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Jacob Fox\* (jacobfox@math.princeton.edu), Fine Hall, Washington Road, Princeton, NJ 08544, and Po-Shen Loh (ploh@math.princeton.edu) and Benny Sudakov (bsudakov@math.ucla.edu). Large induced trees in  $K_r$ -free graphs.

For a graph G, let t(G) denote the maximum number of vertices in an induced subgraph of G that is a tree. We study the problem of bounding t(G) for graphs which do not contain a complete graph  $K_r$  on r vertices. This problem was posed twenty years ago by Erdős, Saks, and Sós. Substantially improving earlier results of various researchers, we prove that every connected triangle-free graph on n vertices contains an induced tree of order  $\sqrt{n}$ . When  $r \geq 4$ , we also show that  $t(G) \geq \frac{\log n}{4 \log r}$  for every connected  $K_r$ -free graph G of order n. Both of these bounds are tight up to small multiplicative constants, and the first one disproves a recent conjecture of Matoušek and Šámal. (Received September 06, 2008)