the percentage of

[^14]women among U.S. citizen mathematics doctoral recipients has been between 24 and 29. The growth in this percentage over the last twenty years is due, about equally, to increases in the number of female doctorates and decreases in the number of male.

Why do so few women (relative to men) pursue advanced degrees in mathematics? Although definitive studies on this problem have not yet been done, anecdotal evidence points to several causes. Often, talented women who are drawn to mathematics find it difficult to believe that they can have effective careers in the field. Few of their professors are women. The male students in the class seem to get more attention from the instructor, and, for whatever reason, seem to dominate the classroom discussions with their questions and responses. Most students demonstrate a lack of awareness about women mathematicians. Thus, women studying mathematics often have to deal not only with the difficulties inherent in the subject itself, but with the psychological and emotional problems caused by studying in such an environment.

While not all women students may be so affected, many are. In a 1984 survey of American graduate students in science, engineering, and medicine at Stanford University, all women and an equal number of randomly selected men in these fields were questioned. A comparison of the male and female students' "always" or "often" responses to the following statements illustrates gender differences in self-esteem. How often do you: fear speaking will reveal inadequacy ( $9 \%$ of males, $33 \%$ of females); question if you can make it in your field ( $9 \%$ of males, $24 \%$ of females). To cite a local example, the women who have participated in the Carleton and St. Olaf Summer Program for Women are among the strongest undergraduate mathematics majors in the country, yet many of them reported in post-program evaluations that, before entering the program, they had doubts about their abilities to succeed in a graduate program. It is a tragedy that some of our brightest young women are being lost to mathematics because of such factors.

## The Summer Mathematics Program

Students in our four-week program participate in two classes which meet on alternate weekday mornings from 8:30 to noon. We encourage the instructors to try new ideas, to teach courses the students would otherwise not see, and to challenge these extremely talented young women. The students each year report in post-program evaluations that they had been asked to do things they they did not believe they could do, but with the support of the faculty and of their peers, they had struggled to accomplish something difficult and meaningful.

We keep the students very busy: in addition to the coursework there is a twice-weekly colloquium series. Highlights have included Mary Ellen Rudin's visit (A History of Women in Mathematics), Marge Murray's visit (An Introduction to Clifford Algebras), and our annual visit from Joe Gallian (Breaking Drivers' License Codes) who not only gives a lovely colloquium, but also tells the students about the opportunities available to them in REUs. We always have talks by several local faculty, and each year we have a speaker from the National Security Agency and other non-academics.

In addition to the colloquium series, there are weekly panel discussions intended to give the women some guidance as they plan the rest of their education. The first discussion is about completing an undergraduate math major. We tell the students what courses one needs for graduate school and what kinds of extra-curricular activities they might seek out: the Budapest Semester, the MASS Program, Research Experiences for Undergraduates. The second discussion is about careers that use mathematics, usually at the B.A. or M.A. level. We bring in four or five women who use mathematics in their work. We have had people in operations research, actuaries, a statistician from the Mayo Clinic, computer programmers, business consultants, a computer security expert, and more. The third discussion is about how to apply to and succeed in graduate school; we always have current graduate students and graduate faculty present to act as resources. The final discussion is about the special difficulties and joys of being a woman in a scientific field.

Around and through it all, we weave outings and social activities: at least one picnic per week, movie nights, canoe trips, hikes, the Science Museum. Every weekend features at least one organized excursion. One evening each week we have recreational problem solving. One night each week Deanna visits the dorm lounge for "Deanna chat," a time for her and the students to check that everything is going smoothly. The program closes with a banquet celebrating success and honoring the participants; farewells are difficult for the women, many of whom found long-sought-after sisters in mathematics.

Students do not receive course credit for, or grades in, these courses. Each student does receive, at the close of the program, written evaluations from her instructors of her performance and accomplishments.

The community that develops amongst these students, teaching assistants, and faculty is reinforced through an electronic list-server which we maintain. Participants from past years still update us and each other on their lives through this list-server. Lifelong friendships and
networks have been formed. The directors plan to hold a reunion of participants at the winter Joint Meetings.

## The Students

We choose students who have taken a course in linear algebra and a small number of subsequent theoretical courses and who have demonstrated some interest in mathematics and gained some mathematical maturity from their courses. They are also at a level where professors are able to identify them as potentially benefitting from such a summer program. Typically, these students have just completed their sophomore year at a college or university. The students are selected based upon information contained in: two teacher's evaluations of the student's potential for advanced work in mathematics; the student's grades in mathematics to date; and the student's motivation and interest in mathematics, as evidenced by the personal essay she writes. The directors select students to achieve a good balance of academic backgrounds, considering such things as their home institutions and the classes they have completed. Applications are solicited through advertisements in the journals of the professional societies (FOCUS, AWM Newsletter, Math Horizons) and by mail sent to the chairs of all U. S. mathematics departments. We average 119 applicants per year, of whom 50 to $60 \%$ are very well qualified for our program. It is quite difficult to narrow our choices to the 18 we can accept.

## The Faculty and Teaching Assistants

The courses are staffed by women who are active professionals and outstanding teachers. Each of them has thrown herself into the program, and given of her time and energies in ways that we did not imagine would happen, and would never dare ask of anyone. Each has described her time in the SMP to us as the most fulfilling, rewarding teaching experience of her life. Those instructors (with affiliation and course title) have been: Judy Kennedy (University of Delaware; Dynamical Systems), Gail Nelson (Carleton College; Knot Theory), Tami Olson (Michigan Technological University; Applied Functional Analysis), Laura Chihara (St. Olaf College; Algebraic Coding Theory), Karen Brucks (University of Wisconsin at Milwaukee; Low-Dimensional Dynamics), and Rhonda Hatcher (Texas Christian University; Game Theory).

The teaching assistants are alumnae from an earlier SMP, whose experience with upper-level mathematics courses and the SMP itself has been much appreciated. The teaching assistants help the students with mathematical problems, live and dine with the students, help
organize outings, cheer the sad, and build an esprit de corps amongst the girls.

The faculty and teaching assistants are expected to, and do, serve as role models of professionalism and dedication to mathematics. It is hoped that the faculty will also lay the foundation for a long-term mentoring relationship with each student in the program. This is happening: the students from the first years of the program write to us and to their instructors for advice about courses and graduate school.

## The Outcome

Our 18 young women mathematicians immerse themselves in mathematics, living and working in a supportive community of women scholars (undergraduates, graduates, and faculty) who are passionate about learning and doing mathematics. Our intentions for them are threefold: to excite them about mathematics and mathematical careers, to provide them with some of the tools they will need to succeed in a mathematical career, and to connect them to a network of fellow female mathematicians. We have been successful, as measured by the participants' post-program evaluations and a survey conducted in 1998, in achieving all of these goals.

The students, faculty, teaching assistants, and directors all confess to being profoundly influenced by the program. The following quotes are culled from student's post-program evaluations:

This experience has revived my mathematical soul and charged me up.

Thank you for an awesome experience. It is something I will remember for the rest of my life.

This has been, by far, the most exciting and fun experience in math I've ever had.

The program has certainly confirmed my desire to major in math at the undergraduate level, and it has revealed more options than I previously expected for graduate study and an eventual career.

But the more I listen and learn of myself, I know that I can't stop after college and I won't stop until I feel that I've learned enough. And when I come to your grad school panel in a few years I'm gonna tell some really good stories.

To learn about other womens' experiences, to be encouraged, supported, to have so many people believe in me,
and to connect with such brilliant and fun women was awesome!

I found it inspiring and informative to be around so many knowledgeable people with whom I could easily connect and communicate. The information and contacts that I have gained are invaluable. Everyone should have something like this at some point (sooner rather than later).

| 1995 | 4 Budapest/MASS program 7 REU or summer research 2 Other women's program |
| :---: | :---: |
| 1996 | 1 Budapest <br> 8 REU or summer research <br> 1 Women's program <br> 1 NSA Summer Research |
| 1997 | 1 Budapest <br> 7 REU or summer research <br> 1 NSA Summer Research |
| $\begin{gathered} 1998 \\ \text { (so far) } \end{gathered}$ | 5 Budapest <br> 8 REU or summer research <br> 1 Women's program |

Table 1. SMP alumnae, further enrichment.
The students return to their home institutions eager to plunge into their studies. They have a clearer idea of what mathematics is and how to organize their future plans. Their increased awareness of various topics within mathematics have led many to give talks in their home departments on the mathematics that they learned in the summer program. Most have already participated in REUs, the Budapest Semester, or other enrichment programs (Table 1). All who have done so acknowledge being much better prepared to succeed at, and benefit from, those programs than they otherwise would have been. More important than the knowledge and renewed excitement for mathematics, each of the students has gained confidence in her ability to do mathematics.

It challenged me, but I was able to work through the proof, I really put my heart into it-and I loved it. It was not only good for me alone, but it was so special to have a class full of people who could handle this level of self-motivated, rigorous learning.

But now, given this opportunity, I'm excited for school to start in the fall and I'm excited that I am a smart and intelligent math student. Really I am. And most of all, I don't need to prove it to anyone - just to myself.
[Studying mathematics in a group of women students] is the best! People explained things so that others would understand, and people kept telling each other, "Good call," "great idea," or "you're brilliant." You don't hear that studying with guys. It is very reassuring to discover that almost everyone else has the same insecurities and self-doubts and when you realize everyone else's are unfounded, it starts to chip away at your own. A very positive experience.
Most of all the program has given me the confidence that I can succeed in math, both as a student and as a woman.
I have proved things which I had doubted I could.
The satisfaction one derived from finally completing that proof they had been working on for a week was tremendous. It taught you that you could do things on your own.
The [SMP] has given me the confidence, the mathematical foundation, and the desire to propel myself through advanced studies of mathematics. It has been an immense force driving all of us on to higher plains of mathematics. The courses and people contributed to an environment which has nourished and developed our mathematical souls.
This confidence building is central to the mission of the program. All of these students, and most of the other one hundred who applied, are intellectually capable of achieving an advanced degree in mathematics. Something other than intellectual capacity prevents many women from pursuing one. Heightened self-confidence and a supportive network of colleagues and mentors are two factors which we hope will prevent young women from dropping out. These students return to their home institutions knowing that women can and should be doing mathematics. They will not only be supported by this knowledge, but they also will carry the message back with them to influence their peers and their teachers.

Table 2 gives up-to-date information on where the SMP alumnae went after graduation (or, in the case of those not yet graduated, where they plan to go).

|  | 1995 | 96 | 97 | TOTALS |
| :--- | :---: | :---: | :---: | :---: |
| Grad school in math or related field | 7 | 11 | 13 | 31 |
| Career in math-related field | 8 | 4 | 0 | 12 |
| Other grad school or career | 3 | 3 | 0 | 6 |
| Undecided | 0 | 0 | 3 | 3 |

Table 2. Post-graduation outcomes/plans for SMP alumnae.

## Conclusion

We have an impact on the lives of the young women who come to our program - we see the increased confidence, enthusiasm, knowledge, and mathematical sophistication. We see the electronic messages they post on the program's list-server to let us and each other know what is happening in their lives - mathematical and otherwise. It is less clear to us how to measure this effect. We can never know how many would have gone on to productive mathematical careers without us given the talent level some certainly would. We won't know for some time how long and how far the impetus we give will sustain them in the face of adversity. We do believe that we are making a difference though: last week Cathy (SMP 95) told us that she was headed off to a top-twenty mathematics department to get her Ph.D. She had followed up her SMP experience with two other summer experiences, including an REU. But, she said, neither of those programs had the kind of transformative effect on her that this one did. "If I hadn't been in your program, even with those other two programs, I wouldn't be going to grad school in math."

Carleton College, Northfield, MN 55057
E-mail address: dhaunspe@carleton.edu, skennedy@carleton.edu

# Summer Bridge Program at the College of Charleston 

Dinesh G. Sarvate and Willette S. Burnham

## A Short History

The College of Charleston has offered the Special Transition program SPECTRA since 1983. The program was designed and implemented in response to the college's concern about the recruitment, retention and attrition rates of its African American students and it desire to enhance their academic success. Originally, the program was funded to serve both new freshmen as well as transfer students and both residential or commuting students. In 1993, after assessment of the program's objectives and outcomes, it was revised. With funding for 70 students including room, board and cultural enrichment programming SPECTRA became a mandatory residential program for eligible minority students, requiring enrollment in two courses. Additionally, the program was expanded to include a mentoring component and a tracking system to expand the transitional support throughout the first year experience. The program 4.5 week program is offered each summer. In 1999, 149 students participated in SPECTRA.

In January of 1995, the College of Charleston joined a consortium of schools (University of South Carolina, Columbia, Clemson, SC State, Benedict, Midlands Tech, Claflin and Vorhees) that constitute the South Carolina Alliance for Minority Participation (SCAMP). SCAMP is an NSF funded initiative to encourage minority enrollment in Science, Engineering and Mathematics (SEM) courses. Specific goals of the program are to increase the number of African-Americans, Hispanics, Native American, Pacific Islanders, and other underrepresented

Received by the editor November 11, 1999.
minorities entering technical careers and receiving Ph.D.'s in these areas of study. The College of Charleston was notified in May 1996 that it had been identified by the Quality Education for Minorities (QEM) Network as one of the top institutions in South Carolina in the number of baccalaureate degrees it awarded to underrepresented minority students in mathematics and science.

There are two summer mathematics programs which are sponsored by SPECTRA and/or SCAMP: Summer Bridge Program and Summer Undergraduate Research Experience.

## Summer Bridge Program

This is a program for minority students interested in pursuing an SEM degree. It is combined with the College's SPECTRA program. Students participate in all SPECTRA activities, but enroll in a special section of pre-calculus. This class includes daily two hour math excellence workshops designed to improve the mastery of skills and concepts essential for success in the gateway courses. In the workshop students work in groups and review concepts they do not understand. They are also given more challenging problems to work on. Before the exams, review work is also done in the workshop. Successful students are also enrolled in a calculus class in the Fall. A calculus workshop meets three days per week. The students are required to attend the workshop in order to receive a $\$ 500$ scholarship from SCAMP. The first SCAMP summer bridge program at the College took place from July 17, 1995 to August 15, 1195. Fifteen minority students, all of whom had been accepted to the College and had indicated an interest in pursuing a degree in science, engineering, or mathematics, were accepted into the program. SPECTRA has funds to serve 25 SCAMP students from the South Carolina schools. The SPECTRA program is totally funded by the College of Charleston and receives no outside fiscal support. Unfortunately only 19 students were supported for the 1999 Summer Bridge Program because of the lack of funds from the SCAMP program for continued support for those students during the academic year.

The purpose of the summer bridge program at the college is:

1. To increase the students' depth of understanding of mathematical concepts needed to succeed in calculus.
2. To improve the students' confidence in their problem solving skills. 3. To introduce the students to college life and orient them to the college of Charleston campus, and to encourage them to form bonds with fellow students and faculty.
3. To assist to students in forming peer study groups and to convince them of the benefits of study groups in all classes.
4. To expose students to college and career options in science and mathematics.

Of course the three primary goals of SPECTRA for its minority participants go hand in hand with the purpose of SCAMP summer bridge program:

1. Develop a successful academic and social network for students of color.
2. Bridge the gap between traditional/stereotypical barriers that students of color face on majority white campuses.
3 . Introduce students to the rigors of the academic program.
Selection: Students are invited to apply for Summer Bridge SEM track as part of their application for SPECTRA. The purpose of SCAMP is to increase the number of SEM students. It is clear that the increase will come from those students who may not major in SEM if proper support is not forthcoming. First, only those students who expressed interest in becoming SEM majors are considered. Then students who have almost the same mathematical preparation but no calculus background are short listed. Those students whose SAT scores are very high are excluded as they will be successful without the extra workshop component or may be ready to go into calculus directly. The students are selected so that all potential majors are represented.

These criteria and the desire to give border line students a chance backfired in 1999. That year, 19 students were selected for the bridge but only 9 were able to pass the course successfully. In 1998, even though 17 students passed the summer bridge program only 4 students were able to get through Calculus in the whole academic year 98-99. In contrast, all nine students who passed summer bridge program in 1999 are enrolled in the Fall 1999. We hope that all these students will pass their calculus class.

The program offers cultural enrichment as well as social programs for its students. These programs include:

1. An historic tour of Charleston.
2. A tour of the Avery Institute for African American Culture.
3. A Charleston harbor evening dinner cruise.
4. Oratorical contest.
5. Talent show.
6. Closing program and banquet.

## Summer Undergraduate Research program

Minority SEM students already at the College are eligible to participate in a 8-week summer directed research and internships supported by SCAMP. The purpose of the Summer Research program at the College of Charleston is to expose students to a brief but intensive scientific research experience in order to:

1. Stimulate further interest in pursuing a graduate degree in their discipline of interest.
2. Allow the students to establish professional relationships with faculty members.
3. Allow students to establish relationships with other students interested in science and math and who are pursuing a graduate degree.
4. Give students the initial tools and skills to begin conducting their own research projects once they begin graduate school.
5. Orient students to writing research papers and making scientific presentations.

In the summer of 1995 three SEM students participated in research program. In the summer of 1996, four, in the summer of 1997, three and in the summer of 1998, three SEM students participated in the research project. In 1999, six students did either summer research or participated in undergraduate summer research workshops. All of these students were able to obtain external summer research fellowships or stipends. There were two students who did academic year research in 98-99. In short, from 1995 to 1999 the number of minority students who had a SEM research experience increased from 3 to 8 . Unfortunately, there are not many opportunities for summer research in mathematics and the number of mathematics related minority majors is smaller than the other sciences at the college. Still, there were three students who did their summer research related to Physics, Computer Science and engineering. Another math major went to a workshop. The two academic year research projects were in Physics and Mathematics. The students feedback about the summer research experience and their presentation of their research at various conferences and at
the college demonstrates that they benefited in a very positive and encouraging way and the purpose of summer research experience at the College is being met.

College of Charleston, Charleston, SC 29424
E-mail address: sarvated@cofc.edu

# Five Years of Summer Programs for Women in Mathematics at The George Washington University 

Murli M. Gupta, Director, SPWM

During the summer of 1995, The George Washington University Mathematics Department hosted a 4 -week pilot program for 10 outstanding undergraduate women mathematics majors from around the United States. This pilot program was funded by the National Security Agency (NSA) and GWU. The program, by every measure, was a terrific success. All of the participants were glowing in support of the program. Every one of the directors, instructors, teaching assistants, and visitors felt that the program provided a tremendous benefit to each participant. In 1996 and 1997, we hosted a similar program for 16 undergraduate women. This program was expanded to 5 weeks in 1998 and 1999. In the five years of existence of our summer program, we have hosted 74 bright undergraduate women at GWU. We plan to continue this highly successful summer program and provide an optimal experience for the participants.

Detailed information on our program is available online at http://www.gwu.edu/ ~math/spwm.html

## OVERVIEW

The Summer Program for Women in Mathematics (SPWM) is a 5 -week program designed for outstanding undergraduate women majoring in mathematical disciplines who have completed their junior year and are considering graduate study in the mathematical sciences. Goals of this program are to communicate an enthusiasm for mathematics, to develop research skills, to cultivate mathematical self-confidence and independence, and to promote success in graduate school. We bring

[^15]the participants into contact with successful women mathematicians in academia, industry, and government. We aim to provide the students with a broad exposure to mathematical culture, illustrating the beauty and attraction of mathematics, the tools necessary for success in mathematics, applications of mathematics to business and industry, and the career opportunities available to mathematicians.

The academic program is centered around two 3 -week courses and two 2-week courses. All of our courses are led by professional women mathematicians, who are assisted by highly qualified graduate students. Topics for these courses aim to complement the typical math major curriculum and are focused to permit the students to reach interesting open problems in a relatively short time. The plan is to lead the students to the forefront of current research, so that they might learn to appreciate the mathematical research enterprise.

Throughout the five weeks, the program provides extensive contact with guest speakers who give expository talks on the areas of their research interests. Some of the guest speakers also address mathematical history, mathematical ethics, and mathematical philosophy. The guest speakers are available to participate in discussions about their careers and personal and professional experiences.

We organize panel discussions on the issues of careers and the job market, graduate schools, and gender issues. We arrange field trips to visit with women mathematicians at work in the many centers of mathematical activity in the Washington area. A series of mathematical films is offered, and the program allows ample time for self-paced work as well as for reflection, recreation, and relaxation.

Student participants, graduate assistants, and faculty are housed in dormitories on the GW campus, in order to foster an atmosphere of community and camaraderie.

## GOALS AND OBJECTIVES

The GW Summer Program for Women in Mathematics aims to:

- provide an immersion program representative of key aspects of graduate school and professional mathematical practice
- promote active mathematical thinking
- underscore the beauty and enjoyment of mathematics
- foster a camaraderie among the participants that emphasizes collaboration and peer support
- bring the participants into contact with active mathematical researchers through a program of guest lectures and field trips
- provide interaction with a wide variety of successful women in mathematical sciences who serve as role models
- illustrate the role of mathematics as the foundation of the sciences and the wide range of mathematical applications in government, business, and industry through first-hand contact with applied mathematicians
- provide students with information about graduate schools and careers in mathematics.


## SETTING

SPWM participants are housed on GW campus in downtown Washington, DC. All student participants and teaching staff (including graduate students and professional mathematicians who lead the instructional program) reside in a dormitory. All participants have access to the library, computer, and recreational facilities on the GW campus.

Washington area is an ideal location for a program to immerse students into mathematics. There is, around Washington, a thriving community of pure and applied mathematicians working at federal government agencies and laboratories, at major universities, and in high-tech industry. The nation's single largest employer of professional mathematicians, the National Security Agency, is also located near Washington. We take advantage of our unique location by visiting mathematicians at many such sites.

## INSTRUCTIONAL ACTIVITIES

During each of the five weeks of the summer program, we devote four days to activities based on campus and one day to a field trip associated with careers and applications of mathematics, with weekends left for independent and group study, consultation with faculty and graduate students, rest, and relaxation.

The activities based on campus are centered around two 3 -week courses and two 2-week courses. Each course is led by a professional woman mathematician and a graduate teaching assistant. Each course focuses on an accessible area of current research and involves the participants in group work, problem solving, mathematical writing, speaking, library research, and computation. Each course provides a learning environment in which lecturing plays a minimal role, with the faculty members doing more questioning than answering, more guiding than revealing. Teamwork is strongly encouraged.

The students have access to computer labs on campus, providing a flexible environment for symbolic and numerical calculation. They have access to the internet and through it to the world- wide web; they also have complete access to the Gelman Library.

The students take an active role at all times. Working individually and in groups, the students explore, experiment, discover, formulate, conjecture, and prove significant mathematical results. Oral and written presentation are an important component; on the last days of each course, the students present written and oral reports summarizing various aspects of the course. (Often, proceedings of such presentations are also prepared.) In this environment, students discover that they have the power to do mathematics on their own. They develop the self-confidence to engage in independent work and the necessary communication skills to engage in the kind of collaborative efforts that produce much of today's new research.

## GUEST LECTURES AND EVENINGS

Our program of guest speakers is intended to bring the participants into contact with a wide variety of mathematical professionals. We invite several guest speakers who inspire, stimulate, and inform the participants. We coordinate the topics for the guest lectures with the mathematical content of our classroom activities, both by preparing the students beforehand and by allowing time for discussion afterwards. The speakers interact with the participants before and after their lectures, and entertain discussions on their background, education, and their careers. Our participants found that they obtained "more glimpses into the wide, diverse world of math," and that they enjoyed the opportunity to interact with the speakers before and after the lectures.

Many evenings are focused around problem sets, which are designed to intrigue students. The problems allow students to explore examples, conjectures, fallacies, paradoxes, definitions, theorems, and generalizations. The faculty and teaching assistants are available during these evening hours to stimulate or encourage student participants and to assist in the development of collaborative problem solving groups.

Some evenings are devoted to mathematical films, selected on the basis of their mathematical content, the quality of their presentation, and their cultural and philosophical insights. The films supplement our guest lectures and the curricula in our classrooms; each show is followed by a group discussion. Our participants find these films to
be entertaining and informative, and gain historical and cultural perspectives which are not customarily present in a typical mathematics classroom.

## FIELD TRIPS AND PANEL DISCUSSIONS

One day each week is devoted to a field trip. These trips bring the participants into contact with women mathematicians in their own workplace and expose the participants to current issues at the forefront of mathematics, the wide variety of applications of mathematics, the depth and complexity of the kinds of mathematics involved, and the possibilities for careers related to mathematics.

Our field trips have been to the following institutions:
National Institutes of Health, Bethesda, Maryland
NASA Goddard Space Flight Center, Greenbelt, Maryland
U.S. Naval Observatory, Washington, DC

Naval Surface Warfare Center, Carderock, Maryland
National Institute of Standards and Technology, Gaithersburg, Maryland

National Academy of Sciences, Washington, DC
Smithsonian Institution, Dibner Library, Washington, DC
National Security Agency, Ft. Meade, Maryland
Center for Computing Science of the Institute for Defense Analysis, Bowie, Maryland

We organize several panel discussions to address several issues associated with the mathematics community, including careers, the job market, and graduate schools.

## SELECTION OF PARTICIPANTS

Our participants are women undergraduates who have completed their junior year and are considering graduate study in mathematics. We are especially interested in attracting students who might not have access to, experience with, or information about graduate study in mathematics. We expect students to have some experience with mathematics beyond the typical first courses in linear algebra and differential equations.

We mail program announcements and application packets to all degree granting institutions in the United States. We also make program announcements through electronic networks, print journals, and through electronic mailings to previous participants and other individuals. The program webpage at http://www.gwu.edu/ ${ }^{\sim}$ math/spwm.html
is visited by a large number of interested students who often download the application material directly from the program homepage.

In our pilot year in 1995, we received 71 applications from which we selected 10 participants. In 1996, we received 103 applications; in 1997, we received 148 applications; in 1998 we received 150 applications; and in 1999 we received 106 applications; we selected 16 program participants in each of these four years. We continue to receive a large number of mail, phone and email inquiries throughout the year. We also distribute program fliers and other program information at AMS and other meetings. The selection process is based upon the students' college transcripts, personal statements, and letters of recommendation.

It is clear that SPWM has been very successful in its goals and objectives, and is providing a much needed resource to the nation's women undergraduates. We expect to be able to continue this program at GW for many years to come.

Student comments about the GWU Summer Program
We carry out program evaluations through extensive formal and informal feedback from the participants. During our programs in 19951999, we asked the participants to provide written comments at the midpoint and at the end of the program. The typical comments are:

- "I gained a wealth of information and insight. I'm now convinced that I can succeed in grad school."
- "Just being around peers and faculty and other professionals helped a great deal and I've made a [career] decision."
- "I feel the strongest part of this program was the interaction between us the students with faculty, guest speakers, and TAs. I have learned so much from it and it has made this experience one which will have a strong impact on my life hereafter."
- "It was great reading the research papers. It was very encouraging to realize that I know enough to read other people's work."
- "Although we worked really hard, I think it was a good prep for grad school."
- "I would say that this has been the best experience of my undergrad career in many ways."
- "I am amazed at how much more aware I am now than I was a month ago."

Here is what an advisor wrote in 1997 about a program participant from our 1995 summer program:
"I believe that Karen's summer at GWU changed her life. When she returned to Stonehill in September, she was unable to contain her
enthusiasm. After her summer experience, she was certain that she wanted to pursue a career as a professor of mathematics." This alumna of our pilot program in 1995 is now successfully pursuing her graduate studies at the University of Virginia.

We plan to obtain participant feedback in a similar manner in the future years. In addition, we are carrying out follow-up surveys to determine the long-term impact of our summer programs on the program participants. Here is a sampling of the responses received from our program participants:

A 1997 alumna now a graduate student at Pennsylvania State University recently wrote:
"Regarding the summer program, well, it had a major impact on my life. Before I didn't have any idea what grad school was even about and I wasn't sure it was for me. The program was very informative and gave me self confidence in my abilities. It also prepared me for the reality of a first year in graduate school. I knew what to expect. I would not be where I am today without the program! You should definitely continue the program. I am definitely interested in attending the SPWM reunion if it works out."

Another 1997 alumna now a graduate student at the University of North Carolina wrote:
"Your program was pretty much the foundation for me to start thinking about grad school and what I wanted to do after graduation. Especially for people from smaller schools, there really is nothing that compares to SPWM for learning about the possibilities for mathematics students. I also think the opportunity to form friendships with all the others was wonderful."

A 1996 alumna now a graduate student at the University of Kentucky wrote:
"Everything about SPWM has my resounding support. I completed my undergraduate degree at a small college in Illinois. Though I enjoyed my undergraduate mathematics courses, my exposure to present day research was limited to say the least. At that point in my life, mathematics was interesting but dead. SPWM changed all that. For the first time in my life, I was exposed to the fascinating world of mathematical research. SPWM not only made graduate school possible for me, it made graduate school an exciting possibility."

Another 1996 alumna now a graduate student at the University of Wisconsin- Madison wrote:
"With regards to the summer program, I think that it was very helpful. It gave me some idea of the kind of work and work load to expect in graduate school. It also encouraged me to try graduate school

- basically, that I could do the work and succeed. I think that it was a good thing for me to do, and I think that it can benefit a lot of other women as well."

A 1995 alumna now working for Lockheed Martin wrote:
"Even though your program did not have any direct impact on my life, it has made me a more rounded and better educated person. If I had it do all over again, I would still attend your program. It was very nice to meet other women who excelled in math. In most of computer science and math classes I was one of a few women and the only one who excelled. I hope you continue your program."

Here are some more comments received from two other participants:
"The program helped give me more confidence in my mathematical abilities and gave me a better idea of what there is out there to do within this field. ... I do feel some of what was expected of us was very intense but it did help prepare me for what to expect in graduate school."
"The summer program definitely had an impact on me. It made me more enthusiastic about mathematics in general. It also made me more aware about opportunities for grad school. I think this program should definitely continue."

[^16]
# The Mills College Summer Mathematics Institute and The Berkeley Summer Institute for the Mathematical Sciences 

Deborah Nolan

## 1. Introduction

From 1991 to 1997, 155 undergraduate women participated in the Mills College Summer Mathematics Institute (SMI) and the Berkeley Summer Institute for Mathematical Sciences (SIMS). By many accounts these programs were highly successful: to date, 20 of the participants have received $\mathrm{PhDs}, 24$ have received Masters degrees, and 49 are attending graduate school in the mathematical sciences. However, many of the benefits that these women received from the program can not be measured by these numbers alone.

We found that SMI and SIMS provided a valuable short-term intervention for its participants at a crucial stage in their development. The program brought together a critical mass of talented women math majors. These women found it exciting to study mathematics with other women. They encouraged each other, they saw that women can and should study advanced mathematics, and they felt part of a growing network of professional women mathematicians. The program provided them with valuable role models and mentors, practical knowledge of what to expect in graduate school, and it provided them with an experience they could carry with them, an experience that helped prepare them mathematically, pragmatically, and emotionally for graduate school.

Received by the editor September 28, 1999.

## 2. History

The Mills SMI was conceived in 1990 during a student strike at Mills College. The Trustees of the college had launched an unsuccessful attempt to make the college coeducational. Student opposition to the proposed change kindled discussions on the benefits of being educated at a women's college, and from one such discussion Steven Givant, Mills College, and Leon Henkin, U.C. Berkeley, hatched the idea for the SMI. The Mills SMI was modeled on a program for minority students that Henkin and Uri Treisman, U.T. Austin, had started at Berkeley the preceding summer. Givant and Henkin worked with Lenore Blum, International Computer Science Institute, and Diane McIntyre, Mills College, to develop the Mills SMI, which began in the summer of 1991.

Although, the program originated at Mills, it was held on both the Berkeley and Mills campuses. In the first year of the program, the Mills dorm, where the participants were housed, was closed for renovations in the fourth week of the six week summer session. Out of necessity, the program moved to the Berkeley campus for its final two weeks. There it was discovered that the students thrived on the research environment at Berkeley with its numerous summer visitors, excellent library and computing facilities, and vibrant graduate student life. The next year, the program was split again with four weeks on the Mills Campus and two on the Berkeley campus. After an overly quiet summer program held entirely on the Mills campus in 1993, the program moved permanently to Berkeley in 1994. At Berkeley, the students were housed in the international graduate student dorm, and SIMS activities took place in the Statistics Department and the Mathematics Department, which are both in Evans Hall.

The first year of the program was funded by the National Science Foundation (NSF). After that, we received funding for three years from the NSF. However, the funding for running the 1993 and 1994 programs was cut successively by about $20 \%$. At that time, the point of view of the NSF was that they wanted to provide seed money for programs with new ideas. Established programs should seek funding from other sources. In response, we secured two generous donations from Genentech, a biotechnology company in the Bay Area. We also secured supplementary grants from the National Security Agency (NSA). From 1994 through 1997, the program was supported by yearly grants from the NSF and NSA. When the program officially came under the aegis of U.C. Berkeley in 1996, we also received funds from the University to help cover administrative costs.

As a result of the yearly fluctuations in funding, the size of the program varied from year to year. It was largest in 1992 and 1994 with 25 students, and it was smallest in 1993 with 18 students.

The directing body of the program also changed over the years, as instructors were invited to join in directing the program. Svetlana Katok taught in the first year of the program, and then directed the program with Givant in 1992. Deborah Nolan became a director after teaching in 1992. Ani Adhikari taught in the 1993 program, and later joined the directors. Helene Barcelo taught in the 1991 and 1994 programs, and helped direct the program in its final year in 1997. Henkin and Givant left the directorship in 1995, 1996, respectively.

The directors of the SMI also organized a conference in 1994 for mathematicians who were interested in starting their own summer math programs for women. The conference was held concurrently with the SMI, and was funded by the NSF. Through the conference, we identified and helped initiate the Summer Mathematics Program at Carleton and St. Olaf Colleges, and the Summer Program for Women in Mathematics at the George Washington University. Another outgrowth of the conference was the publication of MAA Notes 46, Women in Mathematics: Scaling the Heights (Nolan, 1997). This book contains seminar descriptions from SMI/SIMS faculty, other program descriptions from conference participants, and perspectives from mathematicians who have been active in the promotion of women in the field.

## 3. Program Description

Seminars. The heart of the summer program consisted of four seminars - typically two were in classical areas of pure mathematics and two were in areas of applied mathematics. Each student took two seminars, one pure and one applied. The topics of the seminars were usually in areas of mathematics that are not part of the traditional undergraduate mathematics curriculum.

The character of the work was very different from that which is encountered in typical undergraduate courses. Challenged by their instructors, students immersed themselves in the material of their seminars to discover and prove results, without the aid of textbooks. They were given many hard problems, and were encouraged to work on them in small groups as well as individually. They gained experience in the process of searching for and writing up proofs, and they learned how to obtain and express mathematical ideas verbally and in writing. Students were assigned two projects per seminar. In one project, they worked individually, and in the other they worked in small groups of 3 or 4 . The project may have included reading a journal article or doing
independent research. Each student presented her findings both orally and in a short paper.

Seminars met two times a week, each time for two hours. There were also two section meetings a week, where students received help from the teaching assistant in mastering ideas presented in the seminar and facilitating group projects.

In addition, faculty met twice individually with each student in their seminar. One meeting took place near the beginning of the program, to learn about the student's background and to help her choose appropriate projects. The other meeting was near the end of the program, to discuss the student's progress and help her plan for the future.

FACULTY. Each seminar was led by a woman who was an active research mathematician and a talented teacher. Teaching assistance was provided by women who were graduate students in the mathematical sciences. The teaching assistants were drawn mainly from U.C. Berkeley and Stanford University.

The SMI/SIMS faculty were: A. Adhikari, Stanford; H. Andréka, Math. Inst. Hungarian A.S.; D. Wallace, Dartmouth; H. Barcelo, Arizona State, Tempe; A. Bluher, NSA; M.L. Bonet, U. Pennsylvania; L. Butler, Haverford; A. Etheridge, Cambridge U.; E. Flapan, Pomona; S. Katok, Penn. State; C. Kenyon, C.N.R.S. Lyon; N. Mackey, Michigan S.U.; M. Murray, Virginia Polytechnic; D. Nolan, U.C. Berkeley; V. Pless, U. Ill. Chicago; U. Porod, U.C. Berkeley; A. Radunskaya, Pomona; L. Schneps, C.N.R.S. Paris; A. Thompson, U.C. Davis; L. Walling, U. Colorado, Boulder; and S. Wang, Mills.

Of the 21 faculty who have led seminars, 5 returned to lead seminars for a second year; and 3 have directed the program. In addition, 8 have contributed descriptions of their seminars to MAA Notes 46 (Nolan, 1997).

COLLOQUIUM. In addition to the seminars, a series of colloquium talks were held twice a week. The talks introduced students to a vast array of advanced mathematical topics. In 1996 the speakers included Ken Ribet on Diophantine problems and Hellegouarch-Frey curves; Galia Dafni on the Fourier transform and analytic number theory; Abby Thompson on knots and 3-manifolds; David Aldous on shuffling, sorting and randomness; David Blackwell on Polya urns; Elinor Velasquez on the dynamics of springs and particles; Valentin Rybenkov on topological equilibrium in DNA; Larry Gonick on cartooning mathematics; Charles Pugh on Brouwer's fixed point theorem; and Dana Randall on random tilings on lattices.

