

1023-H5-282

David J Pengelley* (davidp@nmsu.edu), Mathematical Sciences, New Mexico State University, Las Cruces, NM 88003. *Dances between continuous and discrete: Euler's summation formula in his own words.*

Euler developed and used his summation formula to estimate the sum of reciprocal squares to 14 digits — a value mathematicians had been competing for since Leibniz's astonishing discovery that the alternating sum of the reciprocal odd numbers is exactly $\pi/4$. This competition became known as the Basel Problem, and Euler's approximation probably spurred his spectacular solution, $\pi^2/6$. Subsequently he connected his summation formula to Bernoulli numbers and many other topics, masterfully circumventing that it almost always diverges. He applied it to estimate harmonic series partial sums, the gamma constant, and sums of logarithms, thereby calculating large factorials (Stirling's series) with ease. He even commented that his approximation of pi was surprisingly accurate for so little work.

I will illustrate Euler's achievements in his own (translated) words, and discuss an undergraduate teaching unit of original sources about the search for formulas for sums of numerical powers in relation to integration, seen through writings of Archimedes, Fermat, Pascal, Jakob Bernoulli, and Euler. I will show Euler's idea for deriving his summation formula, how he applied the formula, and discuss exercises for students, including use of computing software. (Received September 02, 2006)