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**Thomas J. Osler\*** (osler@rowan.edu), Prof. Thomas J. Osler, Mathematics Department,  
Rowan University, Glassboro, NJ 08028. *A proof of the continued fraction expansion of  $\exp(1/M)$ .*

This paper gives another proof for the remarkable simple continued fraction  $\exp(1/M) = [1; M - 1, 1, 3M - 1, 1, 5M - 1, 1, 7M - 1, 1, 9M - 1, \dots]$ . Here  $M$  is any positive number. We use the notation  $x = [a; b, c, \dots]$  for the simple continued fraction  $x = a + (1/(b + (1/c + \dots)))$ . In 1744 Euler proved that  $e = [1; 0, 1, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, \dots]$ , and in 1873 Hermite gave another proof that arose while he was showing that  $e$  is transcendental. This proof involves three unexpected integrals. Recently Cohn gave a short and eloquent variation of Hermite's proof that appears on his webpage: "<http://research.microsoft.com/~cohn/Papers/e.pdf>". This paper is a generalization of Cohn's proof. (Received September 24, 2005)