PANELS, INFORMATION SESSIONS, and SITE VISITS. Four evening events were planned each summer. First was an information
session on the process of applying to graduate school and finding financial aid. A second evening was set aside for a panel discussion on graduate programs, where mathematicians from several universities spoke about graduate schools, the different programs available, the graduate school experience, and the difficulties and rewards one might encounter in graduate school. A third event was an informal discussion held with the seminar leaders, where they give a brief history of their professional career, including how they decided to go to graduate school, why they chose their field of interest, what it is like to do research, what an academic career is like, and how they balance family and career. Finally, a panel was held where representatives from organizations that conduct mathematical research outside the university discussed different career opportunities in the mathematical sciences. In 1996 and 1997, we organized site visits to some of these organizations. Students visited the Lawrence Berkeley Laboratories, Genentech Corp., and Barra Inc.

Each year, we also organized visits to: the Mathematical Sciences Research Institute at Berkeley, Stanford University, and U.C. Davis.

## 4. The Students

Each year, we sought 18 to 24 women who had completed, with top grades, two or three years of undergraduate mathematics courses (including some upper division work involving substantial exposure to proofs). To evaluate applicants, we requested two faculty evaluations, a math course report that included the textbook used in each course, an official transcript, and a personal statement.

We primarily admitted students from small institutions which are unable to offer the diversity and strength of courses needed to make their students competitive for admission to a strong Ph.D. program. Other students were from large regional state universities, places that might not provide enough encouragement to students to continue their studies. Some students admitted to the program were studying at well-known research institutions with strong mathematics programs. We found that students from these institutions also benefited from our program, and that these students often inspired the others to continue their studies in mathematics.

Of the 67 participants in the 1991, 1992, and 1993 programs, 20 have received PhDs and are now working as research mathematicians; 13 are currently in graduate school, in the mathematical sciences; and 10 have received Masters degrees in the mathematical sciences. Of the remaining 24 students, 12 are employed in quantitative fields as consultants, actuaries, high school math teachers, and software developers.

Of the 68 participants in the 1994, 1995, and 1996 programs 37 are currently in graduate school; 15 have obtained Masters degrees in the mathematical sciences; and 6 of the remaining 16 with Bachelors degrees are employed in quantitative fields.

## 5. Evaluation of the Program

${ }^{1}$ We undertook a comprehensive evaluation of the program in the summer of 1996. The evaluation had three components:

- Surveys of the 1993 and 1994 participants, for information on the influence the program had on them.
- Surveys of the faculty who wrote letters of recommendation for students admitted to the 1994 and 1995 programs, for their perceptions of the effect the program had on the student.
- A brief survey of the graduate advisors of the 1991 and 1992 participants who were in graduate school in 1996 and working on a thesis, to see whether the student is making satisfactory progress towards her degree.
We include a summary of some of the main responses to these surveys here.

Results from the Student Survey. Altogether, $80 \%$ (34 out of 43) of the students responded to the survey. The questions in the surveys for the 1993 students and the 1994 students were similar; however, the 1994 students were asked about the effect of the program on the following year of undergraduate school. Over $60 \%$ of these students said that the program had a great deal of effect on their choice of advanced undergraduate courses. (Others pointed out that their schools did not offer many advanced courses.) The students also engaged in mathematical activities outside the classroom: over half gave talks at their home institutions, and over half participated in math clubs or conferences. Students appreciated the wealth of information provided by the program on the process of applying to graduate programs and applying for fellowships.

Questions about the effect of the program on their graduate experience were asked of both 1993 and 1994 students. Students are admitted to the SMI only if they have strong math records. Thus, one might expect them to be strongly predisposed towards graduate school before coming to the program. Nevertheless, the responses to the survey make clear that the program plays an important role in shaping their decisions about graduate school.

[^17]About half of the students said the program had a great impact on their motivation to do graduate work, and about half said that it provided them with a sense of what grad school would be like.

I feel that I was better prepared to handle the demands of graduate school ... The atmosphere of the program opened my eyes as to what would be expected of me.
The strongest influence of the program is on the students' estimation of their own capabilities. Over two-thirds reported that their self-confidence was greatly enhanced by the SMI, and these results were confirmed by the undergraduate and graduate advisors. According to one student,

Perhaps the program's main advantage for me is that I feel very comfortable being in grad school. That is, I feel that I belong here, as opposed to some of my female peers who have many doubts about their ability and place in this environment.

Over $60 \%$ of the students strongly agreed with the statement, "My work in the program showed me I enjoyed doing challenging math." It is worth noting that two of the students who strongly disagreed also said that the program convinced them that graduate school was not their goal; this is a valuable lesson, even though in a sense it is negative. In addition, over half the students strongly agreed with the statement that the program showed them "how to learn advanced math." This percentage is surprisingly high, given that students in the program are selected for their ability to do mathematics, and it underscores the difference between work in the SMI and in standard undergraduate classes.

In end-of-program evaluations, the students have been consistently and overwhelmingly positive about the "all women" nature of the program. It is now clear from the survey that this effect is long-lasting. Over $80 \%$ of the students have stayed in touch with fellow students from the program, and over $70 \%$ with their professors in the program. Over half the students asked for letters of recommendation from their SMI professors. A student sums up the opinion of the vast majority as follows:

Until attending the SMI, I had only had one female math professor. Ever. I think I now have a great advantage in having discovered some positive female role models in mathematics. ... I found the program ... to be extremely helpful to seeing myself as a mathematician. I don't recall ever seriously being told that girls don't do math. But on
the other hand, until attending SMI, I rarely actually saw them doing it.

## 6. Results from Undergraduate Faculty Survey

The response from the undergraduate faculty who wrote letters of recommendation for the SMI participants was very positive. Three quarters (33) of them responded. Six were unable to judge the effect of the program, because they had no contact with the student after her return.

For the faculty who did have contact with the student after her return, more than $80 \%$ said the program was very beneficial. Also, about half said their student's participation in the program had an effect on the whole department. Two themes recurred throughout their comments: they noticed a tremendous increase in the self-confidence and in the mathematical maturity of the student upon her return. One respondent commented,

Clearly, the major benefit to [her] was realizing that she was the mathematical equal of some of the most talented women of her age. While her background in course work was not as strong as some, her mathematical training and ability let her participate as an equal. This was exceptionally helpful to her and did wonders for her self-confidence.
Concerning a different student, another wrote,
[She] mathematically matured a great deal as a result of your program. I noticed that the analytical and topological concepts ... meant a lot more to her than they did to other students.
The students with whom the six respondents had no contact were from Princeton, Stanford, Berkeley, and Johns Hopkins. All of these students did stay in close contact with their SMI professors, receiving letters of recommendation and advice about graduate school. Two of them even wrote undergraduate theses under the long-distance supervision of their SMI professors. We interpret this as evidence of the benefit of the SMI to undergraduate women at major research universities: the SMI provides them with invaluable support that is missing at their home institutions.

The faculty respondents also provided information on the record of their institution in sending women to graduate school in mathematics. At roughly one-quarter of the schools, at most one female student goes to a graduate program in mathematics in any five-year period; at onequarter, one female student goes every other year; at one-quarter, one
goes about every year; and at the remaining quarter, about two go each year. These numbers make it clear that undergraduate women usually have little contact at their home institutions with other women who plan to attend graduate school in mathematics. By way of contrast, at the SMI, students find themselves part of a significant group of women dedicated to math, over two-thirds of whom go on to do graduate work in the mathematical sciences.

## 7. Results from the Graduate Advisor Survey

In 1996, sixteen of the 1991-92 students who responded to our survey had begun work on a thesis, and of these, 13 allowed us to contact their thesis advisors. (Fourteen of these 16 have completed their Ph.D.s and the remaining two expect to finish in the next year). The advisors' responses were unanimous: each student is making satisfactory progress toward her degree. Many advisors responded with accolades such as: "one of our best in the past five years," "the most dedicated student that I have ever seen," "a model graduate student," and "a lot of self-motivation." We see a confirmation of these opinions in the quantitative part of the survey. According to the advisors, the students have a great deal of self-confidence and motivation, and they arrived at graduate school knowing what to expect.

When asked "How does the student compare to other women in your program in terms of adjusting to the demands of graduate work?" the advisors had insightful comments, and offered great encouragement. For example, one advisor wrote:

As an undergraduate, [she] attended a small state college and received an education that really didn't give her the necessary background for graduate school. Nevertheless, she arrived here in graduate school imbued with confidence and the desire to work hard ... She has passed all her qualifying examinations and is now writing a dissertation under my direction. She is a joy to have as a student; she's talented and energetic. If your program had anything to do with this, then you should certainly consider it a success.

## 8. What Happens to the Faculty that Teach in the Program

We asked 12 seminar leaders from 1991 to 1995 (excluding faculty who had directed the program) to tell us what effect the program had on them. All responded to our request. They resoundingly confirmed that the SMI is a valuable opportunity for them as well as for the students. Here are excerpts from three statements.

From a 1991 seminar leader:
[The program] had a big and beneficial influence on me. I gained a unique teaching experience. The experience was beneficial for me from the point of view of my mathematical research also. Collaboration with Steven Givant in the theory of relation algebras turned out to be very fruitful and is marked with several joint papers since then. I had mathematical collaborations with other researchers in the Bay Area (e.g. William Craig, Richard Thompson).

I would recommend working in this program to any good mathematician, because I find it very precious and unique, and because it is really enjoyable for creative people.
From a 1993 seminar leader:
I cared about every single one of the women I had in my seminar at the SMI (they were all potential future women professors in the mathematical sciences), so I was highly motivated to make the seminar a wonderful experience for every student. This is precisely the attitude that . . . every liberal arts college professor must cultivate. ...

The SMI has created a network of women mathematicians nationwide. This year I have an NSF Visiting Professorships for Women grant. When I think about whom to invite to talk about doing mathematics with women graduate students, ..., I automatically think of professors who have taught in the SMI. My professional ties with several colleagues in my field have been strengthened because we share the experience of teaching in the SMI and related summer programs for women.

I think it is a wonderful service opportunity for women mathematicians with established research reputations.
From a 1994 seminar leader:
I have always tried to get students to participate actively in the classroom. However, while I was teaching in the SMI, I was able to engage students more both in my presentations of new material and through problems that they worked on in groups. After my experience in the program, I developed a clearer idea of how to ask questions in class which would get the students involved in thinking about the material, and telling me how the proofs should be done. Now almost all the proofs which are presented in
my classes are constructed by the students. Also, I began putting deeper problems on my homeworks, and making more of an effort to get students to work in groups. As a result I think that the students are learning the material better and becoming more interested in the material in my classes.

## References

1. Nolan, D. (1997) Women in Mathematics: Scaling the Heights, MAA Notes 46.
2. Adhikari, A., Givant, S., and Nolan, D. (1997) The Mills College Summer Mathematics Institute. In Women in Mathematics: Scaling the Heights, MAA Notes 46, pp97-104.

Department of Statistics, University of California, Berkeley, Berkeley, CA 94720-3860

E-mail address: nolan@stat.Berkeley.EDU

## Part III

## STUDENT EVALUATIONS OF SUMMER PROGRAMS

# Cornell 1999 REU Program Evaluation Summary 

Nora L. Balfour

Alex Raichev e-mail: ar41@cornell.edu
Home College: Cornell Major: Math
REU presented me with a great opportunity to do meaningful research with knowledgeable and helpful people. I've learned to work more independently and develop greater self-motivation, to concentrate on a problem for an extended period of time, and to communicate ideas more effectively. These skills are not only pertinent to mathematics, but to all aspects of life. Also, I have met great friends with whom I laughed and played. I thank everyone for their kindness.

Clark Good e-mail: clarkg@mail.utexas.edu Home College: UT Math BS

Overall the Cornell REU was a very positive experience. It provided an excellent opportunity to not only get exposure to different mathematics but also to learn more about what mathematics research is all about. The experience is valuable as an indication of what lies ahead if graduate school is what I pursue in the future. Lastly, the exposure to other people, both my age and in faculty, made the program worthwhile. If given a second chance I would do the program over again.

Arjun Raj e-mail: arjun@drumandbass.com
Home College: UC Berkeley Math and Physics
Well, I guess my overall reaction was positive. I felt that it really

Received by the editor September 15, 1999.
gave me a good overall understanding of how professional math operates, and thats really the main reason I came. In particular, I felt the seminars really let us into a lot of different fields of math at a level beyond what we see in classes. Also, the research part taught me a lot about the nature of math, and that math isnt always as pretty as it seems. However, I really didnt enjoy living at the lodge. Although I enjoyed living with the other members of the math REU, I really didnt like the conditions at the lodge all that much. As far as the research went, I was satisfied overall. Our project worked out, and was quite interesting towards the end. However, I felt that a lot of it was sort of scripted in that from beginning to end, we knew what we had to do; it was just a matter of doing it, which boiled down to a large amount of programming. Still, we were able to get some interesting results, and it seems that out work could prove useful in further research. Anyway, I had a pretty good time, and feel that Prof. Strichartz has done a good job in organizing a program which allows thought but still has enough structure to keep us moving along.

Vikki Kowalski e-mail: vkowals@entropy.uark.edu Home College: Univ of Ark, soon CalTech Math

I can't imagine a way I would have rather spent this summer. My project was intriguing, all the professors were enthusiastic (if idiosyncratic), and all my fellow students were just wonderful. In addition to enjoying what I was working on, I also loved living under the same roof as so many other math majors.

Michael Gibbons e-mail: mgibbons@student.manhatten.edu Home College: Manhattan College Math

My overall take of the program is that it was a great experience both for learning and socially. I got to see what is out there in the field of mathematics, with the smorgasbord seminars and interaction with the other two REU groups. The best experience of all is my effort in the research. Diving into an area in which I had very, very little previous knowledge and getting as far as I did, makes me happy. Working with someone was a great help with everything, and just forming a certain bond. This also goes back to the social experience, where living in the lodge with ten math students from all over the U.S. and becoming close, and just talking about mathematics. As far as future plans, I am unsure whether I will attend grad school right away or full time or go full-time and speed through and get a Ph.D. One thing I know
grad school will appear in my future, Im just not sure when. Teaching on a college level looks like something I would enjoy, if I feel I could have an impact on students. Otherwise, computer programming seems like an industrious choice of career, only it would be boring compared to teaching. So, in closing, I am very glad that I came to the REU program at Cornell, and Im sure its something I wont forget.

Cindy Chang e-mail: cmc.entropy@mail.utexas.edu
Home College: Univ. of Texas at Austin Math
If I were not given the opportunity to switch projects (from Dynamics to Geometry) I fathom I would have been somewhat unhappy, and I am thankful for the permission to have done so. Dr. Bezdek himself did an excellent job. He was very organized, compassionate and sensitive to the ability levels of each of his students. My actual project was intriguing and the reading material contributed positively. I have a newfound interest in Discrete Geometry because of the program. I do wish, however, that the project descriptions on the Web were more detailed. I had misjudged my abilities to perform in the Dynamics group. If a list of prereqs were listed (complex analysis, Diff Q...) or a more specific description of the project(s) was given, I probably would have applied to a different project. All in all, Cornells REU turned out to be an excellent program. I was pleased with most aspects and definitely sharpened my presentation skills. I would recommend this program to other undergraduates. Thanks.

Kai Ju Liu e-mail: kjliu@leland.stanford.com
Home College: Stanford Math
Initially, the learning curve was wonderful and I learned a lot in the first 3 weeks. The pace lessened somewhat as I moved gradually into research, but the final result was extremely satisfying. It was very nice to live together in a small house because it fostered a focused environment and, at the same time, gave us company to do things with. It would have been less of a bonding experience if we had lived in a larger dorm. All the professors were organized and helpful, as well as the office staff. Regular smorgasbord seminars and jam sessions also flavored the program nicely. I learned both what math research was like and how to present. Grade: A+

Grigoriy Blekherman e-mail: gb255@is8.nyu.edu
New York University Math

Working with Professor Bezdek was a very enjoyable experience. The program was a lot of work and this was perhaps the hardest working group of students I have ever been associated with. The research pace was quite intense this summer and since I was interested in the topics I spent a lot of time on the beginning and the middle of the program working but I couldnt sustain this pace throughout. Overall I think this program is an excellent opportunity for students to find out what mathematics research is really like, although I did feel that the two other groups work was a bit too computer oriented, but now, before applying to graduate school, I think I have a clear picture of what mathematical research is and what it entails and levels of concentration and dedication that it requires.

Scott Corry e-mail: corrys@read.edu
Reed College Math
The Cornell REU program provided a very stimulating environment in which to study new mathematics and conduct research. I now feel I have a firm grip on the subject matter of the dynamics of rational quadratic maps, and Ive also gained much programming experience over the course of the summer. The living accommodations were very nice and helped to foster a sense of community among the REU students. The bi-weekly mathematics seminars provided an informative smattering of various branches of mathematics. All in all, this summer was a very worthwhile experience.

Robert Meyers e-mail: rmeyers@virtu.sar.usf.edu New College Math

I will reiterate once again that I was challenged. At times I would know exactly where the research was going and at other times I was totally lost. Sounds like a research experience to me. So what more could I really ask for. It drove me to do my best and that must reflect positively on the setup of the participants in the program. I would suggest this program to another undergraduate in the future.

Scott Wilson e-mail: swilson3@ic3.ithaca.edu
Ithaca College Math
For me, this program was a wonderful experience. Not only did I learn a lot, but I had a lot of fun, too. The Professors and other students
were great to work with and I enjoyed going in every day to, perhaps, make a new discovery. The entire faculty was very helpful and learning seemed to come quite easily. Each student (or a group) was given problems to work on and encouraged to think of new problems and solutions on their own. Lastly, there were always some presentations going on that gave all the students a taste of what really goes on in mathematics. In all, the program was really great.

Cornell University, Ithaca, NY 14853
E-mail address: nlb11@cornell.edu

# The SCAMP Summer Program 

Blair Bradley

The SCAMP summer program was a blessing for me because I benefited so much from it. By working hard during the program I received a $\$ 500.00$ scholarship. The course was very fast paced, therefore you had no time to even blink. The professors were what made class so interesting to me. We were thankful to have Mrs. Prazack, a very prestigious professor at the College of Charleston. Teaching the SCAMP program was her last class that she would ever teach. The class and I were honored to have her. Dr.Sarvate was a unbelievable teacher. He would bend over backwards to help and make sure that you understand the material. He provided us with many extra help sessions. During the sessions he reviewed for tests that we would have. Any question you had would be answered at the extra help sessions. This was not the only extra help provided. Mrs. Jordan also had her own workshop which gave us the helpful hints and tips that made Precal so much easier. She provided us with practice handouts which she made herself. She made these handouts a little harder than what was expected. By doing these handouts along with your assigned work you do not have to study as hard for a test. The key to learning is practice. "By doing all you can do, when you can do, success is bound to be due." One important thing I learned during my SCAMP experience is the true value of time. Time is so precious, therefore you have to manage it properly so you do not waste any. By the end of our month stay I knew how to master my time. The SCAMP program prepared me for college life by giving me a taste of the academics at the College of Charleston. I have decided to attend summer school ever summer to get ahead. I plan to keep my own SCAMP program running on my own. In order to be able to do so, I had to attend the 1999 SCAMP program.

Received by the editor September 9, 1999.

# Summer Mathematics Research Experiences 

Stephen G. Hartke

## Introduction

For the past two decades, summer mathematics research experiences for undergraduates have become increasingly more common. These summer programs are an excellent opportunity for math majors to see if research in math is for them. From my experiences, I found that I really enjoy math research, and have since entered graduate school for mathematics.

I have participated in three summer programs, the REU at Lafayette College ('97), the REU at the University of Minnesota, Duluth, ('98), and the Director's Summer Program (DSP) at the National Security Agency ('99). All three were very worthwhile experiences that gave me different perspectives on a career in mathematics. The following comments are based on my experiences in these three programs, as well the experiences of students I have known in other programs.

## The Research

The primary objective of summer research programs is for undergraduate math majors to do research-something usually outside of the classroom experience that comprises the bulk of undergraduate education. If a student does have some research experience at his or her institution, this is usually in the form of a senior thesis-which occurs too late for determining whether or not to attend graduate school. The summer research experience thus fills a necessary part in the professional development of an undergraduate math major.

Received by the editor September 14, 1999.

That the summer program be a research program is very important. If the answers to the problems posed are already known, this is no different than a homework problem. On the other hand, the problems must be attackable. Giving the Riemann Hypothesis to a student to solve in ten weeks will probably lead to frustration, failure, and a sense that real research is simply beyond the abilities of the student.

The matching of a research question to a student is very difficult, and a good method is usually the reason for the success of the longerrunning programs. However, no one makes these assignments perfectly. If a problem is solved in a week, or is way too hard and no progress can be made, or even if the problem just does not spark the student's interest, then he should have the option of switching to a new problem. Although a program might concentrate in one specific area (say, graph theory), it should not be focused solely on one question (what is the maximum degree growth rate of the iterated line graph?).

The most important aspect of the problem selection is that the student must feel ownership of the problem. The student must really seize the problem, so that he thinks about it when he's brushing his teeth at night and when making breakfast in the morning. If the student doesn't have his problem to think about, then he will not experience the excitement of mathematics. I have seen many students doing research, both during the summer and during the academic year, where the research is only a small part of the advisors' larger research plans. These students feel as if they have no control over their work, since they are not interested in the research. They are merely cogs in the larger research machine of their advisors.

The need for ownership of a problem does not preclude groups of students working on a problem. Both my group experiences at Lafayette and in the DSP turned out very well. However, the use of a group of students to work on one problem is very risky. If the skills of the group are widely varied, then the members might complement each other quite well and learn a great deal from each other. Or, those less prepared might feel the pressure of keeping up with the group, while those more advanced might feel that the group is holding them back. The group might get along well and enjoy working together. Or there might be personality conflicts that have a debilitating effect on the group's productivity. Group work can be very successful, but it must be undertaken carefully and with an awareness of the hazards.

The advisor plays a crucial role in the summer program. As a more experienced researcher, the advisor can suggest approaches to the problem or find the needed background material. He can critique
proofs that are constructed and can challenge the students to generalize their methods to other problems. In my experience, the advisor has also been a great help when I have gotten stuck and frustrated (something that invariably happens at some point in every research). There the advisors were critical in maintaining a positive outlook and in showing that in research one cannot be easily discouraged.

## The Product

Stated alongside the primary goal of gaining research experience is usually a goal of communicating the research that is conducted. One common objective is submitting a paper to a research journal based on the research conducted. Another is presenting the results at a regional or national math conference, as well as giving an end-of-the-summer presentation to all of the participants and advisors in the program. Even though the results of a student's summer research might not be profound, or even complete, it is important to realize that the student's efforts were not wasted. The methods tried and any results obtained, whether partial or complete, is a valuable contribution to the body of knowledge that exists about a problem. Even if the contribution is only a deeper understanding of why a problem is hard, it can be very useful to other mathematicians who later consider the same question.

Most students do not have an opportunity to develop their oral and written mathematical communication skills during their normal undergraduate education. I found that giving periodic talks during the summer on my work was extremely helpful. Not only did I strengthen my speaking skills, but I was also forced to clearly explain what I had been able to prove, what I was currently working on, and what was the sticking point at the moment. The talks also provided a strong incentive to work hard the entire summer.

The preparation of a final report in the form of a research paper is also extremely helpful. In my case, I learned a great deal about the difficulty in writing clearly, and that many, many revisions was the key to overcoming this problem. I also learned the useful skill of writing in $\mathrm{IAT}_{\mathrm{E}} \mathrm{X}$. Summer programs need to be long enough to allow sufficient background to be developed, research to be conducted, and then a final communication of results obtained. All three goals should be reached by the end of the summer so that a sense of closure can be obtained. After the end of the program, students should continue with the research only if they feel interested, and not because they still need to write the final paper.

## The Lifestyle

All three summer programs that I participated in were all very worthwhile from a mathematical standpoint, but they were also a tremendous amount of fun. Eating, sleeping, hanging out with the other students, and doing math-what could be better? The environment that students find themselves in determines how productive they can be mathematically. The availability of comfortable dorms, close proximity to a grocery store, and a large enough stipend to utilize both make for a pleasant summer. Sadly, the low stipends that many summer programs provide discourage many students from participating, since they can make much more working in industry (while learning less).

Socially, having all of the students live together in apartments or dorm rooms that are near each other is a must. The participants are thus able to bond and do things together while not doing mathematics. Students that do not live with the other participants often find themselves not part of the group. Speaking from my experiences, living with a small number of people for several weeks is one of the most enjoyable aspects of the summer programs.

A very successful idea is having planned events, such as picnics or field trips, for the students to get to know each other and the advisors, and to have a break from mathematics. Having the planned events frees the students from attempting to learn the happening places in the region and from needing to organize the adventure. Field trips, such as kayaking and walking on the shore of Lake Superior, are particularly popular at the Duluth REU.

The Duluth program is also notable in that the advisors each year are one or two previous program participants. Several other previous program participants also visit each year to lend their prospectives and insights to the current participants. This concept gives a much greater range of experiences and knowledge for a student to consult than a single advisor, and has the added benefit of providing continuity between successive years of the program. The advisors and the visitors live with the program participants, which adds even more dynamic to the entire social group.

## Conclusion

The three summer research programs that I participated in were great experiences. Not only did I learn a lot of math and develop my communication skills, but the experiences confirmed my desire to continue pursuing mathematics. I would strongly encourage any student who is
considering a career in mathematics to participate in a summer research program first.

I would also encourage any faculty member to become involved with such a program. Without the efforts of faculty, these wonderful programs would not exist.

E-mail address: hartke@math.rutgers.edu
Department of Mathematics, Rutgers University, Piscataway, NJ 08854-8019

# Research Experience at the University of Maryland Eastern Shore 

Andrea Height

The Research Experience for Undergraduates program was enlightening. When I decided to join this program I had not taken linear algebra, or programming in C but I did have C++. Dr. Okunbor thought I could handle it so I stayed in the program to see what I could learn. I was introduced to parallel programming. Since the Department of Mathematics and Computer Science does not have parallel computers, we had to telnet to the University of Missouri-Rolla campus to run our parallel programs. The parallel distributed system consists of 16 processing nodes with a frontend and a file server. Sometimes it was a hassle to get on and when you did someone might have all the processing nodes. We all found that the best time to go to the computer lab was early in the morning or late at night, your program would run quickly then. The research group I participated in did conducted research on Lanczos algorithm. This method takes a symmetric matrix and makes it a tridiagonal system. We had to find an algorithm that worked best for what we were trying to do. When that was accomplished, coding was the next step. Many errors came with writing the code in parallel. Finally, the program worked, the paper was written and the speeches were made. I had the job of describing the orthogonal similarity transformation, where $\mathrm{AQ}=\mathrm{QT}$. That is, matrix A times an orthogonal matrix Q equals Q times the tridiagonal matrix T. During the program the other participants were very patient in helping me to understand what was going on. Each of them had something to contribute to my learning experience. Although lots of work was involved it was not all work and no play. Dr. Okunbor took us on a trip to Washington, D.C., while there we had fun. We saw the sites and after a long day of fun we came back. Another weekend we went to Ocean

[^18]City, to the beach. We had a picnic in Salisbury Park, right beside the zoo, and even visited the Dr. Okunbor's home. Some of the participants took it upon themselves to sight see while on the Eastern Shore and went to Virginia Beach and to Asseteague Island, where there are wild ponies that run on the beach. Overall this program was as fun as it was demanding. This is something I would advise any one to take who is looking for a challenge in the mathematics and computer sciences, and who is thinking about going on to graduate school. We had a good teacher and helpful participants, the time went by so fast that soon we are going to be presenting our projects in November at the Argonne Symposium for undergraduates in science, engineering and mathematics. I am looking forward to that.

University of Maryland Eastern Shore, Princess Anne, MD 21853

# REU and the Nontraditional Student 

Laura E. Johnson

"Are you kidding? I'm not qualified for something like that!" That was my first reaction when Dr. Suzanne Lenhart, the director of the REU program at the University of Tennessee, Knoxville, asked me if I was interested in applying to the program. Why would she consider me qualified when I did not think so? I am over 40, a wheelchair user, and the single mother of two teenagers. At the time, I was taking only one or two classes per semester. I simply assumed that programs such as the REU were for students closer to my children's ages than to mine. Granted, I had the grades, and I found that I had also completed the classes necessary for eligibility. So with Dr. Lenhart's encouragement and that of my other professors, I applied. And to my amazement, I was accepted for the summer of 1998 !

I had a number of concerns that needed to be settled before I could commit to the program. My physical abilities have limited the number and nature of the classes I can take. For example, I knew that I could not do a project that was completely computer oriented, as my vision difficulties limit my working time on a computer. Would my disability and my physical limitations preclude my participation or limit it in some way? Could we get around the fact that I can only work at school at most a half-day, sometimes much less? I was concerned that I would not be able to spend as much time as was needed for both the research and the short courses. But Dr. Lenhart reassured me that we would work it out.

The solution was simple, but involved effort and commitment from both Dr. Lenhart and Dr. David E. Dobbs, with whom I would be working. As I am already a student here at the University of Tennessee, I began working on my project with Dr. Dobbs six weeks in advance of the official start of the REU. Thus, though I was ahead on research

Received by the editor January 22, 1999.
at the beginning, as I continued at my slower pace, the other students caught up to me by the end of the program. Again, this was only possible because of the efforts of Dr. Dobbs, who gave me a great deal of time out of his much-squeezed schedule during an already hectic semester to get a start on the research and give me direction as I began my investigation.

As I said, by the time the REU program officially began, I was well into my research. But I was nervous about meeting the other REU students-after all, I am not your average math major. Heck, I am older than a lot of the professors, much less the students!

The night before the program officially began, the REU students and faculty advisors got together at a picnic at Dr. Lenhart's house. I arrived early, as did some of the faculty. When the bulk of the students arrived, we introduced ourselves; just names, no titles. One of them asked me, "Are you one of the faculty advisors?" When I said no, that I was one of the students, he seemed a little taken aback. But despite my age and other considerations, the other students treated me simply as one of the group.

## REU COURSES AND LECTURES

Our two four-week classes for the REU were "The Topology of the 2 x 2 Matrix" by Dr. Klaus Johannssen and "Optimal Control" by Dr. Lenhart. In addition, most of the REU advisors presented a lecture on either their specialty, the area of research in which they were currently involved, or something that just interested or bugged them. For example, Dr. Conrad Plaut presented a lecture entitled "DON'T Ask Marilyn [vos Savant]". In these various lectures, many things were discussed, including knots, factorization, elasticity, and rabbits and more rabbits! (The rabbits came up in conjunction with both Fibonacci series associated with the $2 \times 2$ matrices and in Math Ecology, where we used a computer model to encourage the rabbits' birth and population growth, or to kill as many rabbits as we wanted. I particularly excelled at decimating a rabbit population very quickly!)

These lectures were of particular interest to me. I am graduating in the fall of 1998, and going straight onto graduate school here at UT. I still am not sure of the areas I might want to research. Though I am leaning more and more toward algebra, my interest in physics also seems to dictate the study of ODEs and PDEs. So I think that these lectures, as I go through my beginning graduate classes, will help me to identify the area that most 'grabs' me.

## RESEARCH

What was really terrific (and a lot of work) was the research. My problem was to see, given a monic integral polynomial of a certain form, how often that polynomial was determined to be irreducible using Eisenstein's Criterion for irreduciblity. From this point, I was to go on to learn about Galois groups and quintics. But we never got that far; the initial investigation into the frequency of occurrence of "Eisensteinable" polynomials turned out to be more complicated than was anticipated.

I began by looking at polynomials of the form: $x^{3}+a x+b$, such that $|a|,|b| \leq n$ where $a, b \in Z$. (Probability considerations suggested letting $n \rightarrow \infty$ eventually.) I knew how to easily determine if these were reducible or irreducible using the rational root test and synthetic division. Such information would help me to learn more about the pattern of frequency of irreducibility of these polynomials. I could also easily determine the ratio of the number of Eisensteinable-irreducible polynomials to the total number of irreducible polynomials and look at the limit of this ratio as $n$ goes to infinity.

One would think the limit of this ratio would be zero. After all, there are many more irreducible polynomials than there are Eisensteinable irreducible polynomials, looking at all monic integral polynomials of any form. The empirical evidence that developed from this study of cubics, however, seemed to indicate a ratio of about 0.28 , but this was only for very small $n$, less than or equal to 30 . So we began to examine the theoretical aspects of the problem, and things got really complicated.

The number of Eisensteinable polynomials of this form for a prime, $p$, can be described by $2((2[n / p]+1)([n / p]-[n / p 2]))$ where $(2[n / p]+1)$ represents the possible number of $a$ 's, $([n / p]-[n / p 2])$ represents the possible number of $b$ 's, and multiplying by 2 incorporates both positives and negatives. However, if one simply adds the above expressions for all primes $p \leq n$, this method counts some polynomials more than once, since some polynomials are Eisensteinable because of more than one prime. For example, $x^{3}+6 x-6$ is Eisensteinable for both the primes $p=2$ and $p=3$; i.e., 2 divides 6 and -6 , but $2 \cdot 2=4$ does not divide -6 , while 3 divides 6 and -6 , but $3^{2}=9$ does not divide -6 .

By means of techniques of analysis and number theory, the following theorem was proven: Fix a positive integer $m \geq 2$. Let $a_{n}$ denote the number of Eisensteinable polynomials, $b_{n}$ the number of irreducible polynomials and $c_{n}$ the number of polynomials $f \in Z[X]$ such that $x^{m}+a x+b,|a|,|b| \leq n, a, b \in Z$. Then $0.2294 \leq \lim \inf a_{n} / c_{n} \leq$
$\lim \sup a_{n} / c_{n} \leq 0.2784$. If $m \leq 3$, then $\lim \inf a_{n} / b_{n}=\lim \inf a_{n} / c_{n}$, and $\lim \sup a_{n} / b_{n}=\lim \sup a_{n} / c_{n}$. The lower (resp., upper) bound for the lim inf (resp., lim sup) is given as the sum of an infinite series.

I have always hated writers who gave away too much, so if you would like to see these infinite series described, please refer to my paper with Dr. Dobbs, "On the Probability That Eisenstein's Criterion Applies to an Arbitrary Irreducible Polynomial" pp. 241-256, Advances in Commutative Ring Theory, Lecture Notes in Pure and Applied Mathematics, Volume TK, Marcel Dekker, New York, 1999. Incidentally, by considering polynomials with more coefficients (and higher degree), one can use similar techniques to support one's intuition that the "probability of Eisensteinability" is arbitrarily small.

## LOOKING AHEAD

When I first became seriously ill, I could not read or write much; I even had a hard time speaking. As I adjusted to the illness, I was forced to use a wheelchair, and not just limit but curtail most activities. I knew I would never again pursue my old profession in typography. I do not know if I will ever again be able to work.

I can do mathematics in my head. Even when my illness precludes reading and writing, I can still explore new ideas, working out proofs and working on problems in my head. Nowadays, I do not take classes with the overt goal of teaching or researching full time. I do not know whether my body will let me do this. But the REU program has convinced me that I want to do these things if I can. Though my body may not function right, my mind can still scale unbelievable heights. My deepest thanks go to both Dr. Lenhart and Dr. Dobbs, without whom this would all have been impossible.

This REU has helped still some of the self-doubt and fear that accompanies every mathematician moving up in their career, whether they are 20 or 40 (or more). So I highly recommend trying this program, regardless of your age or physical ability. By the time that you are finished, you will know if mathematics is truly the path you want to follow.

University of Tennessee, Knoxville
E-mail address: ljohnson@math.utk.edu

# The Experience of a Lifetime 

Darren A. Narayan

I still remember the day I got the message I had been admitted to an NSF-REU program at the University of Dayton. I knew I would be about to embark on a terrific mathematical journey. Six years later the journey continues.

The REU program was exactly what I needed at the time. It was a time when I began to get frustrated in my courses. There were so many interesting topics that I wanted to explore, but there was not enough time to pursue any one deeply. I found myself working late of homework assignments and "be on a roll", but would reluctantly stop, because I knew I needed a full night's rest. The REU was the first chance I had gotten to work on sophisticated math problems without a deadline. There was the freedom to work as late as I wanted and then sleep in the next day if I needed to. I finally could pursue mathematics to my hearts content, and I did just that.

The REU program gave me my first taste of research. The problems given to me tested not only my mathematical knowledge, but also my intuition and drive. It was the first chance that I got to be mathematically creative. There were stretches of time when I got no results, but I learned to be patient, and keep persevering. Eventually the barren stretches of time gave way to progress and I got to experience the thrill of discovering new mathematical results. The positive energy reaped from these instances encouraged me to work harder.

I spent the summer working on two problems in discrete mathematics. The first was a tiling problem given to me by Professor Allen Schwenk of Western Michigan University. I diligently worked on it for four weeks with encouraging results. Schwenk and I continued to work on the problem for years after the REU and recently submitted a paper

Received by the editor September 8, 1999.
for publication. The second half of the summer I worked on a most entertaining and challenging problem on graph labellings with Professor Anthony Evans of Wright State University. I have continued to work on that very same problem since the REU. Last summer Evans, my PhD adviser Professor Garth Isaak and I submitted our latest findings for publication, and began to plan a sequel. I have given several talks on the topics in these two papers at various professional meetings and universities. I hope that my extremely positive and productive REU experience will motivate more undergraduates to apply.

