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Tariq Qazi* (tqazi@vsu.edu), Department of Mathematics and Computer Science, P. O. Box 9068, Petersburg, VA. *On Bernstein's Inequality for Entire Functions of Exponential Type.*

Let f be an entire function of exponential type $\tau > 0$ such that $|f(x)| \leq M$ on the real axis. Then $|f'(x)| \leq M\tau$ on the same axis. The upper bound for $|f'(x)|$ is attained if and only if $f(z) := ae^{i\tau z} + be^{-i\tau z}$, where $|a| + |b| = M$. It was shown by R.P. Boas [Illinois Journal of Mathematics, Vol. 1 (1957)] that the upper bound $M\tau$ for $|f'(x)|$ can be replaced by $M\tau/2$ if $f(z)$ has no zeros in the upper half-plane and $h_f(\pi/2) = 0$, where $h_f(\theta)$ is the Phragmén–Lindelöf indicator function of f . What can we say about $|f'(x)|$ at a point x of the real axis if f has as many zeros in the upper half-plane as it has in the lower half-plane? We show that for any given $\varepsilon > 0$ we can find an entire function f_ε of exponential type $\tau > 0$ such that $|f_\varepsilon(x)| \leq M$ on the real axis, $h_{f_\varepsilon}(\pi/2) = 0$ and the supremum of $|f'_\varepsilon(x)|$ on the real axis is greater than $M(\tau - \varepsilon)$. (Received September 24, 2006)