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Jane Butterfield, Tracy Grauman, Bill Kinnersley, Kevin Milans, Christopher Stocker
and **Douglas B. West*** (west@math.uiuc.edu). *On-line Ramsey theory in bounded-degree graphs.*

In graph Ramsey Theory, a Builder presents a graph whose edges a Painter must color red or blue. Builder wins by forcing a monochromatic copy of a graph G ; Painter wins by avoiding that. In the on-line version, Builder presents edges one by one, with Painter required to color each edge before seeing later edges. Infinitely many vertices are available, but Builder is restricted to keep the presented graph within a family \mathcal{H} . This defines the game (G, \mathcal{H}) .

We examine such games with $\mathcal{H} = \mathcal{S}_k$, where \mathcal{S}_k is the class of graphs with maximum degree at most k . We show that Builder wins (G, \mathcal{S}_3) if and only if G is a path or $K_{1,3}$. If T is a tree with maximum degree d , then always Builder wins (T, \mathcal{S}_{2d-1}) , and this is sharp: for some trees, Painter wins (T, \mathcal{S}_{2d-2}) . If $\Delta(G) \leq 2$, then Builder wins (G, \mathcal{S}_6) . If G is a cycle, then Builder wins (G, \mathcal{S}_5) . If G is an even cycle, a triangle, or a sufficiently large odd cycle, then Builder wins (G, \mathcal{S}_4) . (Received September 11, 2007)