The REU experience helped me develop many skills I will need as a faculty member. During the program we were required to give two briefings a week to our faculty mentors. This gave me practice in writing up results, preparing a presentation, and practice in explaining my own research. At the conclusion of the REU, we were required to give a formal presentation on our summer's work, which was a tremendous exercise in formulating all the summer's results into a short presentation. As I previously have mentioned learning persisted far beyond the conclusion of the program. Faculty members continued to assist me in formulating my results for a presentation at the 104th Joint Meetings of the AMS and MAA, and also to prepare these papers for publication. Mentoring of this type was extremely valuable and it would have been incredibly difficult to attain these experiences elsewhere.

With seven other students from all over the nation, I learned how math courses were taught elsewhere. I learned several tips from them and also became aware of many opportunities that were out there for students. After the REU it was nice to have a support network during the final stages of our undergraduate careers as we contemplated graduate studies in mathematics.

I have extremely fond memories of my REU experience and am still reaping the benefits of the program. Six years later, I still keep in touch with the contacts from my REU, still continue work on extensions of the same problems, and use the skills developed during those eight weeks almost every day. I cannot emphasize enough how my REU experience continues to play a prominent role in my success as a developing mathematician.

Lehigh University
E-mail address: dan6@lehigh.edu

# Reflections on the NSF/REU Program 

Johnnell Parrish

The National Science Foundation/Research Experiences for Undergraduates Program was a very memorable learning experience. As a student with many career goals and who attends a relatively small institution, I was very excited to do research in parallel numerical linear algebra. Being a part of this program helped me to further focus my goals on going to graduate school, and doing more research. There were several group activities conducted throughout the program, which allowed me to interact frequently with Dr. Okunbor and the other students. Besides learning many things, we also had fun outings.

Because the focus of this particular NSF/REU Program was parallel numerical linear algebra, most of our days were devoted to parallel processing. Since parallel computers are becoming the wave of the future, I am happy that we did research under this topic. Mainly, we were finding out how parallel computers can be efficiently used to solve mathematical problems. Upon discussing parallel processing, Dr. Okunbor initially pointed out the benefits that the use of parallel processors can have in regards to saving time and money for companies. He also initially pointed out the importance of using Message Passing Interface (MPI) because of its benefits in regards to portability. Now that parallel computers are gaining more attention in the field of computer science, I plan on using the experience gained from this research to possibly do further research in parallel computing in the future.

The first month of the program was mainly a review of linear algebra, and a time of learning about parallel programming. Dr. Okunbor first covered the basic linear algebra concepts and then introduced various algorithms for solving systems of linear equations. At this time we also learned about some applications of linear algebra. A typical day in the first month of the program consisted of spending the morning

Received by the editor September 10, 1999.
hours in a classroom setting and then spending the afternoon hours in the computer laboratory. A teaching assistant provided us help in learning and understanding the computer programming exercises. The time spent in the computer laboratory during the first month consisted of practice in writing programs using C and Message Passing Interface (MPI). After getting more acquainted with the computer system and the programming language, we proceeded to write programs for the sequential and parallel algorithms presented in the class sessions. The algorithms used in our practice programs contained both direct and iterative methods of solving linear systems of equations.

The second month of the program was mainly directed towards our specific projects. During the week before July 4th, Dr. Okunbor gave us a set of possible research topics and also encouraged us to find a related topic on our own. After previously being arranged into research groups by Dr. Okunbor, each group chose a different topic for their project and then arranged a day to gather all research articles and materials from the main library. Upon returning from the July 4th holiday weekend, we immediately began working in our groups on the projects. Dr. Okunbor planned the class sessions to be three days a week (Mon., Wed., and Fri.) for the remainder of the program. My research partner and I spent about a week reading the articles and materials used for our project.

We spent the second week of the second month working on implementing the parallel algorithm used for our project. The three class sessions held during the week provided a great opportunity for each group to practice their project presentations. The third and fourth weeks of the second month were spent gathering numerical results from our computer experimentation, and writing our research paper. The research paper drafts and the project presentations were then presented on the final day of the program in the classroom.

Throughout the program, trips and activities were generally done on the weekends. We traveled to various places in Maryland and Washington, D.C. for site-seeing and entertainment. We also had a great picnic and a nice banquet to close out the program.

Overall, I feel that I enjoyed the NSF/REU Program. I felt very comfortable with the program agenda and with working and living with the other students. Even though a lot was learned in the classroom and computer lab sessions, I feel that my greatest benefit from the program was my interaction with the other students and Dr. Okunbor. From
this, I have not only learned several things, but I have also become a better student.

Department of Mathematics and Computer Science, University of Maryland Eastern Shore, Princess Anne, MD 21853

# Experience in Faculty Mentored Research 

Suzanne Sindi and Gareguin Mikaelian

We have been participating in faculty-mentored research with Dr. Martelli at California State University, Fullerton in the area of discrete dynamical systems and chaos since summer 1998. The major outcome of our research with Dr. Martelli were two poster presentations at the Joint MAA/AMS Meeting in San Antonio, Texas in January 1999 and at a local meeting MAA/SIAM at Pepperdine University in March 1999. In addition, a paper will be published sometime in fall 1999. During the summers of 1998 and 1999 we met with Dr. Martelli on a weekly or bi-weekly basis to discuss the progress on the project. During those meetings Dr. Martelli guided us through the research process and suggested problems and proofs, which needed to be done between meetings. During some of those sessions Dr. Martelli lectured us on the special topics we needed to know in order to progress in the research. While much work was completed during these meetings we would work independently between them finalizing and preparing our work for the poster presentation and the paper. We continue to meet during the semester, but with less frequency. The goal of our research was to derive the transversality (sufficient) conditions for four different types of bifurcation using a geometric approach. Our results were presented during the poster sessions and will be published. After solving the problems and answering questions of the original research we discovered that our work is not yet complete; we have asked new questions: "Can we repeat our old proofs using new or fewer assumptions?" "Can we modify our proofs for multiple dimensions?" Our experience in facultymentored research has been very positive in terms of knowledge gained from our subject matter as well as general information about conducting research in the field of mathematics. Our research has allowed us to explore mathematics in a way not possible in a traditional course;

Received by the editor September 15, 1999.
we have had the experience of being able to move beyond exercises in textbooks to thinking intently about the mechanics of mathematics in a more abstract sense. As a result, when we encountered material in our courses similar to what we had been studying on our own with Dr. Martelli, we were able to examine such material in a deeper way than was required. Additionally, as a result of our research experience with Dr. Martelli, we have had the chance to attend lectures and conferences as both observers and participants. As participants our role evolved from thinking about mathematics, to conceptualizing how to explain our research in mathematics to others. As observers we had a unique opportunity to interact with other presenters and other members of the mathematics community. We were able to encounter and examine current research in a variety of different areas of mathematics. This gave us an opportunity to think about other fields of mathematics we might be interested in studying later in our academic careers.

California State University Fullerton Fullerton, CA 92634

## Part IV

## ARTICLES

# Helping Students Present Their Research 

Joseph A. Gallian and Aparna W. Higgins

## INTRODUCTION

With the guidance of an experienced research mathematician, there are many undergraduate students who are capable of professional level work in mathematics. The intent of this article is to assist mathematicians in helping their talented students become fully involved in the research experience. Such an experience will benefit the student, the adviser and the mathematics community.

An essential part of a good research experience is the discipline of writing up the results of the research and presenting them. Presenting one's research provides a good capstone event for a research experience. It helps the student to organize and focus the research results. Fortunately, the number of ways of doing this has increased in recent years, and each student researcher can find one that appeals to her/him personally. Apart from the traditional communication of publication in a printed journal, there are electronic journals, poster sessions and contributed paper sessions at meetings. You can help your undergraduate research students choose the most appropriate methods of communicating their results depending on the depth of the results obtained, the personality of the undergraduate, and the logistical difficulties involved. As an item on a student's resume, communicating the results of a research experience, whether in a journal, or at a mathematics meeting, is valued by graduate schools and potential employers alike.

## WRITING UP RESULTS

A well written paper should be a goal of every research endeavor. Such
Received by the editor December 6, 1999.
a paper tells the mathematical "story" that the student has created the origins, the context, the results, the methods, the applications, and possible future investigations. At the same time the student learns to use mathematical type-setting software, such as LaTeX ([3] is an excellent introduction). We strongly recommend that research students write up a paper documenting their results and their proofs and any interesting motivation or applications. It is a good exercise for students to be able to write their results so that they can be read by others this changes the research from a collection of isolated computations or proofs into a coherent whole. As with any well written document, attention to good grammar, proper punctuation and correct spelling provide a rewarding pay-off in the final product. Whether or not the paper will be submitted for publication, it should be written as though it will be submitted. In particular, it should include a title, co-authors, an abstract, acknowledgments and references.

Such a document is useful in several ways - it provides a starting point for a future publication or poster presentation; it can be included as evidence of accomplishment in reports to administrators who supported the research effort; and it will be a convenient reminder of the student's results when you are called upon to write letters of recommendation for employers, graduate schools and fellowships a year or two later. For those students whose research efforts occur at an institution different from their own schools, the paper is a professional way of showing the department at their home institution exactly what they did during their research experience.

## PUBLICATION

Advisers can assist students in preparing papers in several ways. First, a model of a well written paper should be given to the student as a guide. Emphasize to the student that the introduction should tell the reader what the author has done and how it relates to the existing knowledge on the subject. The introduction should persuade the reader that the entire article is worth reading. Advice should be given about the extent of details needed in arguments and the number and kinds of examples to include. Two excellent sources for advice on preparing manuscripts for publication that should be made available to students are [5] and [6]. Since students are not familiar with the literature, the adviser must play a critical role in the decision as to whether and where the article will be submitted. Our advice is to make a realistic assessment of the chances of the paper being accepted by a few particular journals but be a bit on the conservative side in the final choice. A
rejection is a psychological blow to a student. In many cases a student who has a paper rejected by one journal will not submit it to another journal even when an adviser assures the student that paper is worthy of publication. It is not clear whether or not the author of a paper should be identified to the editor of a journal as an undergraduate student. In some cases editors and referees will take pains to help a student get a paper published or, at least, will write a tactful rejection letter, while in other cases they will take the paper less seriously than they do papers written by professionals. Our feeling is that as it becomes more common for undergraduates to publish professional level papers the former scenario will become the norm.

A widely held misconception is that papers written by students are natural candidates for the MAA journals. On the contrary, the MAA journals desire articles that are of broad interest and are exceptionally well written. Like most papers written by professionals, most papers written by students will not meet these criteria. In fact, the rejection rates for the MAA journals are about $90 \%$.

On the other hand, there are some journals such as the Pi Mu Epsilon Journal, the Pentagon (the official journal of Kappa Mu Epsilon) and the Missouri Journal of Mathematical Sciences that have a policy of welcoming undergraduate work. Several newer journals have been started to showcase student work. Among these are the Rose-Hulman Undergraduate Mathematics Journal, Journal of Undergraduate Sciences, Journal of Young Investigators, and the Furman University Electronic Journal of Undergraduate Mathematics. Information about these are provided at the end of this article.

A good resource article on possible publication outlets is Paul Campbell's "Where Else to Publish" [1].

## TALKS

There are increasingly many opportunities for students to present their results at meetings. If attending a meeting is not a viable option for a student, an adviser can arrange for him to give a talk to his department or even at a nearby school.

Careful preparation is required for a successful talk. The best way of ensuring that an oral presentation is good is to practice giving it, and then to practice some more. An excellent resource with tips on how to give a good talk is the article, "How to Give a Good Talk" [2]. Common mistakes made by students and professionals alike are the "too's": writing too small, assuming too much, talking too fast and trying to do too much.

The first transparency for a talk should include the title of the talk and the speaker's name and affiliation. Students should bring several copies of a one-page description of their talk and have it ready to give to anyone in the audience who expresses an interest in knowing more about the results. The information on the sheet should include the title, an extended abstract and all ways of getting in touch with the presenter. A web site where a preprint of the article is available is desirable.

A talk should tell a carefully thought out story. Ten or fifteenminute talks should be used to disseminate new results, to give a context for the results, to provide an outline of the method of proof, and to suggest future lines of inquiry. Computations should be avoided.

Lack of departmental funds should not deter you from seeking other funds to allow for student travel. Many Deans, Provosts and institutional programs have funds that can be used for academic opportunities such as undergraduate participation in professional meetings. Some Student Governments have funds that they award to clubs. Many math clubs and MAA Student Chapters have fund-raising activities to finance student travel to meetings. Some MAA Sections have funds from mini-grants that provide for student travel to their meetings. Most meetings have drastically reduced registration fees for undergraduates and many MAA Sections waive registration fees for undergraduates.

There are many opportunities for students to present a paper. The summer meeting of the MAA (MathFest) includes Pi Mu Epsilon and MAA Student Paper Sessions. A member of either organization's Student Chapters can present a paper. Some travel money is available for speakers, and each organization gives five to ten cash prizes for the best presentations (made possible with the generosity of the AMS, MAA and NSA). Papers can be presented at the AMS or SIAM meetings. An MAA Section meeting that accepts contributed papers is another opportunity for presentation. There are many regional meetings of groups like PME and Kappa Mu Epsilon. Dates and locations are available from the MAA Online web site (www.maa.org). There are also many conferences whose main purpose is to promote undergraduate research. Some, such as the Hudson River Undergraduate Mathematics Conference allow paper presentations by students and faculty, while others, such as the Michigan Undergraduate Mathematics Conference, accept contributed papers only from students. Some schools such as Eastern Kentucky University and Rose-Hulman Institute of Technology host annual undergraduate conferences. Details are available from the MAA web site.

The National Conference on Undergraduate Research sponsored by CUR (Council on Undergraduate Research) invites student papers, as does the annual Argonne Symposium for Undergraduates in Science, Engineering and Mathematics, although these conferences are not limited to mathematics.

In general, we believe that it is preferable for undergraduate research to be presented in a session that is topic-specific, rather than presenter-specific. That is, it is preferable to present a paper on differential equations in a session devoted to differential equations, rather than to present it in a session devoted to undergraduate research. The main reason we feel this way is that in the former kind of session the audience is interested in the topic and consequently the student will likely meet and have the opportunity to network with others who work in the field. Moreover, we feel that at a conference the undergraduates should be treated like professionals and not be segregated according to experience.

We also believe that it is beneficial for students to present at a meeting where there is far more going on than merely contributed papers by other undergraduates. Attending a national meeting of the AMS, the MAA or SIAM provides students the chance to hear some well-known mathematicians give talks, allows them to witness their professors as learners, and gives them the opportunity to meet and network with a variety of mathematicians in different areas and varied types of institutions.

## POSTER PRESENTATIONS

Increasingly, mathematics conferences are including poster presentations as a method of communicating research results. As a matter of fact, many students are more comfortable informally talking about their work to one or two people at a time, as is the case in a poster session, than they are giving a formal talk in front of an audience of experts. Good advice on poster presentations can be found in the article "How to Prepare a Poster," by Sven Hammerling and Nicholas J. Higham [4].

A poster must tell its story by itself. It is usually best to have several sheets of $8-1 / 2 \times 11$ inch paper with the results, motivation, applications and indications of methods of proof, mounted on contrasting colored construction paper. Much of what was said in the above section on presenting talks holds true here. Despite the fact that the "audience" for a poster stands close to it, font size should be large enough
to allow the poster to be viewed comfortably from a short distance. Pictures and colors are very effective in this visual set-up.

Students should have ready a brief, under-two-minutes presentation that is a synopsis of their results. Such a speech points out the highlights of the work. At the annual joint winter meetings, the MAA sponsors a student research poster session and awards cash prizes donated by the MAA and CUR for the best posters.

In helping their students prepare a poster, advisers should comment on such things as poor grammar, awkward style or displeasing appearance. Anything you do to help the student learn to communicate mathematics better is important. Emphasize to your students that effectively communicating mathematics is as important as getting good results. Any difficulty in understanding the work should be due to the depth of the mathematics, and not due to the author's exposition.

## AFTER THE PRESENTATION

If a student from your own institution gives a talk, presents a poster at a mathematics meeting, or gets a paper published, it is important to publicize this information. Let the campus newspaper know about it. An item in the newspaper will be read by other faculty on campus and will reflect well on your department. Let the Dean, the student's adviser and head of the department know about the presentation or publication. Deans often send congratulatory notes to students, which are appreciated. The student will appreciate a thank you from you for taking the time and effort to make a professional presentation.

It is our experience that most students enjoy presenting their research at a mathematics meeting and having the opportunity to hear other mathematicians speak. Students are delighted when someone at a meeting asks them about their work. It is not uncommon for a student who has presented at a meeting to present at future meetings. Such students often have an enthusiasm for mathematics meetings that is infectious, and fellow students get caught up in the excitement of undergraduate research and its presentation.

## PUBLICATION OUTLETS FOR UNDERGRADUATE RESEARCH

1. Pi Mu Epsilon Journal, Ed: Brigitte Servatius, Department of Mathematics, Worcester Polytechnic Institute, Worcester, MA 01609. (508) 831-5361, Fax: (508) 831-5824. bservat@wpi.edu
2. The Pentagon, official journal of Kappa Mu Epsilon. Ed: Steve

Nimmo, Department of Mathematics, Morningside College, P.O. Box 6400, Sioux City, IA 51106. (712) 274-5466.
3. The Missouri Journal of Mathematical Sciences, http://www.mathcs.cmsu.edu/ ${ }^{\text {mjms }} / \mathrm{mjms} . h t m l$
4. Rose-Hulman Undergraduate Mathematics Journal, http://www.
rose-hulman.edu/mathjournal/
5. The Journal of Undergraduate Sciences, Harvard University Science Center, 1 Oxford Street, Cambridge, MA 02138, jus@hcs.harvard.edu, http://www.hcs.harvard.edu/~jus
6. Journal of Young Investigators, http://www.jyi.org
7. Furman University Electronic Journal of Undergraduate Mathematics, http://math.furman.edu/ mwoodard/fuejum/welcome.html

## REFERENCES

[1] Paul J. Campbell, Where else to publish, The UMAP Journal, vol. 17, Jan. 1996.
[2] Joseph A. Gallian, How to give a good talk, Math Horizons, April 1998.
[3] David F. Groffen and Desmond J. Higham, Learning Latex, SIAM, Philadelphia, 1997.
[4] Sven Hammerling and Nicholas J. Higham, How to prepare a poster, SIAM News, May 1996, or http://www.siam.org/siamnews/general/ poster.htm
[5] Nicholas J. Higman, Handbook of Writing for the Mathematical Sciences, SIAM, Philadelphia, 1993.
[6] Steven G. Krantz, A Primer of Mathematical Writing, AMS, Providence, 1997.

Univerity of Minnesota, Duluth; University of Dayton
E-mail address: jgallian@d.umn.edu; higgins@saber.udayton.edu

# MULTI-FACETED UNDERGRADUATE RESEARCH IN MATHEMATICS AT THE UNIVERSITY OF DAYTON 

Aparna W. Higgins

The Department of Mathematics at the University of Dayton has proved successful in encouraging many of its students to pursue professional careers in mathematics. In data compiled by the Board of Human Resources of the National Research Council, comparing fouryear, private, primarily undergraduate institutions, the University of Dayton ranked fifth in the number of alumni who obtained Ph.D.s in mathematics during 1920-1986. The department has tried to stimulate and sustain interest in mathematics among its students in several ways outside the existing curricular channels. Students have been encouraged and directed in the research and presentation of papers at regional and national meetings of Pi Mu Epsilon (PME), which is the national mathematics honorary society, the Mathematical Association of America (MAA), and the American Mathematical Society (AMS). In addition, each year, members of the University Honors Program who are mathematics majors receive guidance and support in the research and writing of their Honors theses in mathematics. Furthermore, in the summers of 1989-1993, the University of Dayton ran a summer mathematics research program, internally funded in 1989, and funded by the National Science Foundation's Research Experiences for Undergraduates (REU) program in the four years thereafter. I have been actively involved with all three of these forms of research activities at the University of Dayton, having helped many students research topics and present them at meetings, having directed ten Honors theses, and having been a co-director of the REU at the University of Dayton from 1989 to 1991.

Received by the editor September 16, 1999.

The University of Dayton is a Catholic university of about 6000 undergraduates and about 4000 graduate students in a few selected graduate programs. The Department of Mathematics has about ten graduating seniors each year. Many of our mathematics majors simultaneously receive secondary school certification, although their degree is a B.A. or a B.S. rather than a B.Ed. On average, about four of our majors each year go on to graduate school in mathematics. We graduate a fair number of double majors, some of whom go on to graduate school in their other major.

## PRESENTING TALKS AT MEETINGS

We are fortunate in Ohio to have at least two opportunities each year for undergraduates to present talks at professional mathematics meetings. In the fall, Miami University of Oxford, Ohio, hosts a regional Pi Mu Epsilon meeting in conjunction with an annual mathematics conference, and in the spring, the Ohio Section of the MAA includes a student paper session. For a few years, prizes were given for best student papers at the Ohio Section meeting, and about ten years ago, our students routinely won these prizes. One year, we won four of the six prizes awarded at that meeting. A few faculty members would actively recruit students showing talent and interest in mathematics from our regular classes, and then invite them for a chat and talk to them about mathematics as a major, and inform them about the mathematics meetings. We worked with students in finding suitable topics and papers and in working through the mathematical questions. Sometimes students came to us with interesting mathematics questions of their own, while at other times they merely had a notion that they would like to present, but didn't quite know where to go from there. My colleagues and I developed a stack of interesting articles we had seen in say, the American Mathematical Monthly or the Mathematics Magazine, or articles in research journals that did not require tremendous amounts of background reading. After determining the students' broad interests (number theory, or groups), we would ask them to peruse a few of the articles and come back in a few days with those they would like to pursue. It was important for us to establish that next appointment. Although many of these talks were not original research, we insisted that they be much more than an expository talk. The students were encouraged (with our help) to work on an open question contained in the paper, or to present a new proof of a theorem in the paper, or at the least, to consider what happens when one of the conditions in a theorem is relaxed.

We also worked hard to prepare a professional talk. We would time a first run-through, then ask difficult questions of the student, enabling her to pinpoint what was really the essence of the talk, and how that would need to be emphasized in the transparencies, and hence in the talk. A few cosmetic changes and a few more timed trials in front of a critical but supportive home audience, and the nervous student would be transformed into a confident speaker. We completed the experience by arranging the transportation to the meeting. Since our department has no line-item in its budget for undergraduates attending meetings, we would rent a college van to transport both faculty and students, and charge it to faculty transportation. At the Ohio Section meeting, the host university arranges for visiting students to be put up free of charge with their own students, so going to the meetings cost our students very little money, but it did entail their time. Typically, our students had a wonderful time. There is an exhilaration that we all, as mathematicians, feel when we have struggled with a problem for many days and understood it well and can tell someone else about it. In addition to this research "high," our students loved the recognition at meetings, loved the fact that faculty from other schools came up and said kind words about their presentations, or asked interesting questions about the topic. Also, students take to networking very naturally. We introduced them to many faculty at these meetings - names that were merely printed words in MAA literature now had faces associated with them. And students enjoyed meeting like-minded students from other institutions. One of the most important benefits of going to a meeting was enabling them to hear the invited speakers - some of whom were great names in mathematics, known for their research prowess, or for their expository skills, or for both. Upon returning to campus, I would do a little publicity on behalf of the students - submit a news item in the campus newspaper and in their hometown newspapers, inform their academic advisors and the deans about their "dedication to their discipline" and their "professional presentation." Usually students were eager to present again at a subsequent meeting, and willing to do the work so they would have something new to present. The students' enthusiasm was contagious, and junior students began expressing interest in going to math meetings.

## HONORS THESES

Forty students from each incoming class of about 1600 students at the University of Dayton are selected to be in the University Honors

Program. The program is designed to provide opportunities for academically gifted undergraduates to develop their skills and talents. The students are selected on the basis of their high school performance and are chosen across the spectra of majors, race and gender. Individual departments do not participate in the selection. The selected students are required to take some Honors seminars and to write a thesis in their major.

Directing Honors theses has been the most professionally satisfying work I have done. I have directed ten Honors theses, ranging from universal algebra and differential geometry to the more recent ones in graph theory, specifically on pebbling and iterated line graphs. The students are given academic credit of three semester hours for each of two semesters for their Honors thesis, which is supposed to be original work. I have developed a system of intense mentoring for students who choose me as their thesis advisor. All ten of my Honors thesis students went on to graduate school. Six of these have earned, or are working towards, their doctoral degrees. Some of the universities they selected for graduate work are the University of Michigan, the University of Notre Dame and Rutgers University. Two received NSF graduate fellowships, two received Barry M. Goldwater Scholarships while undergraduates, with the essays for these awards based on their Honors thesis work. Two of my students' Honors theses results were published in refereed mathematics journals [1],[2].

The primary motivation for me to engage students in undergraduate research is because this is an introduction to our profession. I enjoy the long-term research experience because it provides time for the problem to be truly assimilated by the student. I insist that my students have a major input in choosing their research problem. I will give interested students an overview of my current research interests, along with open questions, and provide them with the basic papers on the topic. If they are interested in the material in a week or two, we settle down to regular weekly meetings, with guided readings (so that I can monitor how well the student reads mathematics), and some easy assignments to write out (so that I can monitor how well the student writes mathematics). If they are not delighted with my choice of topics, we start reading from a standard book and they can point out topics that interest them as we go. Then I work at finding papers of appropriate difficulty through Math Reviews or through survey articles on topics of interest to the student. The student and I generate questions together and we explore various topics until the student really takes a shine to one or two of them. I like my thesis students to have done more than just solved one problem in graph theory by the end of the
year, so the general readings continue until the student gets so involved in her/his own research that there is no time for them. The readings also provide some constant work for the student during those inevitable periods of being stuck in the progress of the research problem. I am not too sympathetic when my students get stuck during their research. I believe strongly that a real research experience is, in fact, one of starts and stalls, and that part of the maturing process of a researcher is knowing when to keep slugging through the details, when to backtrack and try a different approach, and when to quit. I insist on setting up a weekly meeting with my Honors thesis student, even if (s)he has done no work on the thesis during the week. I use that time to explore the obstacle, suggest some new approaches, or discuss some related mathematics, and offer some new reading material. My goal is to let the ideas simmer and be stirred around often enough so that cobwebs don't collect. I also use that time to ask questions about their thoughts about the future. Do they plan to go to graduate school or to have a career in industry? I then counsel them on relevant courses to take. In fact, most of my Honors thesis students use me as their primary (although not official) academic advisor. I insist on my Honors students making at least one off-campus presentation (at a mathematics meeting) on the work of their thesis, and one colloquium to our department faculty and undergraduates at the end of their senior year. This entails the professional training of writing and submitting an abstract, preparing a talk, preparing good transparencies, delivering the talk with good diction, and a sense of timing, and within the time limit set. The Honors Program has some funding available for Honors students to present the results of their research at professional conferences. Many of my students have presented their results at national AMS/MAA meetings. All Honors thesis students are required to submit a thesis for the Honors program, so that a written component is automatic. Given the uneven mathematical talent of the Honors thesis students, not all of them produce results that are publishable in mainstream mathematics research journals. I encourage those who do to publish their work, and help them write a paper of reasonable length. I advise my Honors thesis students on graduate schools, on summer research opportunities, I try to create networks for them by introducing them to people in the areas they are interested in, or to former students who have considered the schools in which the current students are now interested. Of course, I write detailed letters of recommendation of them, chronicling their progress on the thesis, and describing the thesis topic and their chosen techniques of solving the problems posed in the thesis.

## SUMMER RESEARCH EXPERIENCES

Stemming from my enjoyment of the Honors theses that I directed in my early years at the University of Dayton, I decided to conduct undergraduate research in the summers with students who are not necessarily in our Honors program, but are interested in mathematics beyond the classroom. Harold Mushenheim and I co-directed programs in the three summers of 1989-91. The first year, we were funded entirely by the University of Dayton for a pilot program (due to the extraordinary vision of the Provost at the time, Bro. Joseph Stander), the success of which enabled us to get funding from the National Science Foundation (NSF) for their Research Experiences for Undergraduates program. Since the reader is probably familiar with the general tone of NSF-sponsored REUs, I will only describe some special features of ours. The first two weeks consisted of intense lectures in graph theory and combinatorics for half a day, with the other half of the day devoted to time for library research for interesting problems that would ultimately become the participants' research projects for the summer. This was followed by five weeks of research, with twice-a-week meetings with Harry and me to guide their progress, once-a-fortnight talks by students to update the group of their progress. The students performed individual research, although there were several closely-related projects and there was much desirable mathematical interaction between them. In fact, when Harry Mushenheim received an NSF-REU grant for the summers of 1992-93, he invited two "graduates" of our REUs to come and spend the summer with him and the new participants. These graduate students lived with the REU participants and helped provide mathematical input during hours when Harry was not on campus.

We invited colloquium speakers - mathematicians from academe, business, industry and government to expose the students to other areas of interesting mathematics and to inform them of careers in mathematics that result from graduate work in mathematics. The REU participants were required to go to the talks and then join the speaker for a meal and for informal socializing. Our colloquium speakers always included William Dunham (mathematics historian and author of popular books in mathematics such as Journey through Genius), Ronald Graham (Chief Scientist, AT\&T Bell Labs), and someone from the National Security Agency.

We were one of the few REUs at the time to incorporate giving a talk at the national summer meetings as a capstone experience for our participants, and to budget for it in our grant proposal to the NSF.

Since speaking at a professional meeting requires somewhat less effort than preparing results for publication on a refereed journal, we felt that it was a more realistic goal for the eight-week summer program. The rewards were immediate, when questions from the audience helped to provide continued stimulation to the researcher's interest in the problem. We also felt that participating in the rest of the mathematics meetings was a valuable enterprise for budding mathematicians.

The University of Dayton provided the usual cost-sharing for the NSF grant. Two interesting items picked up by the University of Dayton were: each participant got three semester hours of credit, compliments of the University of Dayton; and the University of Dayton paid all the expenses for one non-U.S. citizen participant (who had to be a University of Dayton student), since the NSF had a citizenship requirement.

We had four students in our pilot program and eight or nine in each of the subsequent four years. Participants in the REUs at the University of Dayton have gone on to prestigious graduate schools in mathematics, such as the University of Chicago, the University of Michigan, the University of California, Berkeley, the University of Notre Dame, Rutgers University, Stanford University, Brown University and Dartmouth University. One student published his REU work in a mainstream research journal [3].

## CONNECTIONS

Not surprisingly, there are many connections between the three kinds of undergraduate research described above. Our Honors students comprise a group of regulars for giving talks at mathematics meetings; many students who get interested in going to meetings participate in an REU the following summer, and then give a talk on that research. I have encouraged all my Honors thesis students to participate in an REU in the summers, even if the REU was in their other major. The external validation of their interest and prowess in mathematics is important. An REU director also serves as a valuable reference for graduate school, or a national fellowship or a job application, a reference that is different from the faculty at the student's college. Five of my Honors thesis students participated in REU experiences - four of them away from the University of Dayton. The students all returned rejuvenated, with increased confidence and greater enthusiasm for mathematics in general and for their thesis problem in particular. A couple of students who participated in the University of Dayton's REU switched their Honors thesis projects to continue with the work they had done in the

REU. Students enjoy the contrasting challenges of the year-long thesis research experience and the intense eight- or ten-week REUs.

My own research has been affected by my involvement with undergraduate research. My most recent paper [2] is co-authored with Stephen Hartke, who was my tenth Honors thesis student, a 1999 graduate of the University of Dayton. The paper is a section of Hartke's Honors thesis on iterated line graphs. My co-authored paper [4] on the pebbling number of $C_{5} \times C_{5}$ was written at about the time that two of my Honors thesis students worked in pebbling. There is no question that working with the students on their problems stimulated my interest in solving my problem, and that my work on my own research led to my students being better mentored in the art of research. They watched me in action - saw me excited about progress, and disappointed with a setback, and they saw me rally back and finally finish the problem.

Whatever the form of undergraduate research, there is no question that the student benefits tremendously from it. Many former students have told me how valuable that research experience was to their getting into the graduate school of their choice, or how their interviewer for a job was intrigued by this line item and asked many questions about it, allowing the student to make a very favorable impression. One of my former thesis students explained the benefits of undergraduate research in this way: It provides students the ability to read, speak, and continue to learn mathematics outside the curriculum. Speaking from my experience, the benefits to the faculty member are significant. My mathematical interests have been continually enhanced, I have had the privilege of working with some of the brightest minds I know, I have helped shape the futures of some creative mathematicians and caring teachers of mathematics, and I have been rewarded with former students who are now valued friends and colleagues of mine.

## References

[1] Chambers, Kelly Ann, The isomorphism of the lattice of congruence relations on a group and the lattice of normal subgroups of a group, Pi Mu Epsilon Journal, Fall 1987.
[2] Hartke, Stephen G. and Higgins, Aparna W., Maximum degree growth of the iterated line graph, Electronic Journal of Combinatorics, 6 (1999) \#R28.
[3] Robertson, Ian, T-chromatic polynomials, Discrete Mathematics (135) 1-3 (1994), 279-286.
[4] Herscovici, David S. and Higgins, Aparna W., The pebbling number of $C_{5} \times C_{5}$, Discrete Mathematics, 187 (1998) 123-135.

University of Dayton
E-mail address: higgins@saber.udayton.edu

# MASS Program at Penn State 

Anatole Katok and Svetlana Katok

The MASS program-Mathematics Advanced Study Semesterswas founded in 1996. It is an innovative, intensive program for select groups of up to about 25 undergraduates recruited every year from around the country and brought to Penn State's campus for one semester. This program, currently in its fourth year of operation, provides a unique and mutually reinforcing blend of learning and research activities for its participants. Learning serves as a basis and provides background for research endeavors, and research projects generate the need and desire for deeper and more extensive learning.

The major components of MASS are:

- Three courses designed exclusively for MASS students on topics chosen from the areas of Algebra/Number Theory, Analysis, and Geometry/Topology. Each course features three, 1-hour lectures per week, a weekly meeting conducted by a MASS Teaching Assistant, weekly homework assignments, a written midterm exam, a final project, and an oral final examination/presentation.
- Individual student research projects which range from theoretical mathematics research to computer implementation. Some projects are related to the core courses while others are developed independently according to the interests and abilities of the student. Research projects are supervised by the MASS director, faculty, and teaching assistants.
- A weekly working seminar run by the director of the MASS program, which helps to unify all the other activities.
- The MASS Colloquium, a weekly lecture series by distinguished mathematicians, Penn State research faculty and visitors. These lectures are very instrumental in focusing interest of the MASS participants on various research areas of mathematics

Received by the editor September 8, 1999.
both during their participation in the program and later in their selection of graduate programs.

- Individual guidance by the director of the program and teaching assistants. As a result of the commitment of the director, TA's, and the students, this goes far beyond the ordinary notion of office hours. In particular, the director works with each student to design an individual plan which covers all aspects of the MASS experience and is adapted to the student's background, interests, and talent. The director regularly meets with each student to revise this plan according to the student's progress in the program.


## Participants

The participants are selected from applicants currently enrolled in US colleges or universities who are juniors or seniors in the given academic year. Students who are sophomores may be admitted in exceptional cases. All participants are expected to have demonstrated a sustained interest in mathematics and a high level of mathematical ability and to have mastered basic techniques of mathematical proof. The expected background includes a full calculus sequence, basic linear algebra, and advanced calculus or basic real analysis.

The search of the participants is nationwide. We maintained a website for the program at http://www.math.psu.edu/mass/. Participants are selected based on their academic record, recommendation letters from faculty and an essay. In the course of 4 years we had participants from University of Nebraska, Lincoln, Mount Holyoke College, SUNY at Stony Brook, University of Missouri, Rolla, University of Texas, Austin, Rensselaer

Polytechnic Institute, University of Illinois, Urbana at Champaign, University of Wisconsin at Madison, Michigan State University, Clarkson University, Arizona State University, Juniata College, University of Missouri at Kansas City, University of California at Santa Cruz, Iowa State University, SUNY at Geneseo, Sara Lawrence University, Chapman College, University of Maryland, Wayne State University, College of New Jersey, and Penn State.

## Funding and Financial Arrangements

Penn State provides fellowships for out-of-state students that reduce their tuition to the in-state level. Further support comes through the National Science Foundation VIGRE grant. In particular, MASS participants whose tuition in their home institution is lower than Penn

State in-state tuition receive grants for the difference. The balance of the VIGRE funds is used to further decrease out-of-pocket expenses of the participants and is distributed individually based on merit and need. Additional funding of the research component of the MASS program in the form of small individual stipends is provided through REU supplements to individual NSF grants.

## MASS Courses

The following courses have been offered in the MASS program:
MASS 1996:
"Explorations in Number Theory" (George Andrews, Evan Pugh Professor of Mathematics);
"Introduction to Dynamical Systems" (Anatole Katok, Raymond N. Shibley Professor of Mathematics);
"Linear Algebra in Geometry" (Victor Nistor, Professor of Mathematics).

MASS 1997:
"Arithmetic and Geometry of the Unimodular Group" (Svetlana Katok, Professor of Mathematics);
"Real Analysis" (Nigel Higson, Professor of Mathematics);
"Explorations in Geometry" (Dmitri Burago, Assistant Professor of Mathematics, 1997 Sloan Fellow).

