Meeting: 1003, Atlanta, Georgia, SS 6A, AMS-ASL Special Session on Reverse Mathematics, I

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Carl Mummert* (mummert@math.psu.edu), Department of Mathematics, Penn State University, McAllister Building, University Park, PA 16802. *Representing Second Countable Topological Spaces in Second-Order Arithmetic.* Preliminary report.

I address the problem of formalizing general topology in second-order arithmetic (Z_2). This cannot be done in a general manner because of countability requirements, but a wide class of spaces can be represented in Z_2.

The coding method uses a form of Stone duality. Let $(P, <)$ be a countable poset. A filter on P is a nonempty, upward-closed set F such that if $a, b \in F$ then $c < a$ and $c < b$ for some $c \in F$. Let $\mathrm{MF}(P)$ denote the class of maximal filters on P. We give $\mathrm{MF}(P)$ the topology generated by $\{ \{f \mid f \in \mathrm{MF}(P) \wedge p \in f\} \mid p \in P\}$. We say that a second-countable topological space X is representable if X is homeomorphic to $\mathrm{MF}(P)$ for some countable poset P.

Representable spaces form a rich class: all Polish spaces are representable, as are many nonmetrizable spaces. I will discuss the formalizations of certain theorems in descriptive set theory, provable in Z_2, which show that representable spaces are quite similar to Polish spaces. Z_2 also proves a formalized Urysohn metrization theorem, which says that a regular representable space is metrizable. I will also discuss Reverse Mathematics aspects of these results.

This work is part of my Ph.D. thesis, with advisor Stephen G. Simpson. (Received October 03, 2004)