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Alison B Miller* (miller5@fas.harvard.edu), 320 Dunster House Mail Center, Cambridge, MA 02138. *Asymptotic Bounds for Permutations Containing Many Different Patterns.*

We say that a permutation $\sigma \in S_n$ contains a permutation $\pi \in S_k$ as a pattern if some subsequence of σ has the same order relations among its entries as π . We improve on results of Wilf and Coleman that bound the asymptotic behavior of $\text{pat}(n)$, the maximum number of distinct patterns of any length that can be contained in a permutation of length n . We prove that $2^n - O(n^2 2^{n-\sqrt{2n}}) \leq \text{pat}(n) \leq 2^n - \Theta(n 2^{n-\sqrt{2n}})$ by estimating the amount of redundancy due to patterns that are contained multiple times in a given permutation. This settles a conjecture of Bóna by showing that $\lim_{n \rightarrow \infty} \frac{\text{pat}(n)}{2^n} = 1$. We also consider the question of superpatterns, which are permutations that contain all patterns of a given length k . We give a simple construction that shows that L_k , the length of the shortest k -superpattern, is at most $\frac{k(k+1)}{2}$. This may lend evidence to a conjecture of Eriksson et al. that $L_k \sim \frac{k^2}{2}$. (Received September 24, 2006)