MASS 1998:
"Number Theory: From Fermat's Little Theorem to his Last Theorem" (Ken Ono, Assistant Professor of Mathematics);
"The Exponential Universe" (John Roe, Professor of Mathematics);
"Functions and Dynamics in One Complex Variable" (Greg Swiatek, Professor of Mathematics).

MASS 1999:
"Topics in Number Theory" (Robert Vaughan Professor of Mathematics);
"Geometric Structures, Symmetry and Elements of Lie Groups" (Anatole Katok, Raymond N. Shibley Professor of Mathematics)
"Mathematical Methods in Mechanics" (Mark Levi, Professor of Mathematics).

All courses are assisted by MASS teaching assistants who are chosen from the most accomplished Ph.D. students of the Penn State Department of Mathematics. All MASS TA's significantly exceed the requirements of our Graduate Program. There is a high level of interaction
with the course instructors, the seminar leader and teaching assistants including individual tutoring.

## Research

Some MASS participants have produced significant pieces of mathematical research. Here are several examples:

Mark Schmitt, a MASS-96 participant from University of Illinois, developed his MASS number theory final project into a new theorem about partitions of the terms of Fibonacci-like sequences, which he presented at the 1998 Illinois Number Theory Conference.

An Nguyen, another MASS-96 student and now a graduate student in Computer Science at Stanford, rediscovered the famous value of $\lambda=1+\sqrt{8}$ for the appearance of period three orbits in the logistic family $f(x, \lambda)=\lambda x(1-x)$, and then went on to discover a previously unknown bifurcation point where the second period four orbit appears: $\lambda=1+\sqrt{4+\sqrt[3]{108}}$.

James Kelley, a MASS-98 participant, studied the representation of integers by quadratic forms, a classical problem in number theory. In particular, he studied a well known problem due Irving Kaplansky: What integers are of the form $x^{2}+y^{2}+7 z^{2}$ where $x, y$, and $z$ are integers. Obviously, if $N$ is of this form, then $N k^{2}$ is also of this form (just replace $x, y$, and $z$ by $k x, k y$, and $k z$ ). However, the converse if not necessarily true. James proved, using the theory of elliptic curves and modular forms, that every "eligible" integer $N$ which is not a multiple of 7 and not of this form, is square-free! His paper has been submitted for publication.

John Voight, a MASS-98 participant, now a first year graduate student at the University of California at Berkeley, wrote an extremely impressive paper on odd perfect numbers. A number is perfect if it equals the sum of its proper divisors. For example, 6 is perfect since it equals the sum of 1,2 and 3 , the proper divisors of 6 . One of the most famous open problems in number theory is the conjecture that there are no odd perfect numbers. John proved a number of highly technical results regarding the prime factors of any alleged odd perfect number. The conditions he obtains place so many restrictions that his work may be viewed as further evidence that there are indeed no odd perfect numbers.

## Credits

Students receive 15 credits for the courses and the seminar which are transferable to their home universities and a certificate from the MASS Program at Penn State. Additional recognition is provided through prizes and certificates for achievements in solving problems and for best projects. Each student is also issued a Supplement for MASS Certificate which includes the list of MASS courses with credits, grades, final presentations, and special achievements. It also includes the descriptions of MASS courses, the list of MASS Colloquia and the description of MASS program exams. These supplements are very useful for the student's home institution equivalences and enhance the student's applications to graduate schools.

## Administration

The overall supervision of the MASS program is provided by the Scientific Advisory Board which includes senior members of Penn State's mathematics faculty, and several outstanding mathematicians from other institutions.

Currently the MASS program is managed by the Acting Director chosen among senior Ph.D. students with previous experience as MASS-TAs under the supervision of Professor A. Katok, who chairs the Scientific Advisory Board. Organizational matters are attended by the Administrative Assistant. We hope to be able to hire a permanent faculty Director of the MASS program at the tenure or tenure-track level for the Fall of 2000.

## Feedback

MASS students master key tools and techniques of mathematical research and they are exposed to mathematical ideas from many areas of current interest. This is an ideal preparation for a career as a research mathematician, and we are pleased to report that MASS alumni go on to graduate schools in mathematical sciences. We expect them to become active researchers in academia or in cutting-edge industrial and government research establishments. Here is a list of most of the graduate schools where the MASS alumni matriculated. In mathematics: University of Michigan, Cornell University, Penn State, Harvard University, University of Chicago, Boston University, Ohio State University, University of Wisconsin, University of Illinois at Urbana Champagne, University of Utah, University of California, Berkeley; in computer science: Stanford University. Many MASS alumni have had a choice between a number of top rated graduate schools. It is worth
noticing that MASS experience not only helped them to obtain these offers, but was instrumental in helping them to make choices based on well-developed mathematical interests and not simply on general ranking of the departments.

The MASS Program boasts a number of features which make it unique among mathematics programs for undergraduates in the U.S., quite distinct from honors programs, math clubs, and summer educational or research programs. The principal difference is the comprehensive character of the program: all academic activities of the participants for a semester are specially designed and coordinated to enhance their learning and introduce them to research in mathematics. This produces a quantum leap effect: the achievement and enthusiasm of MASS students increases much more dramatically than if they had been exposed to similar amounts of material over a longer time in more ordinary circumstances. A key feature of the MASS experience is the intense and productive interaction that takes place among the students. The environment is designed to encourage such interaction: a classroom is dedicated to MASS and furnished so as to serve as a lounge and a computer lab outside of class times. The students live together in a contiguous block of dorm rooms, they eat together, and they pursue various social activities together. The effect of such conditions is dramatic: the students find themselves members of a cohesive group of like-minded people sharing a special formative experience. They quickly bond, and often remain friends after the program is over. They study together, attack problems together, debug programs together, collaborate on research projects and, most importantly, talk about mathematics all the time. Of course, this is exactly how "mature" mathematicians operate in their professional life. A necessary condition for this environment is the gathering of a "critical mass" of dedicated and talented math students, which is one of the chief accomplishments of MASS.

Proof that the MASS program is working as hoped, is confirmed by a variety of observations gained from the students' assessments.

Suzanne Lynch, who participated in MASS-96 and is now a graduate student at Cornell, wrote this in an unsolicited letter:
The MASS program has been the best semester of my life. I was immersed in an environment of bright motivated students and professors and challenged as never before. I was pushed by instructors, fellow students and something deep inside myself to work and learn about mathematics, and my place in the mathematical world. I loved my time there, and never wanted to leave. I believe the MASS program helped to prepare me for the rigors of graduate school, academically and emotionally. ... The MASS program has been very instrumental
in opening grad school doors to me, and giving me the courage to walk through them.
The impact of MASS on the students is reflected in comments from their post-program questionnaires, which were uniformly laudatory.

I found the MASS program to be a great cutting edge learning experience. To me it was a preview of the things to come, and although I could not always keep up with all my class work I still enjoyed it a lot. ... I had no definite career goal prior to MASS. Now I have decided to attend graduate school and I'm positive that MASS has influenced my decision. Also, I feel, it made me more confident in my learning ability. - Daniel Genin, MASS-96 and MASS-97, now a graduate student at Penn State.
The MASS Program has taught me how to do, live and breathe MATH. I've come out of this program not a brainy robot with courses under my belt, but a deep thinker and philosophical person toward life.... the MASS Program has been the best education in my life, and I recommend anyone who seeks professionalism to take this program at LEAST ONCE!!! - Chris Staskewicz, MASS-97, now a graduate student at University of Utah

The only word that I can use to describe the MASS Program is intense. ... I know that the MASS Program has helped me focus my mathematical interests. After completing this Program, I feel that I can accomplish anything. I feel like I have so much more mathematical ability or at least the confidence that I can tackle the many challenging problems that are ahead. - Colleen Kilker, MASS-97, now a graduate student at University of Illinois at Urbana-Champaign
I would not be in the strong position I am now in my application to graduate school without MASS. My professors back at home were quite amazed with my GRE score, and that had a lot to do with the environment and the challenge provided by the MASS program. It also helped me to figure out that my proclivities lie in number theory, something that would have taken me much longer to figure out absent the intensive approach taken in each of the three courses. I made a lot of friends I will miss dearly. I feel more confident generally in my mathematical abilities. John Voight, MASS-98, now a graduate student at UC Berkeley
The MASS program is a great thing. When I return to my home school, I will certainly recommend it, and tell the people there about my positive math experience at PSU. I have never been pushed this hard, or learned this much in a single semester. The mathematics was fantastic,

I am very excited about mathematics and my background was greatly expanded and my perspective enlarged. Matthew Boylan, MASS-98, now a graduate student at Penn State and a VIGRE fellow.

Department of Mathematics, Pennsylvania State University, University Park, PA 16802

E-mail address: katok_a@math.psu.edu
Department of Mathematics, Pennsylvania State University, University Park, PA 16802

E-mail address: katok_s@math.psu.edu

# How Program Design and Implementation Leads to Achieving Goals 

Herbert A. Medina, Loyola Marymount University

The conference was designed so that participants discussed five major topics:

1. Identifying goals ${ }^{1}$ for mathematics summer programs.
2. How program design and implementation leads to achieving goals.
3. How to assess programs.
4. How summer programs can help to address the underrepresentation of certain groups in mathematics (i.e., women, African Americans, Hispanics and Native Americans.)
5. What is the future of undergraduate summer programs?

This article, as the title suggests, is about item two on the list.
After participants had engaged in small-group discussions with the aim of identifying goals, they partitipated in a plenary session where they came to an agreement on a collection of goals for undergraduate mathematics summer programs. Conference participants also agreed that the list was not meant to be all inclusive nor exclusive. They recognized that there will be programs with certain goals that are not included on the list. Also, it is very unlikely that there is even a single program that has all the items on the list in its collection of goals. That is, an undergraduate summer program's set of goals most likely will have a non-empty intersection and symmetric difference with the set identified at the conference.

The goals identified by conference participants are the following.

1. Provide a mathematically rich and professional environment.

[^19]2. Raise the mathematical maturity level of the program's students.
3. Develop students" skills to communicate mathematics.
4. Recruit a diverse ${ }^{2}$ pool of students for the program.
5. Diversify the pool of students earning graduate degrees in mathematics.
6. Prepare students to apply and succeed in graduate school.
7. Build a sense of community.
8. Get students excited about mathematical research.

## 1. Organizing Program Constructs

After conference participants had identified goals, they set out to identify successful program designs, activities and implementation details that would lead to achieving goals. We will use the term "program construct" to refer to program designs, activities and implementation details for the remainder of this article. This was done in small groups whose facilitators reported the highlights of their session to the whole group.

The biggest challenge in reporting the outcome of these small-group discussions was organizing the information. The aim was to classify an item in a way that easily identified the goal to which it was contributing. One of the small group facilitators suggested organizing the information in a table whose rows would contain the program constructs and columns the goals. The matrix format allows the reader to easily list the goals to which a program construct contributes and also to list the constructs that contribute to achieving a particular goal.

Table 1 is the substantive part of this article as it summarizes the work done by the conference participants in identifying program constructs. Again we mention that its content is not meant to be inclusive nor exclusive in any way. For example, it is clear that there will be programs that will have to "develop students' skills to communicate mathematics" as one of their stated goals but will not engage in all the program constructs contained in the table that contribute to achieving that goal.

The second biggest challenge in organizing the information was classifying each construct as either a program design ("D") or an implentation detail/activity ("A"). There are several rows in the table that easily could be switched from "D" to "A" (e.g., 33 and 35) and vice

[^20]versa. The general sentiment of the conference participants, smallgroup facilitators and the author is that listing a program construct is much more important than classifying it.

In closing, the author wishes to acknowledge the tremendous amount of assistance, guidance and support that he received from the small group facilitators in recording and organizing the dozens of constructs identified by conference participants. He is very grateful to Patrick Flinn from the National Security Agency, Aparna Higgins from the University of Dayton, Ivelisse Rubio from the University of Puerto Rico - Humacao, and Robert Strichartz from Cornell University for all their work during and after the conference.
Table 1: Program Designs, Activities/Implementation Details, and Goals

|  |  | Activity/ Implementation Detail Program Design | Provide a mathematically rich and professional environment | Raise the mathematical maturity level of the program's students | Develop students' skills to communicate mathematics | Recruit a diverse pool of students for the program | Diversify the pool of students earning graduate degrees in mathematics | Prepare students to apply and succeed in graduate school | Build a sense of community | Get students excited about mathematical research |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | Provide students journals, books and good computing facilities | $X$ | $X$ |  |  |  |  | X | X |
| 2 | A | Provide oppotunities for students to work on problems that are industryrelated | X |  |  |  |  |  |  | X |
| 3 | A | Teach students TeX or LaTeX |  |  | $X$ |  |  | X |  |  |
| 4 | A | Have students write a technical report/paper on their research |  | $X$ | $X$ |  |  | X |  | X |
| 5 | A | Mail out physical (e.g., poster, brochure, etc.) announcement of the program |  |  |  | X |  |  |  |  |
| 6 | A | Ask former students to engage in program recruitment |  |  |  | X |  |  |  |  |
| 7 | A | Send program announcement to historically black colleges and universities (HBCU's) and Hispanic serving institutions |  |  |  | $X$ |  |  |  |  |
| 8 | A | Communicate with faculty in departments with a significant numbe students from underrepresented groups and ask them to identify students who would "fit" nicely into the program |  |  |  | X |  |  |  |  |
| 9 | A | Share the names of students from previous summer programs and of those who are not accepted to a program |  |  |  | X |  |  |  |  |
| 10 | A | Organize an activity where faculty visiting the program talk to students about the opportunities for graduate studies at their institution | $X$ |  |  |  | X | X |  |  |

Table 1: Program Designs, Activities/Implementation Details, and Goals
Table 1: Program Designs, Activities/Implementation Details, and Goals

Table 1: Program Designs, Activities/Implementation Details, and Goals

| $\longleftarrow$ Goals |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Activity/ <br> Implementation Detail <br> Program Design | Provide a mathematically rich and protessional environment | Raise the mathematical maturity level of the program's students | Develop students' skills to communicate mathematics | Recruit a diverse pool of students for the program | Diversify the pool of students earning graduate degrees in mathematics | Prepare students to apply and succeed in graduate school | Build a sense of community | Get students excited about mathematical research |
| 28 |  | Lead students from diverse backgrounds through a program that will motivate them to apply and help to prepare them to succeed in graduate school |  |  |  |  | X |  |  |  |
| 28 | D | Establish connections with graduato departments to which program students can be encouraged to apply |  |  |  |  | $X$ | $X$ |  |  |
| 29 | D | Provide an intense environment that will familiarize students with the intensity and work load of graduate school |  |  |  |  |  | X |  |  |
| 30 | D | Build alliances with universities and foundations that will result in fellowships for students |  |  |  |  |  | X |  |  |
| 31 | D | Have students in the same residence hall or near each other so as to facilitate communication and collaboration | $X$ |  |  |  |  |  | $X$ | X |
| 32 | D | Have faculty live close to students |  |  |  |  |  |  | X |  |
| 33 | D | Provide opportunities for students to work in groups |  |  | X |  |  | $X$ | X | X |
| 34 | D | Have graduate students involved in the program | $X$ | $X$ |  |  |  | X | X |  |
| 35 | D | Provide post-summer experiences (e.g. conferences) so that students can come together after the program |  | $X$ |  |  |  |  | $X$ | X |
| 36 | D | Help students maintain a network that they built during the summer |  |  |  |  |  | X | $X$ | X |

# Evaluating Summer Math Programs 

Deborah Nolan

## Introduction

After five years of operation, the National Science Foundation (NSF) asked us to conduct a complete evaluation of the Berkeley Summer Institute of Mathematical Sciences (SIMS) and its predecessor the Mills Summer Mathematics Institute (SMI). We were surprised by this request because the evaluations already in place at that time included mid-program interviews, detailed anonymous end-of-program questionnaires, and brief follow-up surveys to track students after they left the program. Devising a new plan that would provide more convincing evidence of the program's effectiveness was a challenge. Dr. Ani Adhikari ${ }^{1}$ and I laid out a comprehensive evaluation plan with a time line for collecting specific types of information from past participants at different stages in their careers. We designed questionnaires to aid comparisons across participants and over years, and in addition to past participants, we found other sources of information on the impact of our program, such as faculty who were in contact with students before and after participation in our program.

The mid-program and end-of-program evaluations that we had conducted for several years were extremely useful in shaping our program. Input from students and faculty led to many improvements of the program. But, the new evaluation better helped us document our program's achievements. Although SMI/SIMS is different from a typical NSF Research Experience for Undergraduates (REU) program², we

[^21]hope that our evaluation plan will be useful to directors of other summer mathematics programs who are interested in devising evaluations that document their programs' achievements.

## Why Evaluate?

Evaluations can provide evidence to funding agencies that a program works and should continue to be funded, and they can influence others to adopt successful aspects of a program or to start a new similar program. On a larger scale, the information collected can provide evidence that summer mathematics programs work. This information can help demonstrate to the mathematics community the benefits of summer mathematics programs, and it can shape policy made by the federal government about support for such programs.

In designing our evaluation plan and questionnaires, we sought the advise of evaluation specialists, Assistant Vice Chancellor Barbara Gross Davis ${ }^{3}$ and Dr. Flora McMartin ${ }^{4}$. We also found Evaluating Intervention Programs: Applications from Womens' Programs in Mathematics and Science by Davis and Humphreys to be extremely useful for figuring out what information to collect, who (or what) could provide this information, and how and when to get the information.

To determine what to evaluate, Davis and Humphreys suggest considering these questions:

- Who wants to know?
- What is in your grant proposal?
- What are your program goals and objectives?
- How are similar programs evaluated?
- Are there any unanticipated side effects of your program?

The answers to these questions helped us shape our evaluation plan. In our case, panelists who reviewed our proposal for NSF funding were not convinced of the effectiveness of the program given the data presented. They complained that-

Program evaluations include the standard testimony of participants immediately at the end of their summary experiences ...

[^22]They also raised several comments and questions about the effectiveness of the program, and the NSF wanted them addressed:

For such an expensive and selective program should the success rates in turning out top notch female graduate students be higher?

Better comparisons with students who did not participate should be provided.

What advantages, if any, does this program have over an REU and for whom?

Do these people [faculty leading seminars] really need a grand a week?

To put these questions in context, we describe a few aspects of SMI/SIMS. The program goals and objectives included preparing students for graduate studies in the mathematical sciences, and motivating them to enter and successfully complete a Ph.D. program. Some of the students in SMI/SIMS were from elite institutions such as Harvard and the University of Chicago. It was our thinking that these women often did not receive the individual attention and encouragement that students at small schools received, and that they could be big sisters for other less mathematically prepared students in the program. Although a few students each year were from these schools, most were not, and roughly two-thirds of all program participants went on to graduate school. A comparison number or numbers would be useful in judging the effectiveness of the program in encouraging its participants to attend graduate school. But, to find meaningful comparisons is extremely difficult. We are not in a controlled experimental setting where we can easily find similar groups of students who did not receive the benefits of SMI/SIMS (or any other summer mathematics program).

Because SMI/SIMS was different in design from an REU, it was placed in the position of defining its effectiveness relative to REUs. We advocated that there can be more than one model of success for a summer mathematics program. For example, one benefit that SMI/SIMS may have over an REU is that it provides a large number of female peers, graduate students and faculty to serve as role models and mentors. One difference between SMI/SIMS and REUs that caught a lot of attention was the budget. The cost of a student to attend SMI/SIMS averaged $\$ 1,000$ more than typical REUs. In part this was due to the cost of housing in the Bay Area, but it was also due to the cost of supporting the four visiting faculty who led seminars. These faculty were paid $\$ 6,000$ for working in the program (a grand a week) along with
travel and housing reimbursements. Each faculty member also worked with a paid graduate student assistant.

Although it was expensive to have visiting faculty in the program, we found an unexpected benefit in doing so. The enthusiasm of the mathematically talented students in the program and the nontraditional seminar style of teaching helped the faculty and graduate students develop their teaching skills. The faculty also reported a boost in their research while visiting Berkeley, and they were glad to be part of a growing network of female mathematicians who participated in the program.

## Evaluation Questions and Sources

Working from the questions raised by the reviewers, and the goals, objectives, and unexpected benefits of the program, we devised a set of evaluation questions. With these questions to guide us, we determined how to collect data.

- What is the impact of the program on the student's decision to apply to graduate school?
- Does the program improve a student's self confidence?
- Does the program increase a student's knowledge about and preparation for graduate school?
- What is the program's success rate for students entering and completing advance degrees in the mathematical sciences?
- How does the success rate compare to other rates of attendance and completion of graduate school?
- Do students use the network of peers, graduate students and faculty?
- How does the SMI compare to an REU?
- What impact does the program have on the faculty and graduate students?

The primary source for answers to questions on the benefit of the program to the students was the students themselves. However, we also found other external sources for answering these questions, and used these sources to validate student responses.

- A professor who wrote a letter of recommendation for a student to attend the program should know the student well, and have the unique perspective of observing her before and after the program.
- The graduate advisor of a past participant who is now in graduate school can offer opinions on how prepared she was for graduate school, and on progress to degree.
- Peers of a participant at her home institution may be able to provide comparison information.
- Published tables on the number of graduates from the home institution that go on to receive Ph.D.s in the mathematical sciences can be used as comparisons figures for success rates.
More ambitiously, by following all mathematics majors at a few undergraduate institutions over time, a profile of mathematics majors and the benefits of summer mathematics programs may be documented. And tracking a sample of Ph.D. students over time could provide valuable information on the effectiveness of summer mathematics programs. Such studies were beyond the time and funding available and beyond the scope of our project.


## Evaluation Plan

On the first day of the program, we had students fill in a survey that asked a few brief questions. The purpose was to get a base line measurement on the students. We wanted to find out what students knew about graduate school and funding for graduate school; how sure they were that they were going to graduate school; and which graduate schools they were thinking about applying to for what advanced degree. At the end of the program, in addition to requesting program evaluations, we again asked these questions on plans for graduate school.

We planned to keep in touch with students on a yearly basis to update their directory information. We used email and telephone to contact students, or their parents, to update our files.

A more detailed evaluation was to be completed two years out and four years out of the program. Two years out from the program, students were asked, among other things, about their current status with regard to work and/or graduate school, what mathematics activities they had engaged in over the past two years, and what contacts they had made with program faculty and students after the program. Those that attended an REU, were asked to compare and contrast their experiences in the two programs.

Also at this time, we surveyed the undergraduate faculty who wrote letters of recommendation for the students in the program. Some questions on the faculty survey were the same as those on the two-year participant survey in order to corroborate the students' responses. We found that the faculty were at least as positive as the students were
about the program's effect on the student. We also collected from these faculty information about the undergraduate program at their institution, including the size of the major, the number of female students, and the number of students going on to graduate school each year. This information was useful in comparing program participants to peer groups with similar backgrounds.

In the four-year survey, students were again asked what they were up to and, as appropriate, they were asked to reflect on the program's effect on their decision to attend and ability to succeed in graduate school. Those students who were working toward their Ph.D were asked for permission to contact their thesis advisor. The graduate advisors were asked two open ended questions about the student's preparation for graduate school and progress to degree.

Finally, we also surveyed the SMI/SIMS faculty, and asked them to describe the impact of the program, if any, on their careers.

## Survey details

The student questionnaires were approximately two pages long, and the faculty surveys were under one page in length. In addition to several open-ended questions, the questionnaires contained about ten closed questions asking the student (or faculty) to rate some aspect of the program on a scale from 1 (little or none) to 5 (a great deal). Two example questions appear below. We took care in wording the questions to try to avoid bias in the response. Notice the use of the phrase "if any" in the first question, and "From your perspective" in the second. Some questions on the two-year and four-year surveys were the same in order to make comparisons across years and to pool data, and some questions on the two-year and faculty survey were the same in order to make comparisons between a student's perception and her undergraduate advisor's perception.
To what extent, if any, did the program affect your:
self confidence $\quad \begin{array}{lllll}1 & 2 & 3 & 4 & 5\end{array}$
motivation to do graduate work $\begin{array}{llllll}1 & 2 & 3 & 4 & 5\end{array}$
knowledge about what graduate $\begin{array}{llllll}1 & 2 & 3 & 4 & 5\end{array}$
school is like
From your perspective, how important was it that the program involved only women as:

| students | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| graduate students | 1 | 2 | 3 | 4 | 5 |
| faculty | 1 | 2 | 3 | 4 | 5 |

All surveys were conducted via email. For the student surveys, we employed an assistant to contact the students and collect the data in order to encourage honest responses. Although, Adhikari and I did send reminder emails to students who were late in responding. We also contacted the faculty surveyed. The response rate was very high with $80 \%$ of the students and $75 \%$ of the faculty who wrote letters of recommendation for students responding. All graduate advisors and SMI/SIMS faculty responded. We were pleased to find that the new data collected were even more positive than our earlier findings and they confirmed to us the success of our program. Some of our findings appear in Adhikari, Givant, and Nolan (1997) and Nolan (2000), and more detailed results and sample questionnaires can be found at www.stat.berkeley.edu/users/nolan/sims/.

## Summary

We hope that the evaluation plan presented here will provide useful ideas to others planning evaluations of summer mathematics programs. We encourage you to list your evaluation questions and determine new ways to find better, more informative data to answer them. We also encourage you to share your evaluation plan with others. Consider posting your evaluation plan on your website and sending us the url, or send us (nolan@stat.berkeley.edu) your ideas for evaluation and sample questionnaires. We are interested in collecting these materials on the web to serve as a resource for other program directors.

## References

1. Adhikari, A., Givant, S., and Nolan, D. (1997) The Mills College Summer Mathematics Institute. In Women in Mathematics: Scaling the Heights, MAA Notes 46, pp97-104.
2. Davis, B.G., Humphreys, S. (1985) Evaluating Intervention Programs: Applications from Womens' Programs in Mathematics and Science. New York: Teachers College, Columbia University.
3. Nolan, D. (2000) The Mills College Summer Mathematics Institute \& The Berkeley Summer Institute for the Mathematical Sciences. In Proceedings of the Conference on Summer Undergraduate Mathematics Research Programs. American Mathematical Society.

University of California, Berkeley, Berkeley, CA 94720-3860
E-mail address: nolan@stat.Berkeley.EDU

# Scholarly Experiences for all Undergraduates 

Lynn Arthur Steen

Each September someone sends me e-mail listing things that first year college students may never have experienced-things like typewriters, dial telephones, or Ronald Reagan. In that spirit, I remind you of something that many at this meeting may never have experienced: the widespread belief that research in mathematics is beyond the reach of undergraduates.

My reason for being here may have something to do with the fact that I was one of a very small number of pioneers who helped show, in the 1960s, that research in mathematics was not only possible but also vitally important for aspiring undergraduates. Back then the National Science Foundation had a program called Undergraduate Research Participation (URP) that made nearly 500 awards each year. In 1967, only ten of those awards were in mathematics-less than one-tenth of what might have been expected if mathematics had received a proportional share of the awards. (One of the ten was at St. Olaf, and led to the book Counterexamples in Topology, to which undergraduates made many substantial contributions.)

Why were there so few URP awards in mathematics? Not because NSF was biased against mathematics, but because mathematicians were biased against undergraduate research. Back then, mathematicians simply didn't believe that research by undergraduates was possible, much less desirable. Lyle Phillips, director of URP at that time, described URP as a "scientific apprenticeship." Because mathematicians couldn't imagine undergraduates as research apprentices, very few bothered to apply for URP awards in the 1960s. We know now, as Frank Morgan demonstrated so vividly last night, how wrong

Received by the editor October 5, 1999.
they were. What an irony, especially in the face of evidence such as exhibited by this meeting!

With hindsight, a second irony is evident. In the early years of URP, undergraduate research was seen primarily as a means of launching promising students on careers in research. The practical experience in research provided by URP was intended to help students learn the habits of working scientists: patience, persistence, dedication, independence, and creativity. Although the benefits of URP did motivate a few institutions to make research a regular part of the undergraduate curriculum, spin-off benefits that improved all students' education were at best an afterthought.

However, educational benefits were quite evident to students who participated in URP. The nature of scholarship in URP, wrote a 1967 St. Olaf participant, was a "clear departure" from regular coursework in mathematics:

Heretofore I had always played a purely receptive role, gaining information and insight from one or two didactic sources. This time my role was investigative. Consequently, both my opinion of mathematics and my relationship to it changed greatly. I developed a mathematical aggressiveness that I never had before. Although the fears of inadequacy that haunt scholars entering any field still hover around, for me they will never again function as a stop sign.

The power of research experiences to help students avoid the many potential stop signs in a scientific education has now led to a widespread consensus that, from grade school through graduate school, inquiry and investigation are essential to effective education in science and mathematics. Those of us here today know from experience what so few believed when these programs began - that research experience is essential for effective learning.

Fortunately enthusiasts like Joe Gallian persisted with energy and imagination in the face of much skepticism, even through the dark days when NSF undergraduate support vanished entirely. Yet even today people still need to be convinced that activities such as REU are as important as coursework in students' mathematics education.

Cynics are fond of pointing out that only at the extremes of kindergarten and graduate school does science education take seriously the inquiry-based process of scientific practice. But support for "inquirybased, student-active" courses is evident in everything from the NCTM Standards to calculus reform, from the National Standards for Science

Education to Project Kaleidoscope. In fact, NSF now urges departments to build inquiry into every course in order to "reveal the excitement of cutting edge research" [George, p. 65]. We have indeed come a long way since the early days of URP.

Limited Vision. Nonetheless, old attitudes die slowly. Mathematics still lags behind the other sciences in the penetration of research experiences into the undergraduate program - not because of any failure of mathematics students' intelligence or imagination, but because many mathematicians cling to an excessively narrow view of research. In fact, mathematicians' traditional approach to mathematical research is not only a handicap to undergraduate research, but to mathematics itself.

Evidence comes from many sources. A 1995 NSF workshop on graduate education concluded that although mathematicians' skills are widely useful, prospective mathematicians lack practical experience in contexts where mathematics is used [Harris]. A 1996 report on mathematics in industry identified some widespread deficiencies of mathematicians' typical training, including a tendency to believe that the goal of research is to write a paper, a desire to prove theorems rather than deal with messy data, and a determination to tie up minor loose ends long after the central problem has been resolved [Davis, p. 19].

Perhaps the most thorough critique of the effects of mathematicians' traditional approach to research can be found in the 1998 report of the senior assessment panel for U.S. mathematics [Odom]. Academic mathematics, according to this assessment, is "insufficiently connected" to mathematics outside the university. The authors indict the typical division between pure and applied mathematics as "highly destructive." They worry that mathematical scientists have a "limited vision" of their capacity to interact with other scientists, and that they transmit this limited vision to their students. In short, they conclude, the balance between theory and application is "tilted too far toward inwardness."

What's all this got to do with REU programs? Simply this: to serve mathematics well, the apprenticeship experience provided by REU must include not just the internally focused problems of traditional mathematical research, but the full range of mathematical scholarship that the profession is gradually coming to recognize as crucial to its health and survival.

Unlimited Opportunities. Mathematics is a field of unlimited opportunity and unparalleled value to other disciplines. Research experiences help mathematicians learn to ask the right questions, to develop appropriate levels of abstraction, and to identify hidden connections
among problem components. People with these skills are able to deal not only with the challenges of traditional pure and applied mathematics, but also with new problems arising in such areas as bioinformatics, global climate modeling, management of hazardous materials, nondestructive testing, precision agriculture, and optimization of manufacturing processes - to name just a few new areas where mathematically trained people are having a significant impact.

As evidence I offer two examples from both ends of the undergraduate spectrum. The first, at the upper level, are titles of some talks given at a recent meeting on contemporary applications of mathematics:

- Integrating market and credit risk.
- Hollywood effects: clouds, hair, water.
- Dynamics of air bearing sliders in computer disk drives.
- Early detection of disease-induced transitions in tissue.
- Target tracking: From Star Wars to video games.

The second, at the lower level, is from a recent advertisement by a small machine shop seeking to hire machinists who

- Can edit and program CNC (computer numerically controlled) machine tools;
- Have a working knowledge of dimensioning, tolerances, and understanding symbols from blueprints; and
- Are experienced with inspection techniques, statistical quality control, and statistical process control.

These examples and many others illustrate the rich perspective on scholarship set forth in the widely read report Scholarship Reconsidered [Boyer, pp. 15-25]. Boyer argued that the "arrow of causality" between basic research and other functions such as teaching, communicating, and applying points in both directions. In support of this argument, Boyer identifies four mutually reinforcing forms of scholarship-the scholarship of discovery, of integration, of application, and of teaching:

- Discovery advances knowledge, creates excitement, and invigorates both people and disciplines.
- Integration gives meaning to isolated results by connecting, interpreting, and transforming.
- Application involves scholarly service that both utilizes and creates understanding.
- Teaching transforms and extends knowledge and creates the highest form of understanding.

In 1994 the Joint Policy Board for Mathematics [Moore, pp. 39-40], adapting Boyer's definition, argued that scholarship in the mathematical sciences includes:

- Research in core or applied areas that leads to new concepts, insights, discoveries, structures, theorems, or conjectures;
- Research that leads to the development of new mathematical techniques or new applications of known techniques for addressing problems in other fields;
- Research in teaching and learning that leads to new insights into how mathematical knowledge and skills are most effectively taught and learned;
- Synthesis or integration of existing scholarship in surveys, book reviews, and lists of open problems;
- Exposition that communicates mathematics with improved clarity to a variety of audiences including scientists, teachers, and the general public;
- Development of courses, curricula, or instructional materials for teaching mathematics in K-12 as well as the college level;
- Development of software for research in mathematics or its applications, for communicating mathematics, or for teaching and learning mathematics.
Inquiry-based, student-active scholarly experiences such as these should be a standard part of every undergraduate's experience in mathematics. Whereas the primary goal of the formal NSF and NSA REU programs may still be, as was the earlier URP program, to identify and encourage future research mathematicians, the experience of inquirybased scholarship is broadly beneficial for all undergraduates - even for those who will become teachers or technicians, politicians or journalists, doctors or lawyers. All undergraduate mathematics students deserve opportunities for initiative and enterprise that are as broad and inviting as mathematics itself.


## References.

1. Boyer, Ernest L. Scholarship Reconsidered: Priorities for the Professoriate. Princeton, NJ: Carnegie Foundation for the Advancement of Teaching, 1990.
2. Davis, P. W. Mathematics in Industry. Philadelphia, PA: Society for Industrial and Applied Mathematics, 1996.
3. George, Melvin D. Shaping the Future: New Expectations for Undergraduate Education in Science, Mathematics, Engineering, and Technology. Washington, DC: National Science Foundation, 1996.
4. Harris, William. Graduate Education and Postdoctoral Training in the Mathematical and Physical Sciences. Washington, DC: National Science Foundation, 1996.
5. Moore, Calvin C. Recognition and Rewards in the Mathematical Sciences. Washington, DC: Joint Policy Board for Mathematics, 1994.
6. Odom, William E. Report of the Senior Assessment Panel for the International Assessment of the U.S. Mathematical Sciences. Washington, DC: National Science Foundation, 1998.

St. Olaf College, Northfield, MN 55057
E-mail address: steen@stolaf.edu

# e-REU 

Robby Robson

This article is about the virtual REU or, using more modern notation, the e-REU. My intent is to share some personal observations and views about REU programs and the Internet and, based on these, suggest three structures that I believe will catalyze the emergence of an e-REU. I am reasonably sure that something like an e-REU will emerge in the not too distant future but that the details and exact timing remain unclear.

## Three Questions

It is a relatively easy exercise to construct an online version of an on-site REU using today's technology. If one allows for developments such as Internet II, MathML, and collaborative workspaces, the exercise becomes even easier. Lectures can be given using streaming video with capabilities of client feedback; online mentoring and online collaboration can replace their on-site counterparts, and there could even be a virtual lounge where students talk, play games, go on virtual field trips and engage in other social activities. It is a reasonable proposition that an online version (or at the very worst an online substitute) can be found for any given component of an existing REU.

But the result of transferring existing practices to new media is not always a happy one. The real question is not whether the Internet can support an REU program but whether the Internet is intrinsically suited to do so. If it is, and if furthermore (1) there is a value in REU's and (2) the Internet can add value, then e-REU will happen. The Internet emphasizes self-organization, not organization; the focus should not be on making something happen but on allowing something to happen.

[^23]With this in mind, I would like to address three questions.

1. What is the essence of REU?
2. Can the Internet capture this essence and add something to it?
3. What can be done to facilitate this process?

The remainder of this article is devoted to giving a (not the) set of answers.

## Essence of REU

If the Washington REU meeting taught us anything, it is that successful REU programs exhibit astounding diversity in form, format, and even purpose. What, then, are the common threads? What are REU programs really about? Part of our work in Washington focused on these questions and this volume contains detailed answers elsewhere. My own distillation of these answers is this: REU programs are about community and empowerment.

Community and empowerment play two roles in REU programs. First, they are ends. As stated in the REU program announcement (NSF 96-102), the goal of REU programs in the sciences is to "attract a diversified pool of talented students into research careers in these fields, and to help ensure that they receive the best education possible." In other words, to enlarge the community of science and to empower students to succeed in it. I believe that the goals stated by the various mathematics REU directors for their own programs fall neatly into the categories of community and empowerment as well, but following the mathematical tradition I will leave this to the reader to check.

