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Let K be a finite connected polytopal combinatorial $(d - 1)$ -manifold embedded in R^d . That is, K is a complex of polytopes such that the link of every vertex is PL-homeomorphic to the boundary of a $(d - 1)$ sphere. The interior of K is the finite portion of R^d determined by K . The interior angle of a face F in K is $\omega_K(F) = \frac{\text{vol}(B_\epsilon(x) \cap K)}{\text{vol}(B_\epsilon(x))}$ where x is in the interior of F and $B_\epsilon(x)$ is the ball of radius ϵ centered at x for ϵ sufficiently small. Then the i^{th} angle sum of K is $\alpha_i(K) = \sum_{i\text{-faces } F \subseteq K} \omega_K(F)$ for $0 \leq i \leq d - 1$.

It has long been known that if K is the boundary of a d -polytope, then $\sum_{i=0}^{d-1} (-1)^i \alpha_i(K) = (-1)^{d+1}$. This is called the Gram relation and is analogous to the Euler characteristic on polytopes. The Perles relation on angle sums is also analogous to the Dehn-Sommerville relations on faces of polytopes.

We discuss analogs of these results on a larger class of K , including K not homeomorphic to a sphere. These results continue to parallel the Euler characteristic. Specifically, for genus g surfaces in this class, $\sum_{i=0}^{d-1} (-1)^i \alpha_i(K) = (-1)^{d+1}g$. (Received September 28, 2005)