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Eric Schechter* (eric.schechter@vanderbilt.edu), Mathematics Dept, Vanderbilt University, 1326 Stevenson Center, Nashville, TN 37240. *Kelley's specialization of Tychonov's Theorem is equivalent to the Boolean Prime Ideal Theorem.*

Tychonov's Theorem (hereafter abbreviated TT) was proved in the 1930s by several methods, all using the Axiom of Choice (AC). In 1950 John L. Kelley published a proof of the converse, $TT \Rightarrow AC$, thus establishing equivalence of the two principles. His proof contained a very minor error, which was pointed out and corrected by Frank Plastria in 1972. Kelley had argued $TT \Rightarrow K \stackrel{*}{\Rightarrow} AC$, using the intermediate principle

(K) any product of cofinite topologies is compact,

but his proof of (*) was faulty. Plastria repaired the proof of $TT \Rightarrow AC$ by replacing principle K with a slightly different, more complicated principle. That left these questions unanswered: Is the implication (*) true but unproved, or is it actually false? Is the principle K equivalent to AC, or strictly weaker?

In this note we shall show that (*) is false. We shall show that K is equivalent to the Boolean Prime Ideal Theorem (BPI), a principle well known to be strictly weaker than AC. (Received September 28, 2005)