Second, community and empowerment are also means. Despite the diversity of the manifestations, there are only two fundamental activities in all of the mathematics REU programs: being mathematicians and doing mathematics. REU programs provide a safe place for students to be the former and the support necessary for students to successfully do the latter. Again, community and empowerment.

## Relevance of the Internet

Is the Internet a medium that is suited for community and empowerment? In my opinion, the Internet is about community and empowerment. The evidence for this assertion can be found in newsgroups, chat rooms, email, ICQ, and the hundreds of thousands of pages devoted to highly specialized interests. A different sort of evidence can
be found in groups like the Internet Engineering Task Force whose only qualifications for membership are a willingness to work and the knowledge to do so. And still other evidence can be found at sites like amazon.com and eBay.com which have extended consumer capabilities by identifying and mediating communities of interest. The Internet is quite proficient at legitimizing and supporting groups and cultures defined by common interests and goals, and in principle this is what REU programs do.

Simply because the Internet is capable of supporting something does not mean that there is value in using it to do so. The growth of the Internet is strong evidence that something else of value is being added. What is it?

One answer is access. In the United States, at least, the Internet provides low cost and geographically independent access to information and communication. Access is significant for REU programs. Transportation and housing costs can be conservatively estimated at $20 \%$ of the cost of an REU and the opportunity cost incurred by the need to be temporally and physically co-located is high for students and staff alike. The Internet has the potential to significantly reduce costs.

## Three Suggestions

If we agree that community and empowerment are the essence of REU programs, and that the Internet is at least theoretically wellsuited at providing these while adding value in the form of increased access, then what can be done to facilitate the emergence of e-REU? I think there are three fairly obvious and in principle simple things to try. I would go even further and argue that if these three things don't succeed, it is a sign that it is not yet time for e-REU, and that if it is time for e-REU, these three things will serve to catalyze its formation. Here are my suggestions.

1. I suggest developing an online clearinghouse for REU problems. This site should be as open as possible and draw problems from the mathematical community at large, not just from REU specialists.

I will not suggest how problems should be screened or classified. These are crucial details, but the way of the Internet is to find solutions by community experimentation rather than authoritative prescription. I am confident that "we" or "they" will come up with some excellent ideas and converge on a workable solution.
2. I suggest establishing an online student and mentor network, a
virtual REU center where students can find mentors, mentors can find students, and everyone can communicate with each other.

Again, there are many unresolved issues: how will mentors and students be matched, is it necessary to select students and mentors and how might this be done, what are the commitments and responsibilities and what mechanism will encourage their fulfillment, what sort of communication will be most effective, and so on. And again the resolutions must emerge from experimenting with a number of different implementations.
3. I suggest creating an open forum for the electronic exchange of student work.

This, at least, is a suggestion I heard a number of times at the REU meeting in Washington. In concrete terms, with many details left out, this means creating and publicizing a searchable archive for electronic versions of student REU papers.

I believe that suggestions 1 and 2 provide the basic elements need for a research experience. 3 provides both examples and validation. My feeling is that these would suffice to create an Internet firmament in which an e-REU can grow.

## Conclusion

I greatly appreciate the opportunity to express the ideas in this article and hope that they will motivate others to think about the possibilities and eventually act to make virtual REU a reality. I would also like to add a note of gratitude to the sponsors and organizers of the REU conference that led to this article and the volume in which it appears. The meeting was perfect in its conception and organization and was a wonderful and valuable experience for all involved.

Oregon State University (http://OSU.orst.edu/~Robsonr)
E-mail address: Robby@orst.edu

# Opportunities for Undergraduates at the University of Arizona 

Elias Toubassi

Five years ago the Department of Mathematics at the University of Arizona began to look at ways of integrating research and education. The goal is to involve researchers in the effort to communicate mathematical ideas at a level that undergraduate and graduate students can understand, participate in, and contribute to in a meaningful way. With departmental seed money and funding from NSF, we are able to offer our undergraduate students a variety of opportunities. These include:

1. Undergraduate Research Assistantships
2. Undergraduate Teaching Assistantships
3. Internships in government labs or industry.

The first venture in this area began with the funding of the Southwest Regional Institute in the Mathematical Sciences (SWRIMS) by NSF from 1994 to 1998. One of the purposes of SWRIMS is to provide the research community with examples of universities that integrate research and education. The underlying philosophy of SWRIMS is that of mathematical modeling. The study groups formed by the project in mathematical biology looked at predator-prey simulations, plant growth experiments, deer and squirrel populations, and a model to predict the spread of the Africanized bee into Arizona. There were also study groups in the area of cryptography. The basic research unit of SWRIMS is that of a "core group". It consists of 2-3 university faculty, 1-2 graduate students, 1-2 undergraduate students, and 2-3 high school teachers. The core groups met on a weekly basis to discuss the mathematics behind the model under study as well as how to communicate the mathematics to diverse audiences such as high school students. Many of the lessons that were developed were in fact tried in

[^24]high school classrooms. Undergraduate students were an integral part of these projects including the predator-prey simulation, the growth of bacterial colonies, the model on the spread of the Africanized bee, and several ideas from cryptography. For more information on SWRIMS see http://math.arizona.edu/programs/intro.html and two articles, one by W. Y. Vélez in the October 1996 Notices and another by W. Y. Vélez and J. C. Watkins in Contemporary Issues in Mathematics Education, MSRI Publications, No. 36, Cambridge University Press.

The next venture began two years ago with the offer to undergraduates to work with faculty on research projects. These could be done during the academic year or summer, for independent study credit or for a stipend. Faculty who were interested in leading such projects were asked to write a brief description of the proposed activity together with the background needed to carry it out. Some of the projects required little background while others required some knowledge in linear algebra, differential equations or group theory. These were posted on the Math Center web page, distributed to undergraduates, and followed up with informational meetings to answer questions. Currently twelve projects are posted on the web site such as Modeling Microvascular Networks, Simulation of Waves and Shocks, and Patterns in Continued Fractions. In the past two years 21 projects have been completed and 9 more are currently in progress. For more information see http://www.math.arizona.edu/ ${ }^{\text {m m }}$ center/UAMAZ.html.

This program has recently gotten a big boost with the funding by NSF of the department's VIGRE (Vertical Integration of Research and Education in the Mathematical Sciences) grant. This will intensify the existing programs and open up new opportunities for undergraduates. The grant will fund about 25 undergraduate assistantships each year in both research and teaching. The undergraduate research assistantship program is an extension of the project described earlier. It is an opportunity for an undergraduate to work with a faculty advisor on a research project. One of the requirements of this experience is that a written report be submitted for web publication. These assistantships pay $\$ 1250$ per semester or $\$ 1000$ per month during the summer. Grant funds are also available for travel support to participate in conferences on undergraduate research.

The second type of assistantship under the VIGRE program is related to undergraduate teaching. It offers undergraduates the opportunity to learn about teaching mathematics by working with an instructor in a lower division course. The undergraduate student is supervised by the instructor and a program mentor. The duties can vary from classroom demonstrations, to running review sessions, participating in a
tutoring room or grading papers. In spring 2000 we expect to have between 10 and 15 undergraduate teaching assistants working in second semester calculus. The stipend for the semester is $\$ 1250$. One can find information about the teaching and research assistantships at the Math Center web site http://math.arizona.edu/ ${ }^{\text {mcenter/ UAMAZ.html. }}$

Another opportunity that undergraduates are encouraged to participate in is the Arizona Internships in Math, Engineering and the Sciences (AIMES). This affords students the chance to gain handson work experience at some stage of their education. The AIMES is an informal program that serves as an information-clearing house for opportunities in government laboratories, industry and corporations. Meetings with students are held once each semester to explain the program, hear from previous participants and discuss the application process. The program offers students the opportunity to use their skills and knowledge to solve problems, to learn to be part of a team, and to work under a deadline. It also serves as a way to connect mathematical information learned with other branches of science, engineering and business. The AIMES Program Committee alerts external organizations of the university's interest in participating in their internship program. The Committee sends out a letter to major corporations to express an interest in forming partnerships together with background information on our interdisciplinary research program. One of the goals of AIMES is to act as a liaison and point of contact between these organizations and undergraduate students. The students are expected to contact internship sponsors directly and make their own arrangements. For more information on this program and the list of companies, visit the web at http://math.arizona.edu/~restrepo/amii.html.

University of Arizona, Tucson, AZ 85721
E-mail address: elias@math.arizona.edu

## Part V

## SUMMARY OF <br> CONFERENCE SESSIONS

# Notes from the Plenary Sessions 

Robby Robson

## REU Meeting 10/1/99

## John Ewing AMS

Undergraduate enrollments: Calculus dropped $15 \% 637 \mathrm{~K}$ to 539 K ; advanced mathematics dropped 138 K to $96 \mathrm{~K}(-39 \%)$ since 1975 : 18 K majors down to 12.4 K now; grad enrollment: $-18.5 \%,-28 \%$ in group 1, higher decreases in first year graduate students. PhDs $48 \%$ to US citizens. In 1978 was $74 \%$. Last year only 20 African Americans, 46 Hispanic.

These numbers reflect perception of mathematics as unexciting, stale, dead. In reality mathematics is broader and more fundamental than ever. We need to change the perception of mathematics. Mathematicians rediscover public awareness problem periodically. One reason for the dislike of mathematics is that it is vertically organized.

REU is a special kind of public awareness. REUs make it easy to outline the goals and roles of various people. The underlying goal is to convey the excitement and breadth of mathematics to that group of students. We want to entice some of them into mathematics, but also some of them into simply appreciating mathematics, regardless of what they do later in life. This requires long term investments from agencies. The role of the professional societies is to facilitate, with long term view in mind. Part of the role of faculty is to leave behind a legacy. REU programs are effective.

[^25]
## Jim Schatz NSA

REUs are important to students, the host institution, and the nation. Universities may not feel that an REU is doing them any good. The NSA is self-centered in expectations for REUs. The NSA cannot share the excitement of their work with outsiders. The NSA summer program is important for bringing people in and exposing them to the math culture at NSA and problems of national importance. The NSA identifies 10 mission critical problems at the beginning of the summer. The NSA uses the summer program to start research efforts and get solutions.

Each student gets to jump to a new level of mathematics through the experience. Critical aspect is problem set and problem support. Not having a really super problem set is counterproductive. The REU is a wonderful opportunity to lose students from mathematics forever. Good mentorship is needed.

An REU gives students exposure to mentors other than the one advisor ( PhD advisor) they see during the normal education.

The NSA regards it as their greatest success when one of their students ends up taking a key university position somewhere in the country.

## Lynn Steen

The URP (Undergraduate Research Participation) program is the predecessor to the REU program. In 1967 there were 500 URP awards given in all disciplines with ten in mathematics. This is a very tiny percentage. About one tenth of the expected value based on the number of students. URP was viewed as an apprenticeship for the students. Mathematicians could not imagine that an undergraduate could be an apprentice to them in their research. Investigative, inquiry-based student-active programs are far more effective than passive lecture formats. The NSF now recommends that in every course there should be a research-type experience.

How can we get more undergraduates involved in these types of experiences? We need to broaden our sense of what mathematics is, what it is involved in, etc. An assessment done for NSF about research mathematics was critical of isolationist tendencies of mathematicians. The report recommended changes in graduate education to bring about more cooperation and research coordination with other fields.

JPBM advocated that mathematicians should broaden the view of mathematical research.

## Assessing Programs: Deborah Nolan

Why does evaluation matter?

- Gives you knowledge
- Helps plan
- Shapes policy
- Documents achievement
- Attracts funds
- Identifies successful innovations
- Identifies best practices and propagate
- Educates the public and mathematics community

What to evaluate?

- Audience
- What is in original proposal
- Program goals and objectives
- How do similar programs evaluate
- Unanticipated side effects

Goals and objectives

- Motivate students to complete doctorate
- Prepare for grad school
- Provide information on applying to grad school
- Build support network
- Encourage leadership
- Unanticipated: assist teachers

Evaluation questions

- Success rate
- Compared to others
- Impact on decisions
- Use of people network
- Impact on faculty and grad student

Examples of evaluation questions:
To what extent, if any, did the program affect your:

- self confidence
- motivation to do graduate work
- knowledge about what graduate school is like

From your perspective, how important was it that the program involved only women as: students, TAs, faculty. Response rate: $80 \%$ of students (54); 75\% of faculty (32)

## Student Panel

Stephen Hartke (SH) went to Lafayette REU (rejected by all others), then Gallian's program, then NSA-DSP. Am interested in theoretical CS and combinatorics.
Cheryl Grood (CG) is a U. Michigan undergraduate, Ph.D. from Wisconsin, now in second year as professor at Swarthmore. Went to RoseHulman REU and then to Mills/Berkeley. First REU convinced to go to grad school, second "help me stay in grad school." Networked with friends from Mills program at key points in grad school (qualifying exams, etc.)
QUESTION: How did you hear about your first REU?
CG: I heard about first from another student in college. Second from AWM newsletter.
SH: I heard from advisor and talked to others.
QUESTION: How did other faculty members at home institution react to desire to go to an REU?
CG: No one knew.
SH: In general, they were supportive.
QUESTION: Were there negative side effects from REU participation? SH: Generally positive, but I know others who were completely turned off to mathematics. Math was not for them but some ended up bitter. Group work can be a bad experience due to personality conflicts. Directors' attitude is "deal with it, it's the real world."
CG: Students with less background can question themselves, but in the end it is a confidence building experience. It is a shock for students from smaller schools.
SH: Over $90 \%$ had a great experience.
QUESTION: Were problems pre-selected or did you look for your own? CG: They sent papers prior to program. I got a list of problems. I could choose from list but also I could go off in another direction.
SH: At Lafayette we all worked on different aspects of the same problem. At Duluth students are given a problem. They have no choice. At the NSA DSP we choose among problems. All three methods worked well.
CG: The Mills program organized alumni to go to Winter meetings. The AWM helps too. I went back to Mills as teaching assistant.
QUESTION: What are some reasons for negative experiences? A mismatch in mathematical background, personality conflicts?
SH: That's accurate. Also mismatch between level of students and level of problem. Problem with set problems-not as much flexibility. Some students look at problem as exercise, others look at it as a Ph.D. thesis.

QUESTION: Of the students who decided math wasn't for them, was it obvious to you on day one that they were not interested?
CG: It goes both ways-some who I thought would go to Grad School didn't, others who I thought wouldn't, did. Some amazing math students follow other paths.
SH: Some might look good on paper but don't really have depth of understanding. Communicating with prospective students might give better indication of abilities.
COMMENT: It is a good idea to call students on short list.
QUESTION: Is it worthwhile to participate in more than one REU?
CG: My experiences complemented each other. Goals and objectives of programs were different.
COMMENT: The chairs should meet with majors to make them aware of REUs.
SH: Many departments are too busy or too apathetic to do that.
COMMENT: REUs should be emphasize at annual meeting of chairs. EWING: AMS/NSF can't tell chairs what to do. The departments should take interest in undergraduate programs. Also, there is no evidence of a lack of applications to REUs.
QUESTION: What is your view of discovery-based learning-should this be continued all through education?
SH: I have been spoiled last three years working on projects. But class work is important. Taking standard classes is necessary. VIGRE approaches this.
QUESTION: What is your opinion on team work?
CG: Having individual problems is important for sense of ownership. Related problems are good for discussion.
COMMENT: REUs are not for all math majors. We must be honest about this.
QUESTION: What effect did REUs have on your present teaching?
CG: They help in research arena. I learned how to give good research presentation. I am not so sure about impact on teaching students in classroom. Being a student assistant at Mills helped me sort out what was important.
QUESTION: Is there something about REU programs that is really beneficial?
CG: I would not have gone to grad school were it not for the program.
SH: I was certain I was going to grad school. I chose math because of the REUs. It increased my confidence.
QUESTION: Has the REU experience helped your professional development?
CG: Mills had panels about grad schools. I heard a lot (formal and
informal) about Berkeley (at Mills program). I heard a lot about funding opportunities. Mills brings together a group of passionate people students share knowledge.
SH: REUs are instrumental in developing awareness of what mathematicians do-what do they do when not teaching, funding issues, what is graduate school like. It was an eye-opening experience. I got good information from other students, even better, former students.

## Lloyd Douglas (NSF): Why fund mathematics REUs?

There is a Diversity of models for REUs - number of faculty per students, types of students targeted, different funding models, different types of activities. All models work, which is astounding.
REUs have made a real impact on the discipline, other disciplines etc. Why have REUs at all?
Purpose:

- Attract a diversified pool of talented students into research careers
- Too few research experiences are now available
- The REU program is designed to help meet this need

What's in it for the students?
Students get involved in meaningful ways:

- On-going research projects
- Research projects specially designed for this purpose

Expanding Horizons

- Many undergraduates don't know what mathematical research is
- Many undergraduates don't know what mathematicians do for a living


## Quotes from REU Participants

"It was one of the most interesting and most fun summers I have ever had.
"This REU was conducive to thought, creativity and play - which is an excellent combination."
"... I knew I might not want to do biology, now I know I might not want to do math."
In this country we say "only some of you guys can do math and the rest of you can't, so we won't teach it to you." When you think something is important you want to shove it down someone's throat. In math we
do the opposite. We say math is important but we won't tell you about it. Moreover, mathematicians cannot effectively make their own case. Need other disciplines to make our case.

Other Faculty Benefits:

- The Roman Rule: "The one who says it cannot be done should never interrupt the one who is doing it."
- The sheer enjoyment of directing undergraduate research.


## Challenges

- Everyone's last mathematics experience should be a positive one.
- A significant mathematics research experience for every undergraduate who wants one.
- People brag about not being good in math. No one ever goes around and brags about being illiterate. Money: Money is not the problem. DMS has $\$ 105 \mathrm{M}$ budget. Entire NSF budget approaches $\$ 4$ billion dollars. We are roundoff error. Congress appropriated $\$ 10$ billion to aid refugees in Kosovo.


## Report from Discussion Groups on the Future of REUs

- Stipends need to be increased, $\$ 5,000$ NSF figure has been constant, Faculty need to be paid.
- Need other sources of money.
- Need measures to evaluate programs - Numbers of PhDs may not be the only good measure
- Variety of REUs is positive.
- Should increase diversity of applicants and numbers of programs.
- Ask AMS for help in finding more support.
- Need workshop on REUs.
- Need visible journal of undergraduate research.
- Need visible Web site for student articles plus a Web site for directors. Gathering similar summary statistics.
- Should hold meetings like this periodically.
- Establish an AMS session at joint meetings for presenting REU work.
- Need for diversity among programs.
- More of the "recognize them young and untapped resources" types of programs.
- Intuition says there is a lot of unmet demand, but we need responsible analysis to demonstrate this.
- Student funding level too is low. Cannot compete with industrial jobs.
- There is a need for cooperation and communication among programs
- Internet offers opportunities for change: urging REU sites to post reports. Virtual conferences.
- Need funding from industry
- Need to spread REUs around more geographically
- Need to work more cooperatively as a group, more central information
- Directors should forward applications (with permission box)
- Have traveling speakers, sharing resources
- Need change in ethos: more support from administration. Reward faculty with reduced teaching loads
- Ethos of REU-type problems should permeate our entire educational system. Need to use open-ended problems.

COMMENT: Since 1993 part of our professional recommendations has been that open-ended problems be part of every mathematics class. COMMENT: We need to figure out how many students are not placed in any REU. What about permission box asking if we can use their information for demographic study?
COMMENT: Points of information. New journal: Rose-Hulman (refereed journal). Summer meetings offer students opportunity to present their research. Travel money is available. There are prizes for these. Also, can always submit paper to AMS contributed paper sessions.
COMMENT: Need to study demand. Analogy to army recruiting. Don't need evidence. Lack of minorities is prima facie evidence.
COMMENT: Lots of people want to direct REUs who do not get the opportunity.
COMMENT: REUs give students validity. It therefore magnifies our talent pool. The argument is that only students from colleges with good reputations will get into good graduate programs. REUs give others a chance.

## John Ewing (closing remarks)

This conference has been a perfect match for the AMS and NSA. I opened this conference with a pessimistic view with statistics and I should end with some sincere optimism. REUs solve piece of much, much larger problem, but an important piece. Conservative advice:
don't succumb to the world hunger syndrome. Big problems can paralyze people. REUs cannot solve all the problems of undergraduate education. My advice is to reach out as far as you can but don't lose your balance in doing so. My second piece of advice has to do with expanding. Expanding means more money and more people, and people are probably harder to get than money. To do this we need convincing arguments. We must make certain today's REUs run well and we must document that fact. People invest in success. The first priority is to make sure we continue doing what we are doing now.

[^26]
# Discussion on How to Evaluate Summer Mathematics Programs 

Discussion Leaders:<br>Toni Bluher, National Security Agency<br>Barbara Deuink National Security Agency<br>Svetlana Katok Pennsylvania State University Robby Robson Oregon State University<br>Summarized by Deborah Nolan, University of California, Berkeley

## Introduction

One of the organized discussions at the conference on summer mathematics programs focussed on how to evaluate these programs. The discussion included ideas on: how to conduct evaluations to improve program operation and to address the needs of students while they are participating in a program; how to measure whether the goals and objectives of a program are being met; and how to evaluate the effectiveness of summer math programs in general as opposed to the effectiveness of a particular program. These ideas for evaluation are summarized here.

## Program evaluation

The working and living environment of the program-whether students enjoy their work, have healthy living conditions, and interact well with each other-are important aspects of any summer mathematics program. These aspects are important because they play a role in determining a student's happiness and performance, and in the ultimate success of the program. When the quality of the program environment is assessed during the program, actions can be taken, if necessary, to improve the situation while the program is in progress. The same holds

[^27]for the quality of the research program; if assessed during the program, adjustments can be made as needed.

There are several avenues for obtaining information on the quality of life and the quality of the research experience in the program. For example, program personnel can meet one-on-one with students, where the student has the opportunity to raise his or her concerns in private. Alternatively, groups of students can be asked to brainstorm together to provide an assessment of how the program is going, or anonymous midprogram evaluations can be collected. Meeting with students outside of the work area, such as in a dorm, cafeteria, or coffee shop, provides a safe place for students to freely express their concerns. To further encourage open feedback, you might consider using someone who is not directly involved in the program, such as a returning student or a former instructor, to conduct interviews with the students.

End-of-program evaluations can provide a lot of information for improving your program in the future. To obtain the most useful information, questions need to be worded so that they elicit meaningful responses, not just yes/no answers. Also, anonymous evaluations encourage honest responses, and conducting evaluations before students leave the program should increase the response rate. You may want to offer students an incentive for completing a questionnaire before leaving the program.

An end-of-program questionnaire should solicit feedback on operational specifics such as the program pace and schedule, extracurricular activities, quality and diversity of visitors, difficulty and suitability of the research problem, how the program met or fell short of expectations, and accuracy of preprogram advertising. The questionnaire should also attempt to discern the effect of the program on a student's plans for next year, plans for after graduation, and understanding of what it is to be a mathematician. Baseline questions asked on the first day of the program can help document changes in a student's plans and perspectives.

## Goals and Objectives

The conference participants were asked to consider how to evaluate a program to determine how well it has met its goals and objectives. The discussion focussed on evaluation of the goals and objectives that were set forth at the conference in an earlier organized discussion.

Provide a mathematically rich and professional environment

To measure the richness of the mathematics environment that students are introduced to in a summer mathematics program, student and program activities can be assessed.

With regard to students, you can: enumerate student output, such as oral presentations, poster presentations, and written reports; describe the problems, papers, and books that students used in their summer work; quantify the amount of time students spent communicating mathematics; and have students describe the areas of mathematics which they were exposed to in the program. To make these measures of student activities more meaningful, they need to be compared to preprogram activities or to a typical undergraduate experience.

The mathematical richness of the overall program can also be evaluated by examining students' exposure to professionals, and by providing a description of the facilities available to students in the program.

## Raise the mathematical maturity of students

To evaluate how well a program has performed in raising the mathematical maturity of its participants, the change in the student must be measured. Change can be measured in the following ways: the participants themselves can provide a self-evaluation one year or more after the program; the faculty who recommended the students to your program can be asked for their opinion and observation of the change in the student; program visitors can serve as witnesses to in-program change in students' abilities; and a panel of experts can evaluate the quantity and quality of students' technical reports and students' presentations at conferences. These experts would need to see student improvement, possibly through multiple presentations or written documents.

## Diversity

Many programs have as a goal to increase the number of students from traditionally underrepresented groups who enter and successfully complete graduate school. Many programs also attempt to assist mathematically talented students (from all groups) whose mathematical preparation for graduate school may be insufficient.

To measure how effective your program is in achieving these goals, long term tracking is needed. Not only does one need to keep track of the number of students who apply to and get accepted to graduate school, students also need to be followed through graduate school to determine how many obtain advanced degrees. Additionally, a study of
the demographics of the participants in your program will help indicate how well your program is meeting these goals.

## Prepare students for graduate school

Many of the items discussed previously can be used to evaluate a program on its success at preparing students to apply to and succeed in graduate school. Determining how many students apply to, get accepted at, receive fellowships for, and attend graduate school are all different measures of how well this goal is achieved. A comparison to mathematics majors at the participant's home institution would be most informative in documenting success. Student self evaluations may also be informative. Students can describe their knowledge about graduate school before and after the program. Once in graduate school they can attest to whether the program has had an impact on their research, instruction, and knowing what to expect. To see the long term effect of the program, long term tracking is needed. Although following students in time is important, a balance must be struck between keeping track of the students and annoying them with requests for feedback. A few key points in their careers can be identified, and they can be contacted at these times.

## Build community

To measure the utility of this community, past participants may be questioned on the number of post-program contacts they made and whether they were student to student, student to faculty, or student to graduate student. Program faculty may be asked how many letters of recommendation they have written for former students. Gatherings at conferences, repeat applications to the same or other programs, and interviews with returning students may provide evidence of community and its value. Anthropologists could possibly provide assistance for designing an evaluation of the mathematics community built by the program.

## Global evaluation

The discussants pointed out the need to look at the effectiveness of summer mathematics programs overall to determine the general usefulness of such programs. Suggestions were made to track students in graduate programs in the mathematical sciences, and to compare the success of those who had attended summer mathematics programs to
those who had not had such an experience. A large scale evaluation of summer mathematics programs would be very labor intensive. The mathematics community needs to develop and carry out such a study.

## Conclusions

Many outcomes of a summer mathematics program are intangible, and there may be more than one standard to measure success for different programs. For example, a program may be healthy, even if no research papers are published from it, and a good evaluation must distinguish the quality of research from the quality of research experience.

Student assessment of a program is one source of evaluation, but not the only source. We have discussed many avenues for collecting information that would be useful in evaluating summer mathematics programs. Both factual data and subjective impressions should be sought. For example, non-questionnaire evaluations such as observation by non-program faculty can provide valuable information on a program's effectiveness.

The evaluation task is not insignificant, and it is advisable to set aside funds for evaluation and to seek advice on your evaluation plan. Questionnaires can be shared across programs, students can assist in pinpointing useful information to be collected, and evaluation specialists can be consulted. Evaluation is important to the future of all summer mathematics programs.

## Discussion Group on Future of REUs

David Lutzer-Facilitator

1) It is important to maintain a diversity of program types. There needs to be a mixture that includes programs that focus on "the best and the brightest" together with other programs that focus on "untapped resources" and "developing younger talent". Probably there should be an increase in the number of "untapped resources" and "developing younger talent" sites in the near future, while maintaining the number of others.
2) Is REU funding at the right level? There were several subquestions here.

2-a) Our consensus intuition is that there are not currently enough REU student slots to meet existing demand. The NSF should increase the number of student slots available. Because the number of slots in a given program is a delicate matter of balancing local faculty resources, the NSF should increase the number of sites rather than urge existing sites to expand. There seem to be plenty of places that would like to start NSF funding. (We admitted that we did not have concrete proof that there is a shortfall, and suggested that if NSF wants such proof they will need to ask some centralized agency (say AMS) to conduct a study of demand for REU slots. During our presentation, Lloyd Douglas told us that NSF already believes that a shortfall in REU slots exists.)

2-b) The per capita funding level for REU students needs to rise. Already we cannot compete with summer internships in terms of money offered to students. The $\$ 5000$ per student guideline has been in place far too long already.

2-c) Foundations and universities are another potential source for increased REU funding, but there needs to be adequate centralized funding unless NSF wants REU programs to shift to richly endowed private universities who can afford to add large supplements to what

NSF gives. (There wasn't any discussion about the level of support that NSA gives; I don't know whether that is also a problem.)
3) In the future there needs to be much more cooperation between REU sites. Common dates for making offers and for expecting responses were mentioned. Sharing names of outstanding minority students is another. Having joint conferences between different REU sites was a third.
4) Distance learning and the internet as future components of REU. REU sites (except NSA) should put student papers on line so that other students can study them. Perhaps there should be "virtual conferences" among REU programs, involving two-way video over the internet. Speakers can be brought to REU sites over the internet. Some participants warned us not to go too far in this direction, because direct contact between faculty mentors and REU students can't be replaced by less personal long-distance communication. People joked about electronic REU programs in which software faculty were paid with virtual money to talk to simulated students.

# Small Discussion Group on the Future of REUs 

Deanna Haunsperger-Facilitator

1. There is the expectation that the proceedings of this conference will not only highlight the great things that are happening in REUs, but also the areas where we need support, such as administrative support from our home institutions. The conference and proceedings will lend some credence and national recognition to REUs to help convince our home institutions and colleagues of the importance of these programs.
2. We would like to increase the diversity of the applicant pool and increase the number of programs. The AMS should encourage the NSF to increase the number of REU positions available to students.
3. The AMS should look for alternative sources of funding (such as corporations, industry, and foundations) for REU programs.
4. Many of us were concerned about the future of REUs as some program officers suffer burnout; who will take our place directing these programs, and how can we support them so that they don't need to reinvent the wheel? The Joint Meetings should offer a workshop on "How to Run an REU." Available to participants would be sample proposals, program models, program methodologies, assessment tools, and other information to make starting an REU site easier.
5. A new website should be launched to advertise for all REUs, list publications and summary statistics resulting from REU participation (to help convince some of our colleagues who still believe that undergraduates can't do research), and share proposals and assessment tools.
6. AMS should send a poster advertising all REU programs to the math departments across the country. This could be a catchy, professional poster which would point students to web links for the

Received by the editor March 14, 2000.
various sites; it would also recognize that REUs are supported by the AMS.
7. Some believed there should be a visible journal of undergraduate research, but there was disagreement on this point: others believed that it is better to have students publish their results in established journals.
8. Hold conferences such as this periodically (every two years?) in the future to continue these discussions and include new REU directors into this network that we've started.

# Small Group Discussion on Identifying REU Goals 

Anant Godbole-Facilitator

Our group divided REU goals into two categories; long-term global goals, and short-term local goals.

## Long-term Goals:

- Increasing the supply of REU opportunities.
- Increasing internal support for REU activity.
- Broadening the faculty base by incentivizing REU activity.
- Changing the attitudes of reluctant faculty, who are typically senior, towards undergraduate research.
- Increasing awareness that REU involvement can benefit the departments and the institution.
- Advertising the wide array of summer programs and the distinctive nature of different programs.
- Fostering an REU alumni network.


## Short-Term Goals:

- Giving students an accurate sense of research, as well as an understanding of " Math culture" and the "Math process".
- Taking student to "as independent a status as their competence warrants".
- Increasing the pool of women and undre represented minorities in REU activity.
- Identifying and supporting the graduate school potential of students.
- Organizing grad school and GRE workshops.
- Giving students from small schools deeper opportunities.
- Helping meet high industrial and across-the-discipline demand for mathematicians.
- Helping create good problem solvers.
- Increasing students' verbal and written skills. Creating good communicators.
Caveat: Different programs have different goals. The above represented the union of our goals.


# Small Group Discussion on the Future of REUs 

Tim Pennings - Facilitator

- Achieve a more diverse group of student participants.
- Increase the grant allowance to more than $\$ 5000 /$ student to help attract faculty and to help attract students who must opt for better paying summer jobs needed to cover rising expense of higher education.
- Look for funding from industry.
- Increase the number of REUs and spread them around geographically.
- Increase the cooperation and central organization of the various REUs as needed (but no more). Examples include:
- General advertisement to all colleges and universities simply alerting students to the existence or REUs and directing them to a web address for information on specific sites.
- Forward applications from one site to another (easier if applications are electronic) provided permission has been granted by the student (via a box to check on the application form).
- Share resources among REUs including traveling speakers and using faculty from other institutions to lead projects.
- Changes in ethos:
- Support from more administrators. Current support for faculty who devote resources to undergraduate research varies widely from institution to institution. Some institutions expect such research, others enthusiastically (and generously) support it when instigated by the faculty member, and others give it virtually no support.
- Given that the time involved both for preparation and the actual mentoring is substantial, all institutions might consider reducing the teaching load, giving teaching credit, or providing financial support for those engaged in undergraduate research.
- Given the value of open-ended problems in preparing students for real life mathematical challenges (whether in graduate research or industry), the REUs should increasingly be used as an instrument to change the ethos within higher education to embrace these types of problems which are characteristic of REU research. This might be done through dissemination of successful research projects used in REUs and through getting more faculty involved in REUs so that they see the value and possibility through first-hand experience. In this way, whether or not they are involved in a funded summer research project, all students will have "research" problems woven throughout their educational experience.


# Discussion Groups on Goals for REUs 

Thomas A.Garrity-Facilitator

This is a summary of one of the small group discussions held during the conference. There was a lively discussion. We summarized our views and points by splitting an REUs influence and goals into three parts: how an REU effects a student, how it effects the host institution and how it effects society at large.

## 1. Students

Of course, an REU must be primarily concerned with the students. We hope that an REU will

- improve participants' ability to communicate
- teach problem solving skills
- raise students up a level in mathematical maturity
- provide mentoring for talented students
- help students network with their peers
- introduce students to the profession
- expose students to new areas of mathematics (especially for students from small isolated colleges)


## 2. Host Institutions

A department that sponsors an REU program will also gain some advantages. Here we noted that an REU

- can help jump-start an advisor's own research
- can energize advisor's teaching during the academic year
- can energize the host department


## 3. Society at large

An REU can also

- increase public awareness of mathematical research
- help identify mathematical talent
- help train people for other professions that need mathematics


# Group Discussion on Identifying Goals for REUs 

Suzanne Lenhart

List of goals

- to give opportunities to a diverse group
- to give opportunities to extremely talented students
- to teach students to collaborate
- to help students to make well-informed career choices
- for networking of students with each other (includes current summer students and graduate students)
- to provide mentors and role models for students
- to learn to give talks and to write technical papers
- to include an ethics component

The idea of REUs for future math teachers (going into teaching at middle or high school levels) came up.

## Part VI

## SURVEYS OF

## SUMMER PROGRAMS

# Survey Summary 

Joseph A. Gallian

Here are some facts from the survey that are noteworthy.

## Longevity

The following REU or REU-like programs have been running for 10 or more years: Indiana University, University of Minnesota, Duluth, Mount Holyoke College, NSA, Oregon State University, Rose-Hulman Institute of Technology, University of University of Tennessee, University of Washington, Williams College and the College of William and Mary. Among these the University of Minnesota, Duluth is the only program that has had the same program director throughout its history.

Schools that have run programs for 5 to 9 years include: SUNY at Potsdam, California State University at Fullerton, Berkeley-Mills for Women (no longer exists), Carleton Program for Women, Carnegie Mellon, Cornell University, Tulane University, College of Wooster, Hope College, Louisiana State University, George Washington Program for Women, College of Charleston, University of Puerto Rico, University of Dayton (no longer exists), University of Northern Arizona, Michigan Technology University.

## Tracking

Many programs do not long term track their participants. Among those that do, nine report 6 or more participants have received a Ph. D. degree as of 1999. They are University of Minnesota, Duluth (35), Mills (20), Rose-Hulman (18), Washington (17), Williams (14), NSA (12), University of Tennessee (10), Indiana (6) and Dayton (6). The

Received by the editor September 9, 1999.

AMS will attempt to long term track participants of summer programs in 1999.

## Local Funding

Many schools report substantial yearly local support for their programs. Among these are: Cornell (Math/Biology) $(\$ 70,000)$, Puerto Rico (\$40,000), Potsdam (\$40,000), Dayton (\$40,000), Tennessee ( $\$ 20,000$ ), Tulane ( $\$ 17,200$ ), Oregon ( $\$ 15,000$ ), Virginia $(\$ 15,000)$, Wooster ( $\$ 15,000$ ), Washington ( $\$ 12,000$ ), Idaho ( $\$ 10,800$ ), Williams ( $\$ 10,000$ ), Iowa State ( $\$ 8,000$ ), Utah ( $\$ 8,000$ ), Cornell (Math) ( $\$ 7,500$ ), Carnegie Mellon (\$3,700), Fullerton (\$3,500), Northern Arizona (\$3,000).

Many programs receive nonfinancial support from their schools. Potsdam and William and Mary provide room and board for students; Wooster provides housing for students; Auburn, Carnegie Mellon, Duluth, Fullerton, Hope, Potsdam, Rose-Hulman offer release time to faculty; Hope and Rose-Hulman provide funds for faculty salaries.

