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Richard D. Neidinger* (rineidinger@davidson.edu), Davidson College, Box 7002, Davidson, NC 28035-7002. *Algorithms for Multivariable Polynomial Interpolation.*

Does there exist a unique polynomial of degree d in n variables that agrees with values at (correct number) nodes? First, consider nodes as coordinates on a finite grid, using (hopefully few) tick marks on each axis. A general existence and uniqueness theorem always holds where the space of polynomials is the Span of classic Newton polynomials, e.g. a node at grid point (x_2, y_1) would correspond to $(x - x_0)(x - x_1)(y - y_0)$. With special node structure, the Span will be polynomials of degree d and an efficient divided-difference algorithm will produce the coefficients. More generally, the question asks if a multivariable Vandermonde matrix M is invertible. Recent algorithms can be explained by Gaussian elimination on M^T using blocks of the same monomial order. Row reduce to block identity matrices along the diagonal. If successful, these row operations on I produce normalized Newton polynomials that are one at a specific node but zero on all nodes of lower or same order. Then, a simple back-substitution algorithm can find coefficients. If unsuccessful