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1003-03-823            **Alberto Marcone\*** ([marcone@dimi.uniud.it](mailto:marcone@dimi.uniud.it)), Dipartimento di Matematica e Informatica,  
Univ. di Udine, viale delle Scienze 206, 33100 Udine, Italy. *Finite better quasi orders.*

Let  $BQO(n)$  be the following statement: the partial order consisting of  $n$  mutually incomparable elements is a better quasi order. For every  $n$ ,  $ATR_0$  proves  $BQO(n)$ .  $BQO(2)$  is provable in  $RCA_0$ . For  $n \geq 3$  the proof-theoretic strength of  $BQO(n)$  is unknown, and may lie anywhere between  $RCA_0$  and  $ATR_0$ . However the answer to this question does not depend on  $n$ :

**THEOREM.** Let  $T$  be a subsystem of second order arithmetic containing  $RCA_0$  and suppose that  $T$  proves  $BQO(3)$ . Then for any  $n > 0$ ,  $BQO(n)$  is a theorem of  $T$ .

If  $BQO(3)$  is equivalent to  $ATR_0$  than some basic statements about bqos (e.g. the closure of bqos under cartesian products) will also be equivalent to  $ATR_0$ . Such a result would establish beyond any doubt that very little bqo theory can be carried out in systems properly weaker than  $ATR_0$ . (Received September 30, 2004)