## Large Programs

Several programs have a large number of participants per year. These include: Cornell (Math/Biology) (25), Williams (17), NSA (17), Michigan (13), Berkeley (22-no longer exists), Carleton (18), Charleston (18), George Washington (15), The latter four are instructional programs rather than research programs.

## Group vs Individual Projects

Most programs offer a combination of group and individual projects for participants. Those who report the exclusive use of individual projects are: Tennessee, Indiana, Virginia, Northern Arizona, Dayton, Davidson, Duluth. Those who report using group projects only include: Mount Holyoke, Iowa State, Fullerton, Potsdam, Oregon, Cornell (Math/Biology), Charleston, Hope, Williams, Wooster, NSA, Trinity, Tulane, University Maryland Eastern Shore, and University of Washington.

[^28]
# Survey of Undergraduate Summer Programs in Mathematics 

Name of respondent: Johnson, Peter
Program held at what institution? Auburn Univ
Mark here if this program is no longer active: $\square$ If so, last year of operation: 1999

Answers incorporate 1999 data, if known.

1. Briefly describe the kind of summer program you direct:

## $\boxtimes$ REU

REU Program for certain groupOtherDescribe: $\qquad$ The research problems are in discrete mathematics. The program has run only once, with 7 participants, as noted below.
2. How many years has your program existed (including other directors)? 1
3. In all the years of program operation, what is the total number of students who have participated in the program? $\mathbf{7}$ ( $\mathbf{7}$ per year)
4. Of the students who have attended your program, what number were women? 3 ( 43\% )

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: $\bigcirc$ yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 1 ( $14 \%$ )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: $\bigcirc$ yes
6. Is your program:
$\qquad$ instructional in nature orresearch oriented, orboth
Comments: We tried a very unregimented, you're-free-to-do-as-you-please format, with required daily meetings, however, and the results were quite gratifying.
7. How many weeks does your program run?
$\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10$
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
$\boxtimes$ in groups $\boxtimes$ individually (check both if necessary)

Comments: The students were free to work in groups, or alone, as they pleased.

For questions $9-17$, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
are still undergraduates
1 are currently in a graduate program in the mathematical sciences
0 have received a Masters in the math. sci. (and are no longer in graduate school)
0 have received a Ph.D. in the mathematical sciences
0 are currently in a graduate program in the sciences (excluding the math. sci.)
0 have received a Masters in the sciences (excluding the mathematical sciences)
0 have received a Ph.D. in the sciences (excluding the mathematical sciences)
0 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
"still undergraduates" = 5 or 6
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?

| $\boxtimes 0-10 \%$ | $\square 21-30 \%$ | $\square 41-50 \%$ | $\square 61-70 \%$ | $\square 81-90 \%$ |
| :--- | :--- | :--- | :--- | :--- |
| $\square 11-20 \%$ | $\square 31-40 \%$ | $\square 51-60 \%$ | $\square 71-80 \%$ | $\square 91-100 \%$ |

This figure is an estimate $O$ yes
11. On average, how many oral presentations at conferences have resulted from your program per year?
$\qquad$ $x 1$ years $=1$
"1 or 2"
12. On average, how many poster presentations at conferences have resulted from your program per year?

```
0
x 1 years =0
```

13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).

0 $\qquad$ x 1 years : $\qquad$ Zero so far: two submitted, another one or two in the works.
14. What is the total annual budget for your program?
per student AND/OR $\$ \mathbf{6 0 , 7 1 1}$ total ( $\$ 8,673$ per student)
15. Typically, how much direct financial support does your institution provide?
\$0

| amount: | None -- they forego overhead expenses, charging about half what they normally |
| :--- | :--- |
| comments: $:$charge as an indication of support. Also, they give us "release time", one <br> course in spring quarter off, but I don't think this qualifies as "direct" financial <br> support. |  |

16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
Overhead reduction release time staff

+ our colleagues, visitors and graduate students in the department were very generous with their time, unremunerated.

17. Which agency is the principal source of your support (NSF, NSA, others)?

Comments:
18. Briefly describe the goals of your program. (Attach extra page if necessary).

The goal is to give the participants a pretty authentic research experience in mathematics, to give them an idea of what it would be like to do mathematical research as an academic. Mathematical research can be social, and it can be lonely, and it can be sometimes one, then the other. We sought to create an authentic research experience by requiring daily meetings, in the first two weeks of which we introduced the participants to possible problems and lines of inquiry, but otherwise pretty much leaving them to themselves -- with us at the ready to direct them to literature and to encourage their presentations. Maybe it was beginner's luck, but the one time this program ran, it appeared to achieve this goal squarely, with the consequence that two or possibly three of the participants seem to have decided that mathematical research is for them, at least of this stage of their lives; another two seem to have decided that mathematical
research is not for them, at this point; and the remaining two are leaving the matter open, but had a lot of fun and are coming back next summer.

# Survey of Undergraduate Summer Programs in Mathematics 

Name of respondent: Martelli, Mario
Program held at what institution? California State Univ, Fullerton
Mark here if this program is no longer active: $\square$ If so, last year of operation:
Answers incorporate 1999 data, if known.

1. Briefly describe the kind of summer program you direct:
$\square$ REU
$\square$ REU Program for certain group
$\boxtimes$ Other
Describe: $\qquad$ Funds for this program have been provided by the university.
2. How many years has your program existed (including other directors)? 8
3. In all the years of program operation, what is the total number of students who have participated in the program? 20 ( 3 per year)
4. Of the students who have attended your program, what number were women? 12 ( 60\%

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: $\bigcirc$ yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 2 ( 10\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: $\bigcirc$ yes
6. Is your program:
$\qquad$ instructional in nature or
$\qquad$ research oriented, orboth

Comments: The program has two stages: i) acquiring the background needed (instructional part); ii) doing research on the assigned project.
$\qquad$
$\qquad$
$\qquad$
7. How many weeks does your program run?
$\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10$
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:

## $\boxtimes$ in groups $\square$ individually <br> (check both if necessary)

## Comments: The groups are small: 2 or at most 3 students.

For questions $9-17$, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
are still undergraduates
3 are currently in a graduate program in the mathematical sciences
2 have received a Masters in the math. sci. (and are no longer in graduate school)
1 have received a Ph.D. in the mathematical sciences
1 are currently in a graduate program in the sciences (excluding the math. sci.)
1 have received a Masters in the sciences (excluding the mathematical sciences)
1 have received a Ph.D. in the sciences (excluding the mathematical sciences)
7 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?

| $\square 0-10 \%$ | $\square 21-30 \%$ | $\square 41-50 \%$ | $\square 61-70 \%$ | $\square 81-90 \%$ |
| :--- | :--- | :--- | :--- | :--- |
| $\boxtimes 11-20 \%$ | $\square 31-40 \%$ | $\square 51-60 \%$ | $\square 71-80 \%$ | $\square 91-100 \%$ |

This figure is an estimate $\bigcirc$ yes
11. On average, how many oral presentations at conferences have resulted from your program per year?

2

$$
\times 8 \text { years }=16
$$

12. On average, how many poster presentations at conferences have resulted from your program per year?

$$
2 \times 8 \text { years }=16
$$

13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
```
1
x 8 years :8
```

14. What is the total annual budget for your program?
$\ldots$ per student AND/OR \$3,500 total (\$1,750 per student)
15. Typically, how much direct financial support does your institution provide?
$\qquad$
amount:
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
```
clerical
release time
```

17. Which agency is the principal source of your support (NSF, NSA, others)?
18. Briefly describe the goals of your program. (Attach extra page if necessary).

The goal is to select one, two or three of our best undergraduate students when they are still in the sophomore year and start training them on a suitable topic during their junior year. The research part is done during the summer of their junior year, with presentation at the National and Local level during their senior year. In some case the research is completed after the students received their BA, but usually the work is done before their graduation.

## Survey of Undergraduate Summer Programs in Mathematics

Name of respondent: Haunsperger, Deanna B.
Program held at what institution? Carleton Coll
Mark here if this program is no longer active: $\square$ If so, last year of operation:

Answers incorporate 1999 data, if known.

1. Briefly describe the kind of summer program you direct:
REU
$\square$ REU Program for certain group
$\boxtimes$ Other
Describe:
Enrichment program for talented women finishing their first or second year of undergraduate work.
2. How many years has your program existed (including other directors)? 5
3. In all the years of program operation, what is the total number of students who have participated in the program? 88 ( 18 per year)
4. Of the students who have attended your program, what number were women? $88 \quad(100 \%)$

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 1 ( $1 \%$ )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: $\bigcirc$ yes
6. Is your program:
$\boxtimes$ instructional in nature orresearch oriented, or
$\square$ both

Comments: $\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
7. How many weeks does your program run?
$\square 1 \square 2 \square 3 \boxtimes 4 \square 5$6 7$8 \quad 9$ 9 10
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:in groups $\square$ individually
(check both if necessary)
Comments:

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
are still undergraduates
17 are currently in a graduate program in the mathematical sciences have received a Masters in the math. sci. (and are no longer in graduate school) have received a Ph.D. in the mathematical sciences

3 are currently in a graduate program in the sciences (excluding the math. sci.) have received a Masters in the sciences (excluding the mathematical sciences) have received a Ph.D. in the sciences (excluding the mathematical sciences)

16 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?

| $\boxtimes 0-10 \%$ | $\square 21-30 \%$ | $\square 41-50 \%$ | $\square 61-70 \%$ | $\square 81-90 \%$ |
| :--- | :--- | :--- | :--- | :--- |
| $\square 11-20 \%$ | $\square 31-40 \%$ | $\square 51-60 \%$ | $\square 71-80 \%$ | $\square 91-100 \%$ |

This figure is an estimate $O$ yes
11. On average, how many oral presentations at conferences have resulted from your program per year?
$\qquad$ $x 5$ years $=5$
12. On average, how many poster presentations at conferences have resulted from your program per year?
$1 \times 5$ years $=5$
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).

0 $x 5$ years : 0
14. What is the total annual budget for your program?
per student AND/OR \$115,650
total ( $\$ 6,803$ per student)
$\$ 115,650$, but we've run it for greatly varying amounts, usually much less than $\mathbf{\$ 1 1 5 , 6 5 0}$.
15. Typically, how much direct financial support does your institution provide?
$\$ 0$
amount:
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
clerical
17. Which agency is the principal source of your support (NSF, NSA, others)?

NSF principal; NSA secondary

Comments:
$\boxtimes$ NSA
18. Briefly describe the goals of your program. (Attach extra page if necessary).

The students immerse themselves in mathematics, living and working in a supportive community of women scholars (undergraduates, graduates, and post-graduates) who are passionate about learning and doing mathematics. The program's intent is threefold: to excite these young women about mathematics and mathematical careers, to provide them with the tools they will need to succeed in a mathematical career, and to connect them with a network of fellow female mathematicians.

## Survey of Undergraduate Summer Programs in Mathematics

Name of respondent: Williams, William O.
Program held at what institution? Carnegie Mellon Univ
Mark here if this program is no longer active: $\square$ If so, last year of operation:

Answers incorporate 1999 data, if known.

1. Briefly describe the kind of summer program you direct:
$\square$ REU
$\square$ REU Program for certain group
$\boxtimes$ Other Combined REU, special training
Describe: $\qquad$
2. How many years has your program existed (including other directors)? 7
3. In all the years of program operation, what is the total number of students who have participated in the program? 79 ( 11 per year)
4. Of the students who have attended your program, what number were women? 46 ( $58 \%$ )

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: $\bigcirc$ yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 34 ( 43\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: $\bigcirc$ yes
6. Is your program:
$\qquad$ instructional in nature orresearch oriented, or
$\boxtimes$ both

Comments: $\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
7. How many weeks does your program run?
$\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \boxtimes 7 \quad \square 8 \quad \square 9 \quad \square 10$
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
$\boxtimes$ in groups $\boxtimes$ individually
(check both if necessary)
Comments: Individual choice of method

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
are still undergraduates are currently in a graduate program in the mathematical sciences

2 have received a Masters in the math. sci. (and are no longer in graduate school)
$\qquad$ have received a Ph.D. in the mathematical sciences are currently in a graduate program in the sciences (excluding the math. sci.) have received a Masters in the sciences (excluding the mathematical sciences) have received a Ph.D. in the sciences (excluding the mathematical sciences)

8 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?

| $\square 0-10 \%$ | $\square 21-30 \%$ | $\square 41-50 \%$ | $\square 61-70 \%$ | $\square 81-90 \%$ |
| :--- | :--- | :--- | :--- | :--- |
| $\square 11-20 \%$ | $\square 31-40 \%$ | $\square 51-60 \%$ | $\square 71-80 \%$ | $\square 91-100 \%$ |

This figure is an estimate $\bigcirc$ yes
11. On average, how many oral presentations at conferences have resulted from your program per year?

## 2.5

```
x7 years = 18
```

12. On average, how many poster presentations at conferences have resulted from your program per year?
```
x 7 years =0 not known
```

13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
$\ldots \quad$ x 7 years $: 0 \quad$ only one known
14. What is the total annual budget for your program?
per student AND/OR $\$ 90,000$ total ( $\$ 8,182$ per student)
15. Typically, how much direct financial support does your institution provide?
\$3,700
amount:
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
clerical
release time
17. Which agency is the principal source of your support (NSF, NSA, others)?


Comments:
$\boxtimes$ NSA
18. Briefly describe the goals of your program. (Attach extra page if necessary).

## Goals of the Institute:

Undergraduate students often are unclear on exactly what graduate study and research will require of them and what it can offer them; unfortunately, many talented students decide against graduate school in part because of this uncertainty. Our program is designed to help students make rational decisions by giving them a taste of the graduate experience, without excessive cost of time in their careers. The course-work, while at a level appropriate to Juniors and Sophomores, is taught at graduate-level intensity, and the projects offer the chance to discover pleasures, and frustrations, of attacking open-ended research problems. The students are given the opportunity to interact with current graduate students.

We also aim to introduce students to areas of research in applied mathematics with which they may not be familiar, both through the project work and through a series of seminars by research fculty and graduate students.

Finally, the students in the Institute will leave CMU with tangibles: tools from the real analysis and Maple courses which will prove of service in their continuing undergraduate and their graduate studies and intangibles: the pleasure of working with and hanging out with students with similar interests from a very different geographic and cultural backgrounds.

## Survey of Undergraduate Summer Programs in Mathematics

Name of respondent: Lutzer, David J.
Program held at what institution? Coll ege of William \& Mary
Mark here if this program is no longer active: $\square$ If so, last year of operation:

Answers incorporate 1999 data, if known.

1. Briefly describe the kind of summer program you direct:
$\boxtimes$ REU
$\square$ REU Program for certain groupOther
Describe: $\qquad$
2. How many years has your program existed (including other directors)? 10
3. In all the years of program operation, what is the total number of students who have participated in the program? 80 ( 8 per year)
4. Of the students who have attended your program, what number were women? 22 (28\% )

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? $\qquad$ 2 ( ( 3\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here:
6. Is your program:
$\qquad$ instructional in nature or
$\boxtimes$ research oriented, or
$\square$ both

Comments: $\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
7. How many weeks does your program run?
$\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10$
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:

## $\boxtimes$ in groups $\boxtimes$ individually

(check both if necessary)
Comments: About $\mathbf{2 5 \%}$ work in groups of size 2; others work individually

For questions $9-17$, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
are still undergraduates
56 are currently in a graduate program in the mathematical sciences have received a Masters in the math. sci. (and are no longer in graduate school) have received a Ph.D. in the mathematical sciences are currently in a graduate program in the sciences (excluding the math. sci.) have received a Masters in the sciences (excluding the mathematical sciences) have received a Ph.D. in the sciences (excluding the mathematical sciences)

5 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
Figures for 97 \& 98 are: $18 \%$ still undergratures(3); $71 \%$ in math graduate program(12), 6\% in grad program outside math(1); $6 \%$ other(1). Prior to 97 , we estimate that at least $75 \%$ went to graduate school in mathematics.
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?

| $\square 0-10 \%$ | $\square 21-30 \%$ | $\square 41-50 \%$ | $\square 61-70 \%$ | $\square 81-90 \%$ |
| :--- | :--- | :--- | :--- | :--- |
| $\square 11-20 \%$ | $\square 31-40 \%$ | $\square 51-60 \%$ | $\square 71-80 \%$ | $\square 91-100 \%$ |

This figure is an estimate $\bigcirc$ yes
11. On average, how many oral presentations at conferences have resulted from your program per year?

1

$$
\times 10 \text { years }=10
$$

12. On average, how many poster presentations at conferences have resulted from your program per year?
$1 \times 10$ years $=10$
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
__ $\times 10$ years $: 0 \quad$ Approximately 4 per year.
14. What is the total annual budget for your program?
per student AND/OR \$44,000
total ( $\$ 5,500$ per student)
$\$ 44,000$ in 1998; less in earlier years.
15. Typically, how much direct financial support does your institution provide?
amount:
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
clerical
free housing for students
food for receptions for REU students and faculty computer use/computer support
office space
17. Which agency is the principal source of your support (NSF, NSA, others)?
18. Briefly describe the goals of your program. (Attach extra page if necessary).

The REU Program's objective is to provide talented undergraduates with the experience of how research mathematics is done, something quite different from the fashion in which undergraduate mathematics is usually taught. We want to involve students in research on real problems that can result in publication in order to interest students in mathematics graduate school (or, in rare cases, to give them a basis for deciding that research is not what they want to do).

# Survey of Undergraduate Summer Programs in Mathematics 

Name of respondent: Sarvate, Dinesh
Program held at what institution? College of Charleston
Mark here if this program is no longer active: $\square$ If so, last year of operation:
Answers incorporate 1999 data, if known.

1. Briefly describe the kind of summer program you direct:
$\square$ REUREU Program for certain groupOther
Describe: $\qquad$ Underrepresented minority students in SMET enroll in Precalculus summer course along with a required workshop. Students who receive C+ or above are given $\$ 500$ scholarship for the fall semester.
2. How many years has your program existed (including other directors)? 4
3. In all the years of program operation, what is the total number of students who have participated in the program? 73 ( 18 per year)
4. Of the students who have attended your program, what number were women?

50 68\% )

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 73 ( 100\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: $\bigcirc$ yes
6. Is your program:
$\boxtimes$ instructional in nature orresearch oriented, orboth
Comments:
Program is instructional in nature but includes a cultural enrichment component which assists in the transition from the high school to the college.
7. How many weeks does your program run?
$\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \boxtimes 5 \quad \square 6 \quad \square 7 \quad \square 8 \quad \square 9 \quad \square 10$
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
$\qquad$ in groups $\square$ individually
(check both if necessary)
Comments:

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
$\qquad$ are still undergraduates
$\qquad$ are currently in a graduate program in the mathematical sciences
$\qquad$ have received a Masters in the math. sci. (and are no longer in graduate school)
have received a Ph.D. in the mathematical sciences
$\qquad$ are currently in a graduate program in the sciences (excluding the math. sci.) have received a Masters in the sciences (excluding the mathematical sciences)
$\qquad$ have received a Ph.D. in the sciences (excluding the mathematical sciences)
$\qquad$ other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:

## Most are still undergraduates or graduating this year.

10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?

| $\boxtimes 0-10 \%$ | $\square 21-30 \%$ | $\square 41-50 \%$ | $\square 61-70 \%$ | $\square 81-90 \%$ |
| :--- | :--- | :--- | :--- | :--- |
| $\square 11-20 \%$ | $\square 31-40 \%$ | $\square 51-60 \%$ | $\square 71-80 \%$ | $\square 91-100 \%$ |

This figure is an estimate $\bigcirc$ yes
11. On average, how many oral presentations at conferences have resulted from your program per year?

6

$$
x 4 \text { years }=24
$$

12. On average, how many poster presentations at conferences have resulted from your program per year?
$1 \times 4$ years $=4$
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
$\ldots \quad$ _ 4 years $:=\frac{\text { Recently two students have reported that their work will be }}{\text { published very soon. }}$
14. What is the total annual budget for your program?
per student AND/OR \$50,000 total (\$2,778 per student)

Exact number not known, as it is a part of SPECTRA (The Speedy Consolidation \& Transition Program ) Program.
Room and board for 20 students, two course fees for every student; $\$ 500$ per student who makes a C+ or better.
15. Typically, how much direct financial support does your institution provide?

| amount: | Institution provides $100 \%$ of support. Pay for workshop professor; pay for the |
| :--- | :--- |
| comments:-course professors, pay for student workers, director of SPECTRA \& the office. |  |

16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
clerical
office space
computer use/computer support
Peer mentors
17. Which agency is the principal source of your support (NSF, NSA, others)?

College of Charleston provides all the support (NSF provides support
18. Briefly describe the goals of your program. (Attach extra page if necessary).

To increase the number of African Americans, Native Americans, Pacific Islanders, and other under-represented minorities entering technical science and math courses with the ultimate goal of increasing the number of minorities who receive a Ph.D. in SMET disciplines. Summer program is part of SCAMP (South Carolina Alliance for Minority Participation) program of the college.

## Survey of Undergraduate Summer Programs in Mathematics

Name of respondent: Ramsay, John R.
Program held at what institution? College of Wooster
Mark here if this program is no longer active: $\square$ If so, last year of operation:

Answers incorporate 1999 data, if known.

1. Briefly describe the kind of summer program you direct:
$\square$ REUREU Program for certain groupOther summer "consulting agency"
Describe: $\qquad$ Team of students with a fculty advisor work on projets abtained from local business and industry.
2. How many years has your program existed (including other directors)? 6
3. In all the years of program operation, what is the total number of students who have participated in the program? 55 ( 9 per year)
4. Of the students who have attended your program, what number were women?

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 1 ( 2\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: $\bigcirc$ yes
6. Is your program:
$\qquad$ instructional in nature orresearch oriented, orboth

| Comments: $\quad$ Primarily work in applied mathematics \& computer science. There |
| :---: |
| are a number of instructional components included. |

$\qquad$
$\qquad$
$\qquad$
7. How many weeks does your program run?
$\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10$
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
$\boxtimes$ in groups $\square$ individually
(check both if necessary)
Comments:

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:

19 are still undergraduates
3 are currently in a graduate program in the mathematical sciences
1 have received a Masters in the math. sci. (and are no longer in graduate school)
0 have received a Ph.D. in the mathematical sciences
2 are currently in a graduate program in the sciences (excluding the math. sci.)
1 have received a Masters in the sciences (excluding the mathematical sciences)
0 have received a Ph.D. in the sciences (excluding the mathematical sciences)
1 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?

| $\boxtimes 0-10 \%$ | $\square 21-30 \%$ | $\square 41-50 \%$ | $\square 61-70 \%$ | $\square 81-90 \%$ |
| :--- | :--- | :--- | :--- | :--- |
| $\square 11-20 \%$ | $\square 31-40 \%$ | $\square 51-60 \%$ | $\square 71-80 \%$ | $\square 91-100 \%$ |

This figure is an estimate yes
11. On average, how many oral presentations at conferences have resulted from your program per year?

1

$$
\times 6 \text { years }=6
$$

12. On average, how many poster presentations at conferences have resulted from your program per year?
$1 \times 6$ years $=6$
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).

0 $x 6$ years : 0
14. What is the total annual budget for your program?
$\ldots$ per student AND/OR \$50,000 total (\$5,556 per student)
15. Typically, how much direct financial support does your institution provide?
\$15,000
amount:
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.

```
free housing for students
clerical
school facilities
computer use/computer support
```

17. Which agency is the principal source of your support (NSF, NSA, others)?

Primary source is fees from clients
18. Briefly describe the goals of your program. (Attach extra page if necessary).

The primary goal of the program is to provide practical experience for our students. Our curriculum focuses on preparation for graduate programs but an increasing number of our students are choosing employment upon graduation. This program attempts to provide our best students with an experience similar to the kind of employment options they will have upon graduating with a bachelors degree in mathematics or computer science.

# Survey of Undergraduate Summer Programs in Mathematics 

Name of respondent: Strichartz, Robert S.
Program held at what institution? Cornell Univ
Mark here if this program is no longer active: $\square$ If so, last year of operation:
Answers incorporate 1999 data, if known.

1. Briefly describe the kind of summer program you direct:
$\boxtimes$ REU
$\square$ REU Program for certain group
$\square$ Other
Describe:
Each summer, 10 students are involved in 3 research projects directed by Cornell faculty or visitors.
2. How many years has your program existed (including other directors)? 5
3. In all the years of program operation, what is the total number of students who have participated in the program? 50 ( 10 per year)
4. Of the students who have attended your program, what number were women? 14 ( 28\% )

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 1 ( $2 \%$ )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: $\bigcirc$ yes
6. Is your program:
$\qquad$ instructional in nature orresearch oriented, orboth

Comments: | Students usually need to learn new materials, but this is done |
| :--- |
| $\quad$ informally. |

7. How many weeks does your program run?
$\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10$
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
$\boxtimes$ in groups $\boxtimes$ individually (check both if necessary)

Comments: We try to accomodate student preferences.

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
are still undergraduates
17 are currently in a graduate program in the mathematical sciences have received a Masters in the math. sci. (and are no longer in graduate school) have received a Ph.D. in the mathematical sciences

1 are currently in a graduate program in the sciences (excluding the math. sci.) have received a Masters in the sciences (excluding the mathematical sciences) have received a Ph.D. in the sciences (excluding the mathematical sciences)

3 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?

| $\boxtimes 0-10 \%$ | $\square 21-30 \%$ | $\square 41-50 \%$ | $\square 61-70 \%$ | $\square 81-90 \%$ |
| :--- | :--- | :--- | :--- | :--- |
| $\square 11-20 \%$ | $\square 31-40 \%$ | $\square 51-60 \%$ | $\square 71-80 \%$ | $\square 91-100 \%$ |

This figure is an estimate $O$ yes
11. On average, how many oral presentations at conferences have resulted from your program per year?

5

$$
\times 5 \quad \text { years }=25
$$

12. On average, how many poster presentations at conferences have resulted from your program per year?
$\qquad$
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
2.5 x 5 years : 13
14. What is the total annual budget for your program?
$\qquad$
$\qquad$
$\qquad$
15. Typically, how much direct financial support does your institution provide?
```
amount:}\frac{$7,500}{varies greatly from year to year.
comments:
```

16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.

## clerical

17. Which agency is the principal source of your support (NSF, NSA, others)?
$\boxtimes$ NSF
Comments:
NSA
18. Briefly describe the goals of your program. (Attach extra page if necessary).

To engage undergraduates in meaningful research activities. Many of our projects involve a significant component of computational or experimental work, but we also try to involve the students in more traditional theorem proving activities to the extent that they are willing and able. Students are required to give lectures on their work at the end of the program.

# Survey of Undergraduate Summer Programs in Mathematics 

Name of respondent: Castillo-Chavez, Carlos
Program held at what institution? Cornell Univ
Mark here if this program is no longer active: $\square$ If so, last year of operation:

Answers incorporate 1999 data, if known.

1. Briefly describe the kind of summer program you direct:
$\square$ REU
$\boxtimes$ REU Program for certain group Latinos, Native Americans and Chicanos

- Other

We have gradually increased the diversity of the program since its inception.
Describe: $\begin{aligned} & \text { "Mathematical and Theoretical Biology Research Program for } \\ & \text { Undergraduates". Program stresses coursework in dynamic \& } \\ & \text { Utochastic process for first half and independent research in the second } \\ & \text { half. }\end{aligned}$
2. How many years has your program existed (including other directors)? 4
3. In all the years of program operation, what is the total number of students who have participated in the program? 101 ( 25 per year)
4. Of the students who have attended your program, what number were women?

48
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 98 ( $97 \%$ )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: $\bigcirc$ yes
6. Is your program:
$\qquad$ instructional in nature orresearch oriented, or
$\boxtimes$ both

Comments:
7. How many weeks does your program run?
$\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10$
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
$\boxtimes$ in groups $\square$ individually
(check both if necessary)
Comments:

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
are still undergraduates
13 are currently in a graduate program in the mathematical sciences
1 have received a Masters in the math. sci. (and are no longer in graduate school)
0 have received a Ph.D. in the mathematical sciences

18 are currently in a graduate program in the sciences (excluding the math. sci.) have received a Masters in the sciences (excluding the mathematical sciences) have received a Ph.D. in the sciences (excluding the mathematical sciences)

28 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
estimates; "other" also includes unknown
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?

| $\square 0-10 \%$ | $\square 21-30 \%$ | $\square 41-50 \%$ | $\square 61-70 \%$ | $\square 81-90 \%$ |
| :--- | :--- | :--- | :--- | :--- |
| $\boxtimes 11-20 \%$ | $\square 31-40 \%$ | $\square 51-60 \%$ | $\square 71-80 \%$ | $\square 91-100 \%$ |

This figure is an estimate $O$ yes
11. On average, how many oral presentations at conferences have resulted from your program per year?

6

$$
x 4 \text { years }=24
$$

12. On average, how many poster presentations at conferences have resulted from your program per year?
$\qquad$
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
years
14. What is the total annual budget for your program?
_ per student AND/OR \$250,000 total (\$10,000 per student)
15. Typically, how much direct financial support does your institution provide?
\$70,000
amount:
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
computer use/computer support
graduate tutoring
school facilities
17. Which agency is the principal source of your support (NSF, NSA, others)?

NSA 40\%
$\boxtimes$ NSF
Comments: NSF 35\%
$\boxtimes$ NSA
18. Briefly describe the goals of your program. (Attach extra page if necessary).

## Mathematical and Theoretical Biology Research Program for Undergraduates

A multi-summer research experience for undergraduates with strong quantitative training and interest in the application of mathematics to the natural or social sciences is offered. First year students are introduced to dynamical systems theory, stochastic processes, modeling of biological populations, and methods of simulation during the first three weeks of the program. The last four weeks of the program are devoted to the development of mathematical models that address student-generated questions in the natural or social sciences. Students, in groups of 2-5, carry out the mathematical analysis and simulation of their models and use their results to write a technical report (20-30 pages) that responds to their original question. Four to eight students are selected as second
year students. They serve as mentors while participating in a simultaneous 8-week summer program that builds on their first year research experience. Second year students also produce a significant technical group report (2-3 students) that addresses a scientific question. The program recruits extensively from institutions that have access to limited research opportunities and considers ethnicity and gender as part of the admission process. Students present their research at the end of the summer, at scientific conferences, and at the SACNAS national meeting. The success of the program is measured by the quality of the research and by the number of non-traditional students who enroll in graduate school after their participation in our program. The program's staff monitors students' progress throughout the academic year.

# Survey of Undergraduate Summer Programs in Mathematics 

Name of respondent: Swallow, John
Program held at what institution? Davidson Coll
Mark here if this program is no longer active: $\boxtimes$ If so, last year of operation: 1998
Answers incorporate 1999 data, if known.

1. Briefly describe the kind of summer program you direct:
$\boxtimes$ REU
$\square$ REU Program for certain group
$\square$ Other
Describe: $\qquad$ (funding from REU supplement to exisitng NSF grant)
2. How many years has your program existed (including other directors)? 1
3. In all the years of program operation, what is the total number of students who have participated in the program? 2 ( 2 per year)
4. Of the students who have attended your program, what number were women? 1 ( $50 \%$ )

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: $\bigcirc$ yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 0 ( 0\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: $\bigcirc$ yes
6. Is your program:
$\qquad$ instructional in nature orresearch oriented, orboth
Comments: $\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
7. How many weeks does your program run?
$\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \square 8 \quad \square 9 \quad \boxtimes 10$
8. If your program has an instructional component, are students asked to work :

(check both if necessary)
If your program has a research component, do students work on their research projects:
$\square$ in groups $\boxtimes$ individually
(check both if necessary)
Comments:

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
$\qquad$ are still undergraduates
1 are currently in a graduate program in the mathematical sciences have received a Masters in the math. sci. (and are no longer in graduate school)
$\qquad$ have received a Ph.D. in the mathematical sciences
$\qquad$ are currently in a graduate program in the sciences (excluding the math. sci.)
$\qquad$ have received a Masters in the sciences (excluding the mathematical sciences)
$\qquad$ have received a Ph.D. in the sciences (excluding the mathematical sciences)

1 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?

| $\boxtimes 0-10 \%$ | $\square 21-30 \%$ | $\square 41-50 \%$ | $\square 61-70 \%$ | $\square 81-90 \%$ |
| :--- | :--- | :--- | :--- | :--- |
| $\square 11-20 \%$ | $\square 31-40 \%$ | $\square 51-60 \%$ | $\square 71-80 \%$ | $\square 91-100 \%$ |

This figure is an estimate $\bigcirc$ yes
11. On average, how many oral presentations at conferences have resulted from your program per year?
$\qquad$ $x 1$ years $=1$
12. On average, how many poster presentations at conferences have resulted from your program per year?
$\qquad$
$\times 1$ years $=0$
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).

```
1
x 1 years:1
expected
```

14. What is the total annual budget for your program?
$\qquad$
$\qquad$
$\qquad$
15. Typically, how much direct financial support does your institution provide?
```
            $0
amount:
``` \(\qquad\)
comments: \(\qquad\)
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
```

clerica
computer use/computer support
telephone/photocopying/postage

```
17. Which agency is the principal source of your support (NSF, NSA, others)?
NSF
NSA
Comments: \(\qquad\)
18. Briefly describe the goals of your program. (Attach extra page if necessary).
To provide an attractive, social, and authentic research experience in mathematics.

\title{
Survey of Undergraduate Summer Programs in Mathematics
}

Name of respondent: Gupta, Murli
Program held at what institution? George Washington Univ
Mark here if this program is no longer active: \(\square\) If so, last year of operation:
Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\square\) REUREU Program for certain group
\(\qquad\) Other It is designed to give the students a taste of graduate school
Describe:
\(\qquad\) GW Summer Program for Women in Mathematics. We expose the students to classroom work which includes group projects and presentations. We have guest lecturers, field trips and panel discussions.
2. How many years has your program existed (including other directors)? 5
3. In all the years of program operation, what is the total number of students who have participated in the program? 73 ( 15 per year)
4. Of the students who have attended your program, what number were women? 73 ( \(\mathbf{1 0 0 \%}\) )

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 9 ( 12\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: O yes
6. Is your program:
\(\boxtimes\) instructional in nature orresearch oriented, orboth
Comments:
We aim to communicate an enthusiasm for mathematics, to develop research skills, and to cultivate mathematical self-confidence and interest.
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \boxtimes 5 \quad \square 6 \quad \square 7 \quad \square 8 \quad \square 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\boxtimes\) in groups \(\boxtimes\) individually
(check both if necessary)
Comments:

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:

17 are still undergraduates are currently in a graduate program in the mathematical sciences

3 have received a Masters in the math. sci. (and are no longer in graduate school)
0 have received a Ph.D. in the mathematical sciences
1 are currently in a graduate program in the sciences (excluding the math. sci.)
have received a Masters in the sciences (excluding the mathematical sciences)
0 have received a Ph.D. in the sciences (excluding the mathematical sciences)
other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\boxtimes 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(\bigcirc\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?

5
\[
\times 5 \text { years }=25
\]
12. On average, how many poster presentations at conferences have resulted from your program per year?
\(5 \times 5\) years \(=25\)
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).

0 \(x 5\) years : 0
14. What is the total annual budget for your program?
\(\qquad\)
\(\qquad\)
\(\qquad\)
15. Typically, how much direct financial support does your institution provide?
\(\qquad\)
amount:
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
indirect cost sharing clerical and administrative support
17. Which agency is the principal source of your support (NSF, NSA, others)?
NSF Comments:
NSA
18. Briefly describe the goals of your program. (Attach extra page if necessary).

\section*{Goals and Objectives}
The GW Summer Program for Women in Mathematics aims to:
. provide an immersion program representative of key aspects of graduate school and professional mathematical practice, . promote active mathematical thinking,
. underscore the beauty and enjoyment of mathematics,
. foster a camaraderie among the participants that emphasizes collaboration and peer support,
. bring the participants into contact with active mathematical researchers through a program of guest lectures and field trips, . provide interaction with a wide variety of successful women in mathematical
sciences who serve as role models,
. illustrate the role of mathematics as the foundation of the sciences and the wide range of mathematical applications in government, business, and industry through first-hand contact with applied mathematicians, and
. provide students with information about graduate schools and careers in mathematics.

\title{
Survey of Undergraduate Summer Programs in Mathematics
}

Name of respondent: Pennings, Timothy J.
Program held at what institution? Hope Coll
Mark here if this program is no longer active: \(\qquad\) If so, last year of operation:

Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\boxtimes\) REU
\(\square\) REU Program for certain group
\(\square\) Other
Describe: \(\qquad\)
2. How many years has your program existed (including other directors)? 8
3. In all the years of program operation, what is the total number of students who have participated in the program? 46 ( 6 per year)
4. Of the students who have attended your program, what number were women? 16 ( 35\% )

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 0 ( 0\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
6. Is your program:
\(\qquad\) instructional in nature orresearch oriented, orboth

\section*{Comments: Instruction is used during the first 1-2 weeks sometimes to prepare for research.}
\(\qquad\)
\(\qquad\)
\(\qquad\)
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:

\section*{\(\boxtimes\) in groups \(\square\) individually}
(check both if necessary)

\section*{Comments: Assignments are given primarily to groups. Students may decide among themselves to work individually.}

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
are still undergraduates
12 are currently in a graduate program in the mathematical sciences
4 have received a Masters in the math. sci. (and are no longer in graduate school) have received a Ph.D. in the mathematical sciences

2 are currently in a graduate program in the sciences (excluding the math. sci.)
2 have received a Masters in the sciences (excluding the mathematical sciences) have received a Ph.D. in the sciences (excluding the mathematical sciences)

8 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\boxtimes 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(\bigcirc\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?

