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Existence for a model of fracture evolution based on crack fronts.

Although the notion of a crack front is often used in engineering models for fracture, there has not been a well posed mathematical definition. Here, we consider a model of crack growth where the energy dissipation occurs at the crack front in a way that cannot be reformulated without the front. This requires that we first define the front $F(t)$ of a crack and the “velocity” of the crack, v , defined on this front. We will present two definitions, one that is set-theoretic and another that is measure-theoretic, and we will show that these two definitions are essentially equivalent. We will employ these definitions and a certain class of fracture paths to prove existence for a model of crack evolution, where the fracture trajectory $u(x, t)$ is the minimizer of the energy:

$$I[u] := \int_T e^{-\frac{t}{\epsilon}} \left\{ \int_{\Omega} W(\nabla u) dx + \int_{F(t)} v^p d\mathcal{H}^{N-2} \right\} dt.$$

This energy includes both an elastic bulk energy and a dissipation that occurs along the front $F(t)$. We will analyze this variational problem in a two dimensional setting, i.e., where $\Omega \subset \mathbb{R}^2$, finally showing the existence of an optimal crack path. (Received September 26, 2006)