

1023-43-1180

Ismail Kombe* (ikombe@okcu.edu), Oklahoma City University, Department of Mathematics,
2501 N. Blackwelder, Oklahoma City, OK 73106-1493. *Rellich type inequality on Carnot Groups.*

The classical Rellich inequality states that for $n \geq 5$

$$\int_{\mathbb{R}^n} |\Delta\phi(x)|^2 dx \geq \frac{n^2(n-4)^2}{16} \int_{\mathbb{R}^n} \frac{|\phi(x)|^2}{|x|^4} dx$$

for all $\phi \in C_c^\infty(\mathbb{R}^n \setminus \{0\})$ and the constant $\frac{n^2(n-4)^2}{16}$ is sharp. It is well known that the Euclidean space \mathbb{R}^n with its usual abelian group structure is a trivial Carnot group. We ask now the question: What is the most general form of the Rellich inequality on general Carnot groups? Let \mathbb{G} be a Carnot group with homogeneous dimension $Q \geq 3$ and let N be a homogeneous norm associated to Folland's fundamental solution u for the sub-Laplacian $\Delta_{\mathbb{G}}$, $\phi \in C_0^\infty(\mathbb{G} \setminus \{0\})$, $\alpha \in \mathbb{R}$, $Q + \alpha - 4 > 0$. We proved the following Rellich-type inequality

$$\int_{\mathbb{G}} \frac{N^\alpha}{|\nabla_{\mathbb{G}} N|^2} |\Delta_{\mathbb{G}} \phi|^2 dx \geq \frac{(Q + \alpha - 4)^2 (Q - \alpha)^2}{16} \int_{\mathbb{G}} N^\alpha \frac{|\nabla_{\mathbb{G}} N|^2}{N^4} \phi^2 dx.$$

Here dx and $\nabla_{\mathbb{G}}$ denotes the Haar measure and the sub-elliptic gradient on \mathbb{G} , respectively. (Received September 25, 2006)