4
\[
x 8 \text { years }=32
\]
12. On average, how many poster presentations at conferences have resulted from your program per year?
\(1 \times 8\) years \(=8\)
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).

1 \(x 8\) years : 8
14. What is the total annual budget for your program?
\(\ldots\) per student AND/OR \$30,000 total (\$6,000 per student)
15. Typically, how much direct financial support does your institution provide?
amount: \(\frac{\$ 1,800}{\text { housing, mailing/advertising, travel }}\)
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
```

clerical
release time
faculty salaries
summer support for director

```
17. Which agency is the principal source of your support (NSF, NSA, others)?
18. Briefly describe the goals of your program. (Attach extra page if necessary).

Our goal is to provide an opportunity for dedicated mathematical research to qualified students who do not have such opportunity otherwise. The objective is to give them a taste of graduate school research so they can make a better informed choice about their post-college opportunities.

Qualified students are those with requisite background, writing ability, and maturity to work hard for eight weeks on a particular problem with the anticipation of publishing or presenting their results. Writing up their results formally is an essential part of their work.

\title{
Survey of Undergraduate Summer Programs in Mathematics
}

Name of respondent: Maki, Daniel
Program held at what institution? Indiana Univ, Bloomington
Mark here if this program is no longer active: \(\square\) If so, last year of operation:
Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\square\) REU
\(\boxtimes\) REU Program for certain group
MathematicsOther

Describe: \(\qquad\) Our program is based on having one faculty member working with one student on a research problem. To begin, there is a short period of instruction and special lectures occur on other mathematical topics.
2. How many years has your program existed (including other directors)? 7
(+ 33 years previously as URDP)
3. In all the years of program operation, what is the total number of students who have participated in the program? 85 ( 12 per year)
4. Of the students who have attended your program, what number were women? 33 39\% )

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 9 ( \(11 \%\) )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
6. Is your program:
\(\qquad\) instructional in nature orresearch oriented, orboth

Comments: Our program is based on having one faculty member working with one student on a research problem. To begin, there is a short period of instruction and special lectures on other mathematical topics.
[Note about years: 1966-1978 URPP; 1978-1990 Supplemental REU; 1991-1998 REU. Only students since 91 are reported here. In all years there were about 200 students; 75 were women.]
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\qquad\) in groups \(\boxtimes\) individually
(check both if necessary)
Comments:

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:

14 are still undergraduates
32 are currently in a graduate program in the mathematical sciences
8 have received a Masters in the math. sci. (and are no longer in graduate school)
6 have received a Ph.D. in the mathematical sciences
4 are currently in a graduate program in the sciences (excluding the math. sci.)
1 have received a Masters in the sciences (excluding the mathematical sciences)
0 have received a Ph.D. in the sciences (excluding the mathematical sciences)
4 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\square 0-10 \%\) & \(\boxtimes 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(\bigcirc\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?

4 \(\qquad\) We do not collect these figures; we guess about 4 a year
12. On average, how many poster presentations at conferences have resulted from your program per year?
```

3 < 7 years =21 We do not collect these figures; we guess about 3 a year

```
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
```

1
x }\mp@subsup{}{}{7
years:7 We do not keep these figures; we guess about 1 a year

```
14. What is the total annual budget for your program?
per student AND/OR \(\$ 55,000\) total (\$4,583 per student)
15. Typically, how much direct financial support does your institution provide?
\$5,000
amount: Average
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
clerical
faculty time to read and evaluate 200 files
17. Which agency is the principal source of your support (NSF, NSA, others)?

Comments:
18. Briefly describe the goals of your program. (Attach extra page if necessary).

The Indiana University Mathematics REU is designed to provide ten undergraduate mathematics majors, recruited nationwide, with a research topic and a research environment which is as close to that of a working mathematician as possible, given the student's background. Each student will work with a single faculty member. The students will meet privately with their faculty advisors two to three times a week. Throughout the eight weeks the students will give frequent progress reports to the group. One of the most important aspects of the program is the interaction among the students themselves. For some, this will be their first opportunity to get to know others students of comparable mathematical interest and ability. We will encourage this interaction in a number of ways: The students will be living together in a single dormitory and eating in the same cafeteria. There will be a room in Rawles Hall (the Mathematics Department) in which
students can gather to work. During the last week the students will give lectures on the work they have done. They will also prepare written reports on their work. These will be collected into a single bound.

\title{
Survey of Undergraduate Summer Programs in Mathematics
}

Name of respondent: Peterson, Janet
Program held at what institution? Iowa State Univ
Mark here if this program is no longer active: \(\square\) If so, last year of operation:

Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\boxtimes\) REU
\(\square\) REU Program for certain group
\(\square\) Other
Describe: \(\qquad\)
2. How many years has your program existed (including other directors)?

\section*{2 (including this}
3. In all the years of program operation, what is the total number of students who have participated in the program? 17 ( 9 per year)
4. Of the students who have attended your program, what number were women? 4 ( \(24 \%\)

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 1 ( 6\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
6. Is your program:
\(\square\) instructional in nature orresearch oriented, or
\(\boxtimes\) both
Comments: \(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\boxtimes\) in groups \(\square\) individually
(check both if necessary)
Comments:

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:

13 are still undergraduates
3 are currently in a graduate program in the mathematical sciences
0 have received a Masters in the math. sci. (and are no longer in graduate school)
0 have received a Ph.D. in the mathematical sciences
\(\mathbf{0}\) are currently in a graduate program in the sciences (excluding the math. sci.)
0 have received a Masters in the sciences (excluding the mathematical sciences)
0 have received a Ph.D. in the sciences (excluding the mathematical sciences)
1 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
The first 2 catagories are beginning fall ' 99 '.
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\boxtimes 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(\bigcirc\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?
\(\qquad\) \(\times 2\) years \(=2\)
12. On average, how many poster presentations at conferences have resulted from your program per year?
\(\qquad\)
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).

0 \(x^{2}\) years : 0 \(: 0\)
14. What is the total annual budget for your program?
per student AND/OR \(\$ \mathbf{5 0 , 0 0 0}\) total ( \(\$ 6,250\) per student)
15. Typically, how much direct financial support does your institution provide?
\$8,000
amount:
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
clerical
office space
computer use/computer support
17. Which agency is the principal source of your support (NSF, NSA, others)?

Comments:
NSA
18. Briefly describe the goals of your program. (Attach extra page if necessary).

Scientific computing has become a powerful and essential tool in conducting research in science and engineering. For example, in the design of commercial aircraft many engineering issues are resolved through computer simulation rather than through costly and time consuming wind tunnel experiments. Similarly, the use of computational chemistry to design potential new drugs has saved the pharmaceutical industry millions of dollars. Many research universities have recognized the importance of scientific computing and have created graduate programs in this area.

A computational scientist has been defined by the DOE Graduate Fellowship Program in Computational Science to be a scientist who applies high performance computational technology in innovative and essential ways to
advance the state of knowledge in his or her scientific discipline. The National Science Foundation Workshop on the Role of Supercomputing in Education has recognized computational science as an emerging third scientific methodology, complementing theory and experimentation as techniques for studying applications in the sciences. Computational science can be considered as a catalyst across specific scientific disciplines. The future capability of the United States in technological fields will depend on our ability to train today's college students in this emerging field. Because of the rapidity with which computational technology is developing, it is difficult for many of our colleges and universities to accomodate this need for training in computational sciences, especially at the undergraduate level.

We propose to establish a program to first awaken and then reinforce the interest of capable students in research in scientific computing. We especially want to recruit undergraduate students who are from underrepresented groups such as women and African Americans: we have outlined in our proposal how we hope to accomplish this through interactions with existing programs at lowa State University such as the Program for Women in Science and Engineering and Feeder Programs with schools such as Florida A \& M. Through various follow-up activities, our program is intended to continue exposing the students to the exciting world of research, to aid these students as they prepare for graduate school, and even to help guide them through graduate school.

In summary, the general goal of this project is to expose a small group of talented students to various issues in scientific computing by creating an environment which will foster instructional and research activities. A short range goal of this program is to increase the number of students attending graduate school in mathematics while a longer range goal is to attract talented students to careers in mathematics in general and computational science in particular.

\section*{Survey of Undergraduate Summer Programs in Mathematics}

Name of respondent: Madden, James/Stoltzfus, Neal W.
Program held at what institution? Louisiana State Univ, Baton Rouge
Mark here if this program is no longer active: \(\square\) If so, last year of operation:

Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\boxtimes\) REU
\(\square\) REU Program for certain group
\(\square\) Other
Describe:__Students design and carry out a research project in collaboration with a faculty member.
2. How many years has your program existed (including other directors)? 7
3. In all the years of program operation, what is the total number of students who have participated in the program? 58 ( 8 per year)
4. Of the students who have attended your program, what number were women? 19 ( 33\% )

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 3 ( 5\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
6. Is your program:
\(\square\) instructional in nature orresearch oriented, orboth
Comments: \(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\boxtimes\) in groups \(\boxtimes\) individually (check both if necessary)

Comments:

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
are still undergraduates
15 are currently in a graduate program in the mathematical sciences have received a Masters in the math. sci. (and are no longer in graduate school) have received a Ph.D. in the mathematical sciences

4 are currently in a graduate program in the sciences (excluding the math. sci.) have received a Masters in the sciences (excluding the mathematical sciences) have received a Ph.D. in the sciences (excluding the mathematical sciences)
other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
Figures are from Fall 97
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\boxtimes 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(\bigcirc\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?
\(\qquad\) \(\times 7\) years \(=7\)
12. On average, how many poster presentations at conferences have resulted from your program per year?
\(\qquad\)
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
\(\qquad\)
14. What is the total annual budget for your program?
\(\$ 5,000\) per student AND/OR __ total ( \(\$ 5,000\) per student)
15. Typically, how much direct financial support does your institution provide?
\$0
amount:
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
office space
17. Which agency is the principal source of your support (NSF, NSA, others)?

Louisiana Educational Quality Support Fund
\(\boxtimes\) NSF
Comments:
NSA
18. Briefly describe the goals of your program. (Attach extra page if necessary).

The Mathematics Research Experience for Undergraduates program at Louisiana State University gives able college sophomores and juniors the opportunity to engage in research on topics of recognized importance in a professional environment that supplies all the support they require to be successful. Participants develop the ability to set meaningful research goals, work independently and communicate research-level mathematics orally and in writing.

The LSU REU is shaped by an underlying philosophy and by the accumulated experience of 5 summers. We aim not only to give undergraduates a first-rate experience, but also to ensure that the university and the profession absorbs the benefits that accrue from their involvement. Our experience has proved that
good things flow both ways - from the research environment to the students and from the students to the research environment. We treat the participants as true collaborators, who occupy a unique role within the research community. Their needs are special, as are the contributions they make.

Research in mathematics is a social activity, the goal of which is the creation of ideas that are to be shared and upon which others can build. While there is a phase that is individual, all real progress comes about through communication and cooperation. For this reason, we have always devoted special efforts to fostering a strong intellectual community that includes both the student participants and the faculty directors. Mathematical communication, in all its forms and settings, is central to the REU, and we take advantage of every opportunity to help students to improve communication skills. On a large scale, it is communication between research communities and the great currents and traditions of mathematics that raises mathematical research above the simple drive to satisfy curiosity. For this reason, we strive to give the participants a sense of how their work fits into the broader intellectual frameworks, how their work contributes to a senior researcher's overall research program, how this program fits within the discipline and the place of the discipline in mathematics as a whole.

Participants are recruited nationally, and are drawn from all the major regions of the nation. We seek to include groups that are under-represented in the mathematical sciences and regularly achieve a good gender balance. A small number of highly qualified students from racial mionorities have participated, and we are committeed to increasing this number. Over the history of the program, about \(70 \%\) of the funding has come from the State of Louisiana. Because of this, we have in the past made special efforts to attract local talent.

In addition to benefitting the participants directly, the LSU REU is poised to have a major impact of graduate and undergraduate training at LSU by establishing new models for educating mathematicians.

\section*{Survey of Undergraduate Summer Programs in Mathematics}

Name of respondent: Godbole, Anant P.
Program held at what institution? Michigan Technological Univ
Mark here if this program is no longer active: \(\square\) If so, last year of operation:

Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\boxtimes\) REU
\(\square\) REU Program for certain groupOther
Describe: \(\qquad\)
2. How many years has your program existed (including other directors)? 9
3. In all the years of program operation, what is the total number of students who have participated in the program? 66 ( 7 per year)
4. Of the students who have attended your program, what number were women? 32

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 3 ( \(5 \%\) )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
6. Is your program:
\(\square\) instructional in nature or
\(\boxtimes\) research oriented, or

Comments: \(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \square 8 \quad \boxtimes 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\boxtimes\) in groups \(\boxtimes\) individually (check both if necessary)

Comments:

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
are still undergraduates
22 are currently in a graduate program in the mathematical sciences

8 have received a Masters in the math. sci. (and are no longer in graduate school)
3 have received a Ph.D. in the mathematical sciences are currently in a graduate program in the sciences (excluding the math. sci.) have received a Masters in the sciences (excluding the mathematical sciences) have received a Ph.D. in the sciences (excluding the mathematical sciences)

11 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\square 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\boxtimes 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(O\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?

7
\[
\times 9 \text { years }=63
\]
12. On average, how many poster presentations at conferences have resulted from your program per year?

13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).

3 \(x 9\) years : 2 27
14. What is the total annual budget for your program?
\$5,000
per student AND/OR \(\qquad\) total (\$5,000 per student)
\(\qquad\)
\(\qquad\)
15. Typically, how much direct financial support does your institution provide?
\(\qquad\)
amount:
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.

\section*{clerical}
17. Which agency is the principal source of your support (NSF, NSA, others)?
NSF
NSA
18. Briefly describe the goals of your program. (Attach extra page if necessary).
To involve students in cutting edge research in discrete probabliity; to expose them to contemproary techniques such as isoperimetric inequalities and Stein's method; to have them present their work orally, to have them write papers for possible publication.

\title{
Survey of Undergraduate Summer Programs in Mathematics
}

Name of respondent: Blows, Terence R.
Program held at what institution? Northern Arizona Univ
Mark here if this program is no longer active: \(\square\) If so, last year of operation:

Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\square\) REU
\(\boxtimes\) REU Program for certain group
\(\square\) Other
Describe: \(\qquad\) 6-8 students work one-on-one with advisors. Areas: ODE, PDE, \(\qquad\) Combinatorics, Algebra, Topology, Statistics, Stochastic processes
2. How many years has your program existed (including other directors)? 6
3. In all the years of program operation, what is the total number of students who have participated in the program? 42 ( 7 per year)
4. Of the students who have attended your program, what number were women? 17 ( 40\%

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 2 ( 5\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
6. Is your program:
\(\qquad\) instructional in nature orresearch oriented, orboth
Comments: "Research" is to be interpreted broadly. Depends on the strength/background of the student, and the accessibility of the advisor's area. We target recruitment at students from schools without graduate programs - often it is hard to judge how good they will be until they are here on campus.
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\square\) in groups \(\boxtimes\) individually
(check both if necessary)
Comments: Often students have projects "close" to others so some interaction is possible. At other times they work in total isolation from other students.

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
are still undergraduates
16 are currently in a graduate program in the mathematical sciences have received a Masters in the math. sci. (and are no longer in graduate school) have received a Ph.D. in the mathematical sciences are currently in a graduate program in the sciences (excluding the math. sci.) have received a Masters in the sciences (excluding the mathematical sciences) have received a Ph.D. in the sciences (excluding the mathematical sciences)

14 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:

\section*{I have not tracked success in graduate school, only placement in grad programs.}
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\square 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(\bigcirc\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?
2.5 \(\times 6\) years \(=15\) mainly regional MAA meetings
12. On average, how many poster presentations at conferences have resulted from your program per year?

\section*{1 \\ \(x 6\) years \(=6\) \\ 0-2 at National MAA/AMS meetings. Two students won prizes for their presentations}
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
\(\qquad\) \(x 6\) years : 6 \(\qquad\) 1 would be a reasonable number. There are recent submissions as well as others that have been (or are being)
14. What is the total annual budget for your program?
\(\$ 5\) per student AND/OR \(\$ 33,000\) total ( \(\$ 5,000\) per student)
\(\qquad\)
15. Typically, how much direct financial support does your institution provide?
amount:
comments: \(\frac{\$ 3,000}{\frac{\text { For the first few years of the program, nothing. But for past } 2 \text { years } \$ 3000}{\text { allowing faculty support at } \$ 1000 \text { per student. Even this is insufficient for }}} ⿻\)\begin{tabular}{l} 
facul effort.
\end{tabular}
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
clerical tuition waivers
17. Which agency is the principal source of your support (NSF, NSA, others)?
\(\boxtimes\) NSF
NSA
18. Briefly describe the goals of your program. (Attach extra page if necessary).
To give research opportunities to quality undergraduates. We target smaller schools and schools with high minority enrollemnts. Publication is less of a goal than giving opportunities to such students.

\title{
Survey of Undergraduate Summer Programs in Mathematics
}

Name of respondent: Bluher, Antonia
Program held at what institution? NSA
Mark here if this program is no longer active:
If so, last year of operation:
Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\square\) REU
\(\boxtimes\) REU Program for certain group U.S. citizens onlyOther

Describe: \(\qquad\)
2. How many years has your program existed (including other directors)? 10
3. In all the years of program operation, what is the total number of students who have participated in the program? 170 ( 17 per year)
4. Of the students who have attended your program, what number were women? 38 ( \(22 \%\)

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 5 ( 3\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: O yes
6. Is your program:instructional in nature orresearch oriented, orboth

Comments: Students work on current problems, most of which are of mission-critical importance to NSA. Hence the students are fully cleared for classified work. [This is why we are restricted to U.S. citizen participants.]
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \square 8 \quad \square 9 \quad \boxtimes 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\boxtimes\) in groups \(\square\) individually (check both if necessary)
Comments:

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
are still undergraduates
80 are currently in a graduate program in the mathematical sciences
12 have received a Masters in the math. sci. (and are no longer in graduate school)
12 have received a Ph.D. in the mathematical sciences
6 are currently in a graduate program in the sciences (excluding the math. sci.)
0 have received a Masters in the sciences (excluding the mathematical sciences)
0 have received a Ph.D. in the sciences (excluding the mathematical sciences)
10 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\square 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\boxed{31-40 \%}\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(\bigcirc\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?
\(\qquad\)
\(\qquad\) There are numerous [at least 10 each of the past two years] internal presentations, however.
12. On average, how many poster presentations at conferences have resulted from your program per year?
\(\underline{0} \times 10\) years \(=0 \quad \frac{\text { Internally, everyone participates in at least one [usually two] }}{\text { poster sessions. }}\)
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).

14. What is the total annual budget for your program?
per student AND/OR \$20,000 total (\$12,941 per student)
for salaries and travel for the participants - this figure excludes mentors' salaries and equipment.
15. Typically, how much direct financial support does your institution provide?
amount. \$220,000
100\%
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
one full time program administrator
3 senior technical directors [full time for about 6 months]
about 6 problem supporters [varying from about \(20 \%\) to \(100 \%\) during the summer]
17. Which agency is the principal source of your support (NSF, NSA, others)?
NSF Comments:
\(\boxtimes\) NSA
18. Briefly describe the goals of your program. (Attach extra page if necessary).
The premise behind the National Security Agency's Director's Summer Program is that mathematics is the core discipline of NSA; to ensure the future technical health of NSA we must foster a vibrant academic mathematics community in the United States and maintain access to our country's best mathematicians.

\section*{The major goals of the DSP are threefold:}

\footnotetext{
* Expose the very brightest young mathematicians in the country to the excitement of cryptologic mathematics through work on very challenging mission-critical problems.
* Encourage them to pursue an advanced degree in mathematics.
* Establish a deep connection to the future leaders of the U.S. mathematics
}
community and develop a pool of world-class consultants.

\section*{Survey of Undergraduate Summer Programs in Mathematics}

Name of respondent: Garity, Dennis J.
Program held at what institution? Oregon State Univ
Mark here if this program is no longer active: \(\square\) If so, last year of operation:

Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\boxtimes\) REUREU Program for certain groupOther
Describe: \(\qquad\) students. 3 faculty direct projects.
2. How many years has your program existed (including other directors)? 12
3. In all the years of program operation, what is the total number of students who have participated in the program? 114 ( 10 per year)
4. Of the students who have attended your program, what number were women? 53 (46\%

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 5 ( 4\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: Yes
6. Is your program:
\(\qquad\) instructional in nature orresearch oriented, orboth

Comments: Instruction involves introduction to various research areas.
\(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\boxtimes\) in groups \(\square\) individually (check both if necessary)
Comments:

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:

8 are still undergraduates
34 are currently in a graduate program in the mathematical sciences have received a Masters in the math. sci. (and are no longer in graduate school) have received a Ph.D. in the mathematical sciences are currently in a graduate program in the sciences (excluding the math. sci.) have received a Masters in the sciences (excluding the mathematical sciences) have received a Ph.D. in the sciences (excluding the mathematical sciences)
other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
These figures are estimates. I have a mailing out to ex-students and can update later.
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\square 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\boxtimes 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(O\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?

3
\[
x 12 \text { years }=36
\]
12. On average, how many poster presentations at conferences have resulted from your program per year?
\(2 \times 12\) years \(=24\)
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
```

3
x 12 years: 36

```
14. What is the total annual budget for your program?
\(\qquad\)
\(\qquad\)
\(\qquad\)
15. Typically, how much direct financial support does your institution provide? \$15,000
amount: \(\qquad\)
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.

\section*{tuition waivers}
17. Which agency is the principal source of your support (NSF, NSA, others)?
NSF
NSA
Comments: \(\qquad\)
18. Briefly describe the goals of your program. (Attach extra page if necessary).
Introduce students to research techniques. Motivate students to attend graduate school.

\title{
Survey of Undergraduate Summer Programs in Mathematics
}

Name of respondent: Katok, Svetlana R.
Program held at what institution? Pennsylvania State Univ, Univ Park
Mark here if this program is no longer active: \(\square\) If so, last year of operation:

Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\boxtimes\) REU
\(\square\) REU Program for certain group
\(\square\) Other
Describe: \(\qquad\)
2. How many years has your program existed (including other directors)? 1
3. In all the years of program operation, what is the total number of students who have participated in the program? 11 ( \(\mathbf{1 1}\) per year)
4. Of the students who have attended your program, what number were women? \(\mathbf{2}\) 2 ( 18\%

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? ( \(0 \%\) ) NA - we do not collect this data
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
6. Is your program:
\(\qquad\) instructional in nature or
\(\qquad\) research oriented, orboth

Comments: The program will include two mini-courses, a seminar, and individual research work on a choice of projects.
7. How many weeks does your program run?
\(\square 1 \square 2 \square 3 \square 4 \quad \square 5 \boxtimes 6\)68 9\(\square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\boxtimes\) in groups \(\boxtimes\) individually
(check both if necessary)
Comments:

For questions \(9-17\), please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:

11 are still undergraduates
0 are currently in a graduate program in the mathematical sciences
0 have received a Masters in the math. sci. (and are no longer in graduate school)
0 have received a Ph.D. in the mathematical sciences
0 are currently in a graduate program in the sciences (excluding the math. sci.)
0 have received a Masters in the sciences (excluding the mathematical sciences)
0 have received a Ph.D. in the sciences (excluding the mathematical sciences)
0 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\boxtimes 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(\bigcirc\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?
\[
x 1 \text { years }=0 \quad \text { NA - first year }
\]
12. On average, how many poster presentations at conferences have resulted from your program per year?
\[
x 1 \text { years }=0 \quad \text { NA - first year }
\]
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
\(\qquad\) \(x 1\) years :0_NA - first year
14. What is the total annual budget for your program?
\(\ldots\) per student AND/OR \$35,200 total (\$3,200 per student)
\(\qquad\)
\(\qquad\)
15. Typically, how much direct financial support does your institution provide?
```

            $0
    amount:

```
\(\qquad\)
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
clerical graduate tutoring colloquia
17. Which agency is the principal source of your support (NSF, NSA, others)?
18. Briefly describe the goals of your program. (Attach extra page if necessary).

The goal of the program is to provide research initiation and instruction to a group of students seriously interested in mathematical sciences. Former and future participants of the Mathematical Advanced Study Semesters (MASS) program have an opportunity to integrate summer research with a research project pursued during the MASS semester.

\section*{Survey of Undergraduate Summer Programs in Mathematics}

Name of respondent: Broughton, S. Allen
Program held at what institution? Rose-Hulman Inst of Technology
Mark here if this program is no longer active: \(\square\) If so, last year of operation:

Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\boxtimes\) REU
\(\square\) REU Program for certain group
\(\square\) Other
Describe: \(\qquad\)
2. How many years has your program existed (including other directors)? 11
3. In all the years of program operation, what is the total number of students who have participated in the program? 70 ( 6 per year)
4. Of the students who have attended your program, what number were women? 21 30\%

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 0 ( 0\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: O yes
6. Is your program:
\(\qquad\) instructional in nature orresearch oriented, orboth

Comments: \(\quad \begin{aligned} & \text { Instruction is given only in an informal fashion on a need to know } \\ & \text { basis. About 3-5 hours over } 7 \text { weeks. }\end{aligned}\)
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \boxtimes 7 \quad \square 8 \quad \square 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\boxtimes\) in groups \(\boxtimes\) individually
(check both if necessary)

\section*{Comments: Problems are presented. Students choose their own problems and teams.}

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:

10 are still undergraduates
38 are currently in a graduate program in the mathematical sciences
0 have received a Masters in the math. sci. (and are no longer in graduate school)
18 have received a Ph.D. in the mathematical sciences
0 are currently in a graduate program in the sciences (excluding the math. sci.)
0 have received a Masters in the sciences (excluding the mathematical sciences)
0 have received a Ph.D. in the sciences (excluding the mathematical sciences)
4 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\square 0-10 \%\) & \(\boxtimes 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(O\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?

9
\[
\times 11 \text { years }=99
\]
12. On average, how many poster presentations at conferences have resulted from your program per year?
\(1 \times 11\) years \(=11\)
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
\(1.7 \times 11\) years \(: \underline{19}\)
14. What is the total annual budget for your program?
per student AND/OR \$74,000
total ( \(\$ 12,333\) per student)
(last proposal only) NSF \$40,000 Cost share \$34,000 Total \$74,000
15. Typically, how much direct financial support does your institution provide? \$34,000
amount: none, note cost share above comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
release time
summer salary cost share
clerical support
faculty salaries
17. Which agency is the principal source of your support (NSF, NSA, others)?
18. Briefly describe the goals of your program. (Attach extra page if necessary).

GOALS

\section*{1. Nature of the student activities and program}

\subsection*{1.1 Goals of the program}

The goals of the proposed program are:
. for students to achieve independent success at formulating and solving problems that are their own, and that require significant growth beyond "textbook problems";
. to increase their depth and breadth of knowledge in the area of algebra and discrete structures;
. to develop their collaborative work skills;
. to make significant use of computer tools in solving research problems;
. to develop their oral communication skills by making presentations of their work during the program and at a conference after the program;
. to develop their written communication skills by writing a Rose-Hulman Mathematics Technical Report and refining the report into a submitted paper where appropriate;
. to develop a collegial peer relation with one or more professional mathematicians, outside their college; and
. to develop a sense of the culture of professional mathematicians. both academic and applied.

\title{
Survey of Undergraduate Summer Programs in Mathematics
}

Name of respondent: Mahdavi, Kazem
Program held at what institution? SUNY, Potsdam
Mark here if this program is no longer active: \(\square\) If so, last year of operation:

Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\boxtimes\) REU
\(\square\) REU Program for certain groupOther

Describe: \(\qquad\)
2. How many years has your program existed (including other directors)? 9
3. In all the years of program operation, what is the total number of students who have participated in the program? \(\mathbf{3 0}\) ( \(\mathbf{3}\) per year)
4. Of the students who have attended your program, what number were women? \(9 \quad\) ( 30\% )

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 1 ( 3\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
6. Is your program:
\(\qquad\) instructional in nature or
\(\qquad\) research oriented, orboth
Comments: We engage them in research and also in learning advanced materials in mathematics.
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\boxtimes\) in groups \(\square\) individually
(check both if necessary)
Comments:

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
are still undergraduates
13 are currently in a graduate program in the mathematical sciences
7 have received a Masters in the math. sci. (and are no longer in graduate school)
1 have received a Ph.D. in the mathematical sciences are currently in a graduate program in the sciences (excluding the math. sci.) have received a Masters in the sciences (excluding the mathematical sciences) have received a Ph.D. in the sciences (excluding the mathematical sciences)
other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\boxtimes 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(\bigcirc\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?
\(\qquad\) \(x 9\) years \(=9\)
12. On average, how many poster presentations at conferences have resulted from your program per year?
\(1 \times 9\) years \(=9\)
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
\(\qquad\)
14. What is the total annual budget for your program?
\(\ldots\) per student AND/OR \$30,000 total (\$10,000 per student)
15. Typically, how much direct financial support does your institution provide? \$40,000
amount: Approximate amounts for annual budget and direct financial support. comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
```

clerical
release time
free housing for students
free food and entertainment for students

```
17. Which agency is the principal source of your support (NSF, NSA, others)?
18. Briefly describe the goals of your program. (Attach extra page if necessary).

The goal of this program will be to build the participants' confidence in their ability to do research independently and to stimulate their interest in pursuing mathematics.

To expand the students' horizons, there will be one talk each week by a guest speaker. Well known mathematicians are among our speakers.

To familiarize students with advanced topics in mathematics.

\title{
Survey of Undergraduate Summer Programs in Mathematics
}

Name of respondent: Elaydi, Saber
Program held at what institution? Trinity Univ
Mark here if this program is no longer active: \(\square\) If so, last year of operation:

Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\boxtimes\) REU
\(\square\) REU Program for certain group
\(\square\) Other
Describe:_ We get six students; 2 work in Algebra, 2 in dynamical systems and two in difference \(/ \Delta \sum\) equations.
2. How many years has your program existed (including other directors)? 3
3. In all the years of program operation, what is the total number of students who have participated in the program? 16 ( 5 per year)
4. Of the students who have attended your program, what number were women? 5 ( 31\%

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 0 ( \(0 \%\) )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: O yes
6. Is your program:
\(\square\) instructional in nature orresearch oriented, orboth
Comments: \(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \boxtimes 7 \quad \square 8 \quad \square 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
\(\boxtimes\) in groups \(\square\) individually
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\boxtimes\) in groups \(\square\) individually
(check both if necessary)
Comments:

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:

\section*{5 are still undergraduates}

4 are currently in a graduate program in the mathematical sciences have received a Masters in the math. sci. (and are no longer in graduate school)
have received a Ph.D. in the mathematical sciences are currently in a graduate program in the sciences (excluding the math. sci.) have received a Masters in the sciences (excluding the mathematical sciences) have received a Ph.D. in the sciences (excluding the mathematical sciences)
other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\square 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(\bigcirc\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?

2
\[
x 3 \text { years }=6
\]
12. On average, how many poster presentations at conferences have resulted from your program per year?
\(\qquad\) x 3 years \(=0\)
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).

0
```

x 3 years:0

```
14. What is the total annual budget for your program?
\(\qquad\)
\(\qquad\)
\(\qquad\)
15. Typically, how much direct financial support does your institution provide?
```

            $0
    amount:

``` \(\qquad\)
comments: \(\qquad\)
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.

\section*{computer use/computer support}
17. Which agency is the principal source of your support (NSF, NSA, others)?

Comments: \(\qquad\)
18. Briefly describe the goals of your program. (Attach extra page if necessary).

\section*{To actively involve students in research.}

\title{
Survey of Undergraduate Summer Programs in Mathematics
}

Name of respondent: Kalka, Morris
Program held at what institution? Tulane Univ
Mark here if this program is no longer active: \(\qquad\) If so, last year of operation:

Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\boxtimes\) REU
\(\square\) REU Program for certain groupOther
Describe: \(\qquad\)
2. How many years has your program existed (including other directors)? 5
3. In all the years of program operation, what is the total number of students who have participated in the program? 43 ( 9 per year)
4. Of the students who have attended your program, what number were women? 12 ( \(28 \%\) )

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 0 ( 0\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
6. Is your program:
\(\qquad\) instructional in nature or
\(\boxtimes\) research oriented, or
\(\square\) both

Comments: \(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\boxtimes\) in groups \(\square\) individually
(check both if necessary)

Comments:

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:

23 are still undergraduates
18 are currently in a graduate program in the mathematical sciences _ have received a Masters in the math. sci. (and are no longer in graduate school)
have received a Ph.D. in the mathematical sciences
\(\qquad\) are currently in a graduate program in the sciences (excluding the math. sci.)
\(\qquad\) have received a Masters in the sciences (excluding the mathematical sciences)
\(\qquad\) have received a Ph.D. in the sciences (excluding the mathematical sciences)

2 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\square 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\boxtimes 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(\bigcirc\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?
\(\qquad\) \(\times 5\) years \(=5\)
12. On average, how many poster presentations at conferences have resulted from your program per year?
\(1 \times 5\) years \(=5\)
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
```

2
x 5
years:10

```
14. What is the total annual budget for your program?
\(\ldots\) per student AND/OR \$77,200 total (\$9,650 per student)
15. Typically, how much direct financial support does your institution provide? \$17,200
amount: \(\qquad\)
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.

\section*{clerical}
17. Which agency is the principal source of your support (NSF, NSA, others)?
\(\boxtimes\) NSF
NSA
18. Briefly describe the goals of your program. (Attach extra page if necessary).
The goals of our program are to introduce motivated undergraduates to research in geometry and topology and to do so in a way which makes it clear that learning and doing mathematics is a collaborative endeavor which can be fun.

\title{
Survey of Undergraduate Summer Programs in Mathematics
}

Name of respondent: Nolan, Deborah
Program held at what institution? Univ of California, Berkeley and Mills College
Mark here if this program is no longer active: \(\boxtimes\) If so, last year of operation: 1997
Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
REU
\(\square\) REU Program for certain group
- Other

Describe:
Summer program for undergraduate women interested in obtaining an advanced degree in math.
2. How many years has your program existed (including other directors)? 7
3. In all the years of program operation, what is the total number of students who have participated in the program? 155 ( 22 per year)
4. Of the students who have attended your program, what number were women? 156 ( \(101 \%\) )

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 12 ( 8\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
6. Is your program:
\(\boxtimes\) instructional in nature orresearch oriented, orboth
Comments: \(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
7. How many weeks does your program run?
\(\square 1 \square 2 \quad \square 3 \square 4 \quad \square 5 \boxtimes 6\)6\(\square\) 9\(\square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:in groups \(\square\) individually
(check both if necessary)
Comments:

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
are still undergraduates
47 are currently in a graduate program in the mathematical sciences
24 have received a Masters in the math. sci. (and are no longer in graduate school)
20 have received a Ph.D. in the mathematical sciences
2 are currently in a graduate program in the sciences (excluding the math. sci.)
0 have received a Masters in the sciences (excluding the mathematical sciences)
0 have received a Ph.D. in the sciences (excluding the mathematical sciences)
51 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:

\section*{Still working on this part.}
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\square 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(\bigcirc\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?
12. On average, how many poster presentations at conferences have resulted from your program per year?
\[
\text { NA } \times 7 \text { years }=0 \quad \begin{aligned}
& \text { As part of the program we sponsor our students to attend the } \\
& \text { January Joint Meetings, usually } 10 \text { go each year. }
\end{aligned}
\]
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).

0 \(\qquad\) x \({ }^{7}\) years : 0 \(: 0\)
14. What is the total annual budget for your program?
per student AND/OR \$140,000 total (\$6,364 per student)
It varied from year to year. With 24 students the budget was about \$140,000.
15. Typically, how much direct financial support does your institution provide?
amount: \(\frac{\$ 0}{1 \text { year UCB provided } \$ 14,000(1997) \text {, other } 6 \text { years } \$ 0 .}\)
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
17. Which agency is the principal source of your support (NSF, NSA, others)?

NSF about \(\$ 110,000\) (more in the earlier years before NSA support)


Comments: with about \(\$ 30,000\) from NSA a year from 1994-1997.
18. Briefly describe the goals of your program. (Attach extra page if necessary).

The Mills SMI and Berkley SIMS annually provided 20-24 women, selected nationwide, with an intensive mathematics program that was designed to motivate and prepare them to enter and complete successfully a PH.D. program in the mathematical sciences. The SMI/SIMS program aimed to provide participants with a taste of what is is to be a research mathematician; to communicate to the students that women can and should be doing mathematics; to provide students a broad view of current research areas in mathematics; and to inform students on how to apply to graduate school, find financial aid, choose a graduate program, what to expect in graduate school, and to discuss different career opportunities in the mathematical sciences.

We brought together a critical mass of bright undergraduate women math majors, women with diverse backgrounds who could give each other mutual support, encouragement, and inspiration, and who could be research partners and professional colleagues. We provided these women with role models: women faculty and graduate students with whom they could form close bonds. In the program the students participated in seminars where they were challenged to develop the basic theory of an area of mathematics that is not part of the traditional undergraduate curriculum They worked in small groups and individually where: they gained experience and practice in the process of searching for and writing up proofs; they read journal articles; and learned how to obtain and express mathematical ideas in conversation and in writing.

