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Godfried T. Toussaint* (godfried@cs.mcgill.ca), School of Computer Science, McConnell Hall, Room 318, 3480 University Street, Montreal, Quebec H3A 2A7, Canada. *Elementary Proofs of the Hexachordal Theorem*. Preliminary report.

We consider a subset of k out of n points on the circular lattice, called *cyclotomic* sets in the crystallography literature. Every pair of such points determines a (geodesic) distance. The remaining $(n-k)$ lattice points determine a complementary set. Two sets of points which are not congruent but possess the same multiset of distances are said to be *homometric*. The hexachordal theorem states that two non-congruent complementary sets with $k = n/2$ (and n even) are homometric. The earliest proofs of this theorem in the music literature appear to be due to Milton Babbitt and David Lewin, who used topology and group theory. In 1974 Eric Regener found an elementary proof. The music theorists appear to be unaware that this theorem was known to crystallographers about thirty years earlier. It seems to have been proved first (but not published) by Lindo Patterson around 1940. The first published proof in the crystallography literature is due to Martin Buerger; it is based on image algebra, and is non-intuitive. Much simpler and elegant elementary proofs were later found by Juan Iglesias and Steven Blau. We elucidate the proofs of Iglesias, and Blau, and compare them to the proof of Regener. (Received August 30, 2006)