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Jonathan M. Borwein* (jborwein@cs.dal.ca), Faculty of Computer Science, 6050 University Ave, Halifax, NS B3H 1W5, Canada. *Experimental discovery of Apéry-type identities for even zeta values.*

This talk will describe older and very recent work, see [1], in which Bailey, Bradley and I hunted for various desired generating functions for zeta functions and then were able to methodically prove our results.

One example is

$$3 \sum_{k=1}^{\infty} \frac{1}{\binom{2k}{k}(k^2 - x^2)} \prod_{n=1}^{k-1} \frac{4x^2 - n^2}{x^2 - n^2} = \sum_{n=1}^{\infty} \frac{1}{n^2 - x^2} \quad (1)$$
$$\left[= \sum_{k=0}^{\infty} \zeta(2k + 2) x^{2k} = \frac{1 - \pi x \cot(\pi x)}{2x^2} \right].$$

The constant term in (1) recovers the well known identity

$$3 \sum_{k=1}^{\infty} \frac{1}{\binom{2k}{k}k^2} = \sum_{n=1}^{\infty} \frac{1}{n^2} = \zeta(2).$$

As I hope to show, this led to one of the most satisfying experimental mathematics experiences we have had.

[1] David Bailey, Jonathan Borwein and David Bradley, “Experimental Determination of Apéry-type Formulae for $\zeta(2n + 2)$,” *Exp Math*, in press, 2006. [D-drive Preprint 295]. (Received July 25, 2006)