\title{
Survey of Undergraduate Summer Programs in Mathematics
}

Name of respondent: Higgins, Aparna W.
Program held at what institution? Univ of Dayton
Mark here if this program is no longer active: \(\boxtimes\) If so, last year of operation: 1993

Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\boxtimes\) REU
\(\square\) REU Program for certain group
\(\square\) Other
Describe: \(\qquad\)
2. How many years has your program existed (including other directors)? 5
3. In all the years of program operation, what is the total number of students who have participated in the program? 37 ( 7 per year)
4. Of the students who have attended your program, what number were women? 14 ( 38\% )

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 0 ( \(0 \%\) )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
6. Is your program:
\(\qquad\) instructional in nature orresearch oriented, or
\(\boxtimes\) both

\section*{Comments: Minimally instructional - first week or two of intense lectures in graph theory. A week-long visit by an "expert", who may conduct some instructional talks - else, all research.}
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\square\) in groups \(\boxtimes\) individually
(check both if necessary)

\section*{Comments: There is some group suggestions and comments, but each student had his/her own problem.}

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
\(\qquad\) are still undergraduates
2 are currently in a graduate program in the mathematical sciences
3 have received a Masters in the math. sci. (and are no longer in graduate school)
6 have received a Ph.D. in the mathematical sciences
0 are currently in a graduate program in the sciences (excluding the math. sci.)
0 have received a Masters in the sciences (excluding the mathematical sciences)
0 have received a Ph.D. in the sciences (excluding the mathematical sciences)
2 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\boxtimes 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(\bigcirc\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?
\(\qquad\) \(\times 5\) years \(=35\)
One per student for the year they were in our program
12. On average, how many poster presentations at conferences have resulted from your program per year?
```

0
x 5 years =0
posters are a more recent phenomenon at meetings!

```
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).

0 \(x 5\) years : 0
14. What is the total annual budget for your program?
per student AND/OR \$80,000
total ( \(\$ 11,429\) per student)
15. Typically, how much direct financial support does your institution provide?
\[
\begin{array}{ll}
\text { amount: } & \$ 40,000 \\
\text { comments: } & \text { The NSF-sponsored REUs were cost-sharing programs. Our budget charged } \\
\text { student-related stuff to NSF, while the institution picked up the rest. }
\end{array}
\]
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
clerical
reduction of normal overhead
17. Which agency is the principal source of your support (NSF, NSA, others)?
NSF
Comments:
NSA
18. Briefly describe the goals of your program. (Attach extra page if necessary).
. To introduce students to mathematical research, and how to do it! (This was accomplished by allowing students to choose individual projects, and supervising them on that research.)
. To expose students to different areas of math (by inviting scintillating colloquium speakers like Bill Dunham, Richard Guy, Allen Schwenk, etc.) . To make students aware of professional and educational opportunities in mathematics (by inviting speakers from business, industry and government NSA reps, Ron Graham, etc.)

\title{
Survey of Undergraduate Summer Programs in Mathematics
}

Name of respondent: Schaal, Daniel
Program held at what institution? Univ of Idaho
Mark here if this program is no longer active: \(\square\) If so, last year of operation:

Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\boxtimes\) REU
\(\square\) REU Program for certain groupOther
Describe: \(\qquad\)
2. How many years has your program existed (including other directors)? 1
3. In all the years of program operation, what is the total number of students who have participated in the program? 5 ( 5 per year)
4. Of the students who have attended your program, what number were women? 3 ( \(60 \%\) )

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 0 ( 0\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
6. Is your program:
\(\qquad\) instructional in nature or
\(\boxtimes\) research oriented, or
\(\square\) both

Comments: \(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\boxtimes\) in groups \(\boxtimes\) individually (check both if necessary)

Comments: students may choose to work individually or in groups

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
are still undergraduates
are currently in a graduate program in the mathematical sciences have received a Masters in the math. sci. (and are no longer in graduate school)
have received a Ph.D. in the mathematical sciences
are currently in a graduate program in the sciences (excluding the math. sci.)
have received a Masters in the sciences (excluding the mathematical sciences)
have received a Ph.D. in the sciences (excluding the mathematical sciences)
other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\boxtimes 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(\bigcirc\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?

4
\[
x 1 \text { years }=4
\]
12. On average, how many poster presentations at conferences have resulted from your program per year?
```

3
x 1 years =3

```
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
```

3 < 1 years:3 projected

```
14. What is the total annual budget for your program?
\(\ldots\) per student AND/OR \$40,800 total (\$8,160 per student)
15. Typically, how much direct financial support does your institution provide? \$10,800
amount: \(\qquad\)
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
clerical
17. Which agency is the principal source of your support (NSF, NSA, others)?
NSF
NSA
18. Briefly describe the goals of your program. (Attach extra page if necessary).
To expose undergraduates to mathematics research in hope of making it more likely that they will choose a career as a mathematician.
We are targeting students from institutions that do not have opportunities for undergraduate research. Specifically, small colleges without graduate programs and institutions in the states of South Dakota, North Dakota, Montana, Idaho, Wyoming and Alaska, where REU's are not often held.

\section*{Survey of Undergraduate Summer Programs in Mathematics}

Name of respondent: Okunbor, Daniel I.
Program held at what institution? Univ of Maryland, Eastern Shore
Mark here if this program is no longer active: \(\square\) If so, last year of operation:

Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\boxtimes\) REU
\(\square\) REU Program for certain group
\(\square\) Other
Describe:__ NSF REU in ParalleI Numerical Linear Algebra
2. How many years has your program existed (including other directors)? 3
3. In all the years of program operation, what is the total number of students who have participated in the program? 22 ( 7 per year)
4. Of the students who have attended your program, what number were women? \(7 \quad\) ( 32\% )

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 4 ( 18\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
6. Is your program:
\(\square\) instructional in nature or
\(\square\) research oriented, or
\(\boxtimes\) both
Comments: \(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\boxtimes\) in groups \(\square\) individually
(check both if necessary)
Comments:

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
\(\qquad\) are still undergraduates
3 are currently in a graduate program in the mathematical sciences
1 have received a Masters in the math. sci. (and are no longer in graduate school)
_ have received a Ph.D. in the mathematical sciences
\(\qquad\) are currently in a graduate program in the sciences (excluding the math. sci.)
\(\qquad\) have received a Masters in the sciences (excluding the mathematical sciences)
\(\qquad\) have received a Ph.D. in the sciences (excluding the mathematical sciences)
\(\qquad\) other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\square 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate yes
11. On average, how many oral presentations at conferences have resulted from your program per year?

8
\[
x 3 \text { years }=24
\]
12. On average, how many poster presentations at conferences have resulted from your program per year?
\[
\times 3 \text { years }=0
\]
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
\(\qquad\) \(x^{3}\) years : 0
14. What is the total annual budget for your program?
per student AND/OR \$50,000
total ( \(\$ 7,143\) per student)
15. Typically, how much direct financial support does your institution provide?
\(\qquad\)
amount:
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
clerical office space school facilities
17. Which agency is the principal source of your support (NSF, NSA, others)?

Comments:
18. Briefly describe the goals of your program. (Attach extra page if necessary).

The program is for undergraduate juniors who have limited or no opportunities for research, to learn how to do research and to be involved in a parallel numerical linear algebra research project. Participants learn state-of-the-art numerical algorithms in linear algebra, new programming paradigms for use in parallel processing. Participants are given access to an undergraduate research laboratory and computer facilities to do parallel processing and are required to present a research paper at a special parallel processing conference specifically designed for undergraduates.

\section*{Survey of Undergraduate Summer Programs in Mathematics}

Name of respondent: Burns, Daniel M.
Program held at what institution? Univ of Michigan, Ann Arbor
Mark here if this program is no longer active: \(\square\) If so, last year of operation:

Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\boxtimes\) REU
\(\square\) REU Program for certain group
\(\square\) Other
Describe: \(\qquad\)
Not a "site program".
Supplemented by other funds. (note program began in 86 or 87, responses reflect the years 90-98)
2. How many years has your program existed (including other directors)? 9
3. In all the years of program operation, what is the total number of students who have participated in the program? 117 ( 13 per year)
4. Of the students who have attended your program, what number were women? 36 ( 31\% )

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 4 ( 3\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: O yes
6. Is your program:instructional in nature orresearch oriented, orboth

Comments: \(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\boxtimes\) in groups \(\boxtimes\) individually (check both if necessary)

Comments: Both, depending on the project

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
\(\qquad\) are still undergraduates
\(\qquad\) are currently in a graduate program in the mathematical sciences
\(\qquad\) have received a Masters in the math. sci. (and are no longer in graduate school) have received a Ph.D. in the mathematical sciences are currently in a graduate program in the sciences (excluding the math. sci.) have received a Masters in the sciences (excluding the mathematical sciences) have received a Ph.D. in the sciences (excluding the mathematical sciences)
other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
35 of approx. 110 graduates proceeded to Ph.D. programs, across all fields. Don't have records for Math vs. other.
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\square 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\boxtimes 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(O\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?
12. On average, how many poster presentations at conferences have resulted from your program per year?
\(\qquad\)
\(\times 9\) years \(=0\)
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
\(1 \quad x^{9}\) years \(: 9\)
14. What is the total annual budget for your program?
\(\$ 2,000\) per student AND/OR __ total (\$2,000 per student)
stipend (to be raised, '99)
15. Typically, how much direct financial support does your institution provide?
\[
\begin{array}{ll}
\text { amount: } & \begin{array}{l}
\text { Sporadic - use Mathematics Dept endowment funds to support some projects. } \\
\text { comments: }
\end{array} \\
\text { University has provided bridging (or "SURGE") funds when \# of applicants was } \\
\text { too large. }
\end{array}
\]
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
```

waiver of overhead
faculty salaries
computer support
clerical
dedicated computer room
computer use/computer support

```
17. Which agency is the principal source of your support (NSF, NSA, others)?
also individual grants; some NSF GIG lines; some DOD, NIH lines;

\section*{\(\triangle\) NSF}

NSA
Comments: Math Dept. \& U Michigan funds; private fellowship funds (Packard)
18. Briefly describe the goals of your program. (Attach extra page if necessary).

To provide a realistic research experience to our students. It also gains us more time (Summer) to work with them intensively.

\section*{Survey of Undergraduate Summer Programs in Mathematics}

Name of respondent: Gallian, Joseph
Program held at what institution? Univ of Minnesota, Duluth
Mark here if this program is no longer active: \(\square\) If so, last year of operation:

Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\boxtimes\) REU
\(\square\) REU Program for certain groupOther
Describe: \(\qquad\)
2. How many years has your program existed (including other directors)?
```

20

```
3. In all the years of program operation, what is the total number of students who have participated in the program? 89 ( 4 per year)
4. Of the students who have attended your program, what number were women? 27 ( 30\% ) Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 0 ( 0\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
6. Is your program:
\(\square\) instructional in nature or
\(\qquad\) research oriented, orboth

Comments: We strive for professional level research.
\(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \square 8 \quad \square 9 \quad \boxtimes 10\)
8. If your program has an instructional component, are students asked to work :in groupsindividually
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\qquad\) in groups \(\boxtimes\) individually
(check both if necessary)
Comments:

For questions 9-17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:

11 are still undergraduates
25 are currently in a graduate program in the mathematical sciences
5 have received a Masters in the math. sci. (and are no longer in graduate school)
35 have received a Ph.D. in the mathematical sciences
1 are currently in a graduate program in the sciences (excluding the math. sci.)
0 have received a Masters in the sciences (excluding the mathematical sciences)
0 have received a Ph.D. in the sciences (excluding the mathematical sciences)
0 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\square 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\boxtimes 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(\bigcirc\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?

4
\[
\times 20 \text { years }=80
\]
12. On average, how many poster presentations at conferences have resulted from your program per year?
\(\qquad\)
x 20 years \(=0\)
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
```

4

```
```

x 20 years:80

```
```

x 20 years:80

```
14. What is the total annual budget for your program?
_ per student AND/OR \$70,000 total (\$17,500 per student)
15. Typically, how much direct financial support does your institution provide? \$1,500
amount: \(\qquad\)
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
```

clerical
telephone/photocopying/postage
release time

```
17. Which agency is the principal source of your support (NSF, NSA, others)?

NSF \& NSA equally
\(\boxtimes\) NSA
Comments: \(\qquad\)
18. Briefly describe the goals of your program. (Attach extra page if necessary).

Provide a research experience; encourage talented students to go for Ph.D.; develop skills needed to become a professional mathematican; provide networking opportunity; contribute to the body of knowledge; have fun.

\section*{Survey of Undergraduate Summer Programs in Mathematics}

Name of respondent: Lenhart, Suzanne
Program held at what institution? Univ of Tennessee, Knoxville
Mark here if this program is no longer active: \(\square\) If so, last year of operation:

Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\boxtimes\) REU
\(\square\) REU Program for certain group
\(\square\) Other
Describe:
10 students, "one-on-one" style, with 10 research advisors, on a variety of topics.
2. How many years has your program existed (including other directors)? 12
3. In all the years of program operation, what is the total number of students who have participated in the program? 125 ( 10 per year)
4. Of the students who have attended your program, what number were women? 60 ( 48\%

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 4 ( 3\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
6. Is your program:instructional in nature orresearch oriented, orboth

\section*{Comments: Research is the main emphasis but some instruction in 2 short courses is included.}
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\square\) in groups \(\boxtimes\) individually
(check both if necessary)

Comments:

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:

12 are still undergraduates
30 are currently in a graduate program in the mathematical sciences
5 have received a Masters in the math. sci. (and are no longer in graduate school)
10 have received a Ph.D. in the mathematical sciences are currently in a graduate program in the sciences (excluding the math. sci.) have received a Masters in the sciences (excluding the mathematical sciences) have received a Ph.D. in the sciences (excluding the mathematical sciences)
other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\square 0-10 \%\) & \(\boxtimes 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(\bigcirc\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?

4
\[
x 12 \text { years }=48
\]
12. On average, how many poster presentations at conferences have resulted from your program per year?
\[
2 \times 12 \text { years }=24
\]
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
```

2
x 12 years:2
:24

```
14. What is the total annual budget for your program?
\(\ldots\) per student AND/OR \$70,000 total (\$7,000 per student)
15. Typically, how much direct financial support does your institution provide? \$20,000
amount: \(\qquad\)
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.

\section*{clerical}
17. Which agency is the principal source of your support (NSF, NSA, others)?

\footnotetext{
\(\boxtimes\) NSF
NSA
18. Briefly describe the goals of your program. (Attach extra page if necessary).

Our program is based on the "one-on-one" approach. Each student is working with an advisor on a research project. One goal is to give the students enough research experience in order for them to make a decision about going to a Ph.D. program. We also try to educate these students about the variety of research areas in mathematics, the structure of math organizations, and the range of opportunities.
}

\title{
Survey of Undergraduate Summer Programs in Mathematics
}

Name of respondent: Rovnyak, James
Program held at what institution? Univ of Virginia
Mark here if this program is no longer active: \(\square\) If so, last year of operation:

Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\square\) REU
\(\boxtimes\) REU Program for certain group UVA students onlyOther
Describe: \(\qquad\)
2. How many years has your program existed (including other directors)? 4
3. In all the years of program operation, what is the total number of students who have participated in the program? 25 ( 6 per year)
4. Of the students who have attended your program, what number were women? 4 ( \(16 \%\) )

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? \(\qquad\) (0\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
6. Is your program:
\(\qquad\) instructional in nature or
\(\boxtimes\) research oriented, or
\(\square\) both

Comments: \(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :

(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\square\) in groups \(\boxtimes\) individually
(check both if necessary)
Comments:

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
\(\qquad\) are still undergraduates
\(\qquad\) are currently in a graduate program in the mathematical sciences
\(\qquad\) have received a Masters in the math. sci. (and are no longer in graduate school)
have received a Ph.D. in the mathematical sciences
\(\qquad\) are currently in a graduate program in the sciences (excluding the math. sci.) have received a Masters in the sciences (excluding the mathematical sciences) have received a Ph.D. in the sciences (excluding the mathematical sciences)
other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\square 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate yes
11. On average, how many oral presentations at conferences have resulted from your program per year?
\[
\times 4 \text { years }=0
\]
12. On average, how many poster presentations at conferences have resulted from your program per year?
\(\square\)
x 4 years \(=0\)
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
\(\qquad\)
```

x4 years:0

```
14. What is the total annual budget for your program?
\(\qquad\) per student AND/OR \$20,000 total (\$3,333 per student)
\(\qquad\)
\(\qquad\)
15. Typically, how much direct financial support does your institution provide? \$15,000
amount: \(\qquad\)
comments: \(\qquad\)
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
17. Which agency is the principal source of your support (NSF, NSA, others)?
1. UVA private funds.

\section*{Comments: 2. NSF}
18. Briefly describe the goals of your program. (Attach extra page if necessary).

\section*{Survey of Undergraduate Summer Programs in Mathematics}

Name of respondent: Morrow, James A.
Program held at what institution? Univ of Washington
Mark here if this program is no longer active: \(\qquad\) If so, last year of operation:

Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\boxtimes\) REU
\(\square\) REU Program for certain groupOther
Describe: \(\qquad\)
2. How many years has your program existed (including other directors)? 11
3. In all the years of program operation, what is the total number of students who have participated in the program? 98 ( 9 per year)
4. Of the students who have attended your program, what number were women?

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 0 ( \(0 \%\) )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: O yes
6. Is your program:
\(\qquad\) instructional in nature or
\(\boxtimes\) research oriented, or
\(\square\) both
Comments: \(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\boxtimes\) in groups \(\boxtimes\) individually (check both if necessary)

Comments:

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:

18 are still undergraduates
15 are currently in a graduate program in the mathematical sciences
10 have received a Masters in the math. sci. (and are no longer in graduate school)
17 have received a Ph.D. in the mathematical sciences
3 are currently in a graduate program in the sciences (excluding the math. sci.)
have received a Masters in the sciences (excluding the mathematical sciences)
have received a Ph.D. in the sciences (excluding the mathematical sciences)
20 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
"Math Sci" PhD group includes engineering.
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\square 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\boxtimes 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(O\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?
\(\qquad\) \(\times 11\) years \(=11\)
(actual answer was " 8 total")
12. On average, how many poster presentations at conferences have resulted from your program per year?
\(\qquad\)
x 11 years \(=0\)
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
```

1 x 11 years:11

```
14. What is the total annual budget for your program?
_ per student AND/OR \$52,000 total (\$6,500 per student)
15. Typically, how much direct financial support does your institution provide?
\$12,000
amount: for faculty salaries
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
clerical
computer use/computer support
17. Which agency is the principal source of your support (NSF, NSA, others)?

\footnotetext{
\(\boxtimes\) NSF
Comments:
NSA
18. Briefly describe the goals of your program. (Attach extra page if necessary).

Students are introduced to the work of the directors and earlier students in the program. Problems are suggested and problem ideas are solicited for the students. Students then work in groups and with the directors on these problems and write up their work. The main goal of the program is to show students what research is like. We think it is important for students to find out of they want to pursue mathematics as a career. If so, they should understand it is not a classroom activity. Most discover it is a unique experience and many are attracted into mathematics because of our REU program.
}

\section*{Survey of Undergraduate Summer Programs in Mathematics}

Name of respondent: Rubio, Ivelisse
Program held at what institution? University of Puerto Rico at Humacao
Mark here if this program is no longer active: \(\square\) If so, last year of operation:

Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:REUREU Program for certain groupOther
Describe: \(\qquad\)
2. How many years has your program existed (including other directors)? 2
3. In all the years of program operation, what is the total number of students who have participated in the program? 49 ( 25 per year)
4. Of the students who have attended your program, what number were women? 23

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 49 ( 100\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
6. Is your program:
\(\qquad\) instructional in nature orresearch oriented, orboth

\title{
Comments: The first half of the program is devoted to introducing a mathematical topic. The second half is devoted to undergraduate research projects.
}
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \boxtimes 6 \quad \square 7 \quad \square 8 \quad \square 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\boxtimes\) in groups \(\square\) individually
(check both if necessary)
Comments: The first year there were some studentswho worked individually.

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:

29 are still undergraduates
9 are currently in a graduate program in the mathematical sciences have received a Masters in the math. sci. (and are no longer in graduate school) have received a Ph.D. in the mathematical sciences

1 are currently in a graduate program in the sciences (excluding the math. sci.) have received a Masters in the sciences (excluding the mathematical sciences) have received a Ph.D. in the sciences (excluding the mathematical sciences)

10 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\boxtimes 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate yes
11. On average, how many oral presentations at conferences have resulted from your program per year?

9
```

x 2 years = 18

```
12. On average, how many poster presentations at conferences have resulted from your program per year?
20
\(x 2\) years \(=40\)
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).

0 \(x^{2}\) years : 0
14. What is the total annual budget for your program?
per student AND/OR \(\$ \mathbf{2 1 0 , 0 0 0}\) total ( \(\$ 8,750\) per student)
15. Typically, how much direct financial support does your institution provide? \$40,000
amount:
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
clerical telephone/photocopying/postage advertisement transportation
17. Which agency is the principal source of your support (NSF, NSA, others)?

Comments:
\(\boxtimes\) NSA
18. Briefly describe the goals of your program. (Attach extra page if necessary)

Goal: to increase the number of Chicanos/Latinos and Native Americans earning Masters, Ph.D.'s, and pursuing research careers in the mathematical sciences.

\section*{Objectives:}
1. To provide SIMU participants a rich academic and research experience in the mathematical Sciences - an opportunity unavailable to most Chicano/Latino and Native American undergraduates;
2. To familiarize SIMU students with research protocols and techniques, with collaboration between peers, and with untilizing faculty as effective resources while conducting research - skills that will help them succeed in their undergraduate and graduate education;
3. To build a network of academicians and peers that, through mentoring, will help

SIMU participants to excel in their undergraduate education - a necessity if they are to have the option of attending graduate school.;
4. To introduce SIMU students to successful Chicano/Latino and Native American academicians and graduate students so as to encourage and motivate them to pursue graduate studies and research careers in mathematics;
5. To offer SIMU participants workshops that will teach them skills and techniques that will maximize their likelihood of admission to graduate programs best suited to their needs as well as their likelihood of securing financial support for such programs;
6. To enhance the overall academic portfolio of SIMU students by providing them the opportunity to work with and meet leading mathematicians so that their future applications to graduate school and fellowships are strengthened;
7. To monitor the educational progress of SIMU participants for at least five years after their participation in the institute, such monitoring being a measure of the program's success.

\title{
Survey of Undergraduate Summer Programs in Mathematics
}

Name of respondent: Stone, Emily
Program held at what institution? Utah State Univ
Mark here if this program is no longer active: \(\boxtimes\) If so, last year of operation: 1999
Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\boxtimes\) REU
\(\square\) REU Program for certain group
\(\square\) Other
Describe: \(\qquad\)
2. How many years has your program existed (including other directors)? 3
3. In all the years of program operation, what is the total number of students who have participated in the program? \(\mathbf{3 0}\) ( \(\mathbf{1 0}\) per year)
4. Of the students who have attended your program, what number were women? 12 ( 40\% )

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 3 ( 10\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
6. Is your program:
\(\qquad\) instructional in nature or
\(\qquad\) research oriented, orboth
Comments: Along with research projects, students attend a class for 3 semester credits.
\(\qquad\)
\(\qquad\)
\(\qquad\)
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\boxtimes\) in groups \(\boxtimes\) individually (check both if necessary)

Comments: for research, depends on the mentor

For questions 9-17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
are still undergraduates
1 are currently in a graduate program in the mathematical sciences have received a Masters in the math. sci. (and are no longer in graduate school) have received a Ph.D. in the mathematical sciences

4 are currently in a graduate program in the sciences (excluding the math. sci.) have received a Masters in the sciences (excluding the mathematical sciences) have received a Ph.D. in the sciences (excluding the mathematical sciences)

2 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\boxtimes 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(O\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?

2
\[
x 3 \text { years }=6
\]
12. On average, how many poster presentations at conferences have resulted from your program per year?
\(\qquad\)
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).
```

2 x 3 years:6

```
14. What is the total annual budget for your program?
\(\qquad\)
15. Typically, how much direct financial support does your institution provide?
amount: \(\frac{\$ 8,000}{\text { per year }}\)
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
clerical graduate tutoring release time
17. Which agency is the principal source of your support (NSF, NSA, others)?

Comments:
18. Briefly describe the goals of your program. (Attach extra page if necessary).

\section*{Summary of Nonliner Dynamics REU program at USU}

We propose to establish an REU site at Utah State University whose project theme will be the study of the principles and applications of nonlinear dynamics. This project will run through the 8 week summer sessions of 1997, 1998 and 1999. The primary objective is to show young scholars the power of mathematical thinking in understanding their surroundings by taking advantage of the broad spectrum of problems of interest to the interdisciplinary community of researchers in nonlinear dynamics at Utah State University. The site will also make available a very unusual research opprtunity for students of the Western region of the United states.

Each student will be assigned a project and a faculty mentor based on a statement of interests that the student includes in their application to the REU site. While the projects all can be classified as applied mathematics, the faculty mentors are from Physics, Biology, Forest Resources, and Engineering, as well as Mathematics. There will be two or more interdisciplinary teams of faculty mentors who will advise more than one student on a group project, as well as more traditional single faculty/single student projects. Student researchers will be expected to write weekly progress reports, a final research paper and give an oral presentation on their work at the end of the workshop. To enhance the research experience a 3 credit course on topics in nonliner dynamics will be offered. A weekly colloquium series, with outside speakers, is also planned, each followed by a social event that will allow all workshop participants to interact on a less formal level.

By offering this research opportunity we hope to encourage unusually capable undergraduates to continue their studies in mathematics and enhance the mathematical skills of the technical workforce of the Western states.

\title{
Survey of Undergraduate Summer Programs in Mathematics
}

Name of respondent: Adams, Colin C.
Program held at what institution? Williams Coll
Mark here if this program is no longer active: \(\square\) If so, last year of operation:
Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\boxtimes\) REU
\(\square\) REU Program for certain groupOther
Describe: \(\qquad\)
2. How many years has your program existed (including other directors)? 11
3. In all the years of program operation, what is the total number of students who have participated in the program? 184 ( 17 per year)
4. Of the students who have attended your program, what number were women? 61 ( 33\% )

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 5 ( 3\% )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: Yes
6. Is your program:
\(\square\) instructional in nature or
\(\boxtimes\) research oriented, or
\(\square\) both

Comments: \(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \square 8 \quad \boxtimes 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\qquad\) (check both if necessary)
Comments:

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:
are still undergraduates
22 are currently in a graduate program in the mathematical sciences

2 have received a Masters in the math. sci. (and are no longer in graduate school)
14 have received a Ph.D. in the mathematical sciences
5 are currently in a graduate program in the sciences (excluding the math. sci.) have received a Masters in the sciences (excluding the mathematical sciences) have received a Ph.D. in the sciences (excluding the mathematical sciences)

46 other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\boxtimes 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate \(\bigcirc\) yes
11. On average, how many oral presentations at conferences have resulted from your program per year?

8
\[
\times 11 \text { years }=88
\]
12. On average, how many poster presentations at conferences have resulted from your program per year?

13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).

3 x 11 years: 33
14. What is the total annual budget for your program?
\(\qquad\)
\(\qquad\)
\(\qquad\)
15. Typically, how much direct financial support does your institution provide? \$10,000
amount: to support students per year.
comments: \(\qquad\)
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.

\section*{clerical}
17. Which agency is the principal source of your support (NSF, NSA, others)?

Comments: \(\qquad\)
18. Briefly describe the goals of your program. (Attach extra page if necessary).

We want students to experience the excitement of original research, before they
spend 3 years in graduate school.

\section*{Survey of Undergraduate Summer Programs in Mathematics}

Name of respondent: Vernescu, Bogdan M.
Program held at what institution? Worcester Polytechnic Inst
Mark here if this program is no longer active: \(\square\) If so, last year of operation:
Answers incorporate 1999 data, if known.
1. Briefly describe the kind of summer program you direct:
\(\boxtimes\) REU
\(\square\) REU Program for certain group
\(\square\) Other
Describe:

\section*{REU in Industrial Mathematics and Statistics}
2. How many years has your program existed (including other directors)? 2
3. In all the years of program operation, what is the total number of students who have participated in the program? 22 ( 11 per year)
4. Of the students who have attended your program, what number were women? 8 ( 36\% )

Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: yes
5. Of the students who have attended your program, what number were Hispanic/Latino, Native American or African American? 4 ( \(18 \%\) )
Use an integer only, do not give a range or percentage. If this figure is an estimate, mark here: \(\bigcirc\) yes
6. Is your program:instructional in nature orresearch oriented, orboth
Comments: The students work on industrial mathematics and statistics projects generated by the industrial partners of the Center for Industrial Mathematics and Statistics at WPI.
7. How many weeks does your program run?
\(\square 1 \quad \square 2 \quad \square 3 \quad \square 4 \quad \square 5 \quad \square 6 \quad \square 7 \quad \boxtimes 8 \quad \square 9 \quad \square 10\)
8. If your program has an instructional component, are students asked to work :
(check both if necessary)
If your program has a research component, do students work on their research projects:
\(\boxtimes\) in groups \(\square\) individually (check both if necessary)

\section*{Comments: The groups have 2-3 students, a faculty advisor and 1-2 industrial advisors.}

For questions 9 -17, please give an estimate if exact figures are not available. (Indicate when the figures are estimates.)
9. Of the students whose academic career you are aware of since they participated in your program, give the number who fit into the following categories:

17 are still undergraduates
4 are currently in a graduate program in the mathematical sciences have received a Masters in the math. sci. (and are no longer in graduate school) have received a Ph.D. in the mathematical sciences

1 are currently in a graduate program in the sciences (excluding the math. sci.) have received a Masters in the sciences (excluding the mathematical sciences) have received a Ph.D. in the sciences (excluding the mathematical sciences)
other (e.g., did not obtain an adv. degree, or are pursuing graduate studies outside of the sciences)
(Please attach a list of the graduate schools that your program participants have attended, if available. Also send a list of the names of students who have received a Ph.D. in the mathematical sciences, if available.)
Comments:
10. What percentage of your program participants have received national graduate fellowships (NSF, Hertz, NDSEG, Ford, NPSC, GEM, etc.)?
\begin{tabular}{lllll}
\(\boxtimes 0-10 \%\) & \(\square 21-30 \%\) & \(\square 41-50 \%\) & \(\square 61-70 \%\) & \(\square 81-90 \%\) \\
\(\square 11-20 \%\) & \(\square 31-40 \%\) & \(\square 51-60 \%\) & \(\square 71-80 \%\) & \(\square 91-100 \%\)
\end{tabular}

This figure is an estimate yes
11. On average, how many oral presentations at conferences have resulted from your program per year?
\(\qquad\) \(\times 2\) years \(=2\)
12. On average, how many poster presentations at conferences have resulted from your program per year?
\[
3 \times 2 \text { years }=6
\]
13. On average, how many publications in refereed journals have resulted from your program per year? (Please attach a list, if available).

0 \(x 2\) years : 0
14. What is the total annual budget for your program?
per student AND/OR \(\$ \mathbf{5 0 , 0 0 0}\) total ( \(\$ 4,545\) per student)
15. Typically, how much direct financial support does your institution provide?
amount: 2 months salary for the first year only
comments:
16. Typically, what kind of nonfinancial support does your institution provide to your program? Examples might include: clerical, release time, graduate students.
clerical
17. Which agency is the principal source of your support (NSF, NSA, others)?
```

NSF
Comments:
$\square$ NSA
18. Briefly describe the goals of your program. (Attach extra page if necessary).

```

The REU in Industrial Mathematics and Statistics at WPI is an 8 week program that focuses on mathematics and statistics, applied to industrial problems. The program, sponsored by NSF, started in 1998 and provided support for 10 undergraduates and 1 graduate student per year.

Our program benefits from an active mathematics faculty in the Mathematical Sciences Department that has a successful experience in conducting research work with the undergraduates within the WPI project-based undergraduate program. It also benefits from the experience gained by the Center for Industrial Mathematics and Statistics at WPI of running student research projects with local business and industrial partners.

Our goal is to provide a unique experience for students of mathematics by introducing them to mathematical research in an industrial environment. The summer research experience provides students with a glimpse of the ways that advanced mathematics is used in the real world to analyze and solve complex problems. It provides challenges not faced in standard undergraduate programs and thus develops skills not always developed in traditional mathematics education.

One of the key features of the REU experience at WPI is that we put the students into a "professional situation." The students work in teams on problems provided by local business and industry, partners of the Center for Industrial Mathematics and Statistics. They work closely with a company representative, to define and develop solutions for problems of immediate importance to the company and a faculty advisor, to maintain a clear focus on the mathematical issues behind the project. The students are called on to produce more than a solution, they must communicate their solution to the company in a form that the company can understand and use.

Our program has an important impact which cannot be obtained in a standard course or REU experience. First of all, the problems are presented in their original language. The students will not receive the distilled mathematical essence of the problem. The process of taking the problem and identifying the key mathematical structure, or refining and redefining the problem is a crucial part of the industrial mathematical experience.```


[^0]:    Library of Congress Cataloging-in-Publication Data
    Conference on Summer Undergraduate Mathematics Research Programs (1999 : Crystal City, Arlington, Va.)

    Proceedings of the Conference on Summer Undergraduate Mathematics Research Programs / Joseph A. Gallian, editor. p. cm.

    ISBN 0-8218-2137-7 (alk. paper)

    1. Mathematics-Study and teaching (Higher)—United States-Congresses. 2. MathematicsResearch—United States-Congresses. I. Gallian, Joseph A. II. Title.
    QA13.C6523 1999
    $510^{\prime} .7^{\prime} 2073$-dc21 00-029329
[^1]:    Received by the editor October 15, 1999.

[^2]:    Received by the editor September 1, 1999.

[^3]:    Received by the editor December 7, 1999.

[^4]:    Received by the editor September 15, 1999.

[^5]:    Received by the editor October 20, 1999.

[^6]:    E-mail address: vicky@afterlife.ncsc.mil

[^7]:    Received by the editor September 24, 1999.

[^8]:    Tulane University, New Orleans, LA 70118
    E-mail address: reu@math.tulane.edu

[^9]:    Received by the editor September 10, 1999.

[^10]:    Received by the editor September 9, 1999.

[^11]:    Received by the editor September 15, 1999.
    ${ }^{1}$ The populations served by the Society for Advancement of Chicanos and Native Americans in Science (SACNAS), an organization with which the program has close ties.

[^12]:    ${ }^{2}$ The universities that these students are enrolled are Cornell University, New Mexico State University, Stanford University, University of Arizona, University of Colorado, University of Iowa, University of Maryland, and University of PuertoRico - Mayagüez.

[^13]:    ${ }^{1}$ Italicized statements are quotes from student evaluation forms.

[^14]:    Received by the editor $15,1999$.
    ${ }^{1}$ The program's Webpage is located at www.mathcs.carleton.edu/smp/.
    ${ }^{2}$ Notices of the American Mathematical Society, vol. 44, no. 8, p. 926.
    ${ }^{3}$ Notices of the American Mathematical Society, vol. 43, no. 8, p. 850.

[^15]:    Received by the editor September 15, 1999.

[^16]:    The George Washington University, Washington, DC 20052
    E-mail address: mmg@math.gwu.edu

[^17]:    ${ }^{1}$ This section is an excerpt from Adhikari, Givant and Nolan (1997) Women in Mathematics: Scaling the Heights, MAA Notes 46, pp97-104.

[^18]:    Received by the editor September 10, 1999.

[^19]:    ${ }^{1}$ It is clear that most of the items that are classified herein as goals are exactly that. With others, some conference participants felt that the term "objective" may have been a better classification. For the most part, conference participants felt that identifying items was more important than classifying them and chose to use the term "goal."

[^20]:    2 "Diverse" meaning students from schools of different types that offer different levels of opportunites to engage in mathematical research, students from underrepresented groups, students with different levels of mathematical maturity, etc.

[^21]:    Received by the editor January 18, 2000.
    ${ }^{1}$ Visiting Professor, University of California, Berkeley
    ${ }^{2}$ From 1991 to 1997, SMI/SIMS was a six-week summer program for 20 undergraduate women. While in the program, students participated in two seminars in advanced mathematics, a colloquium series given by research mathematicians, and

[^22]:    panels on various aspects of graduate school. For more information on the program see Nolan (2000)
    ${ }^{3}$ Office of Student Life-Educational Development, University of California, Berkeley
    ${ }^{4}$ Synthesis Coalition, University of California, Berkeley

[^23]:    Received by the editor December 7, 1999.

[^24]:    Received by the editor November 16, 1999.

[^25]:    Received by the editor December 7, 1999.
    These are personal notes of the plenary sessions of the REU conference. I am happy to share them and feel they are reasonably accurate transcriptions of what was said, but it should be noted that the speakers have not reviewed them. They appear here in essentially raw form.

[^26]:    Oregon State University
    E-mail address: Robby@orst.edu

[^27]:    Received by the editor November 21, 1999.

[^28]:    University of Minnesota, Duluth, Duluth, MN 55812
    E-mail address: jgallian@d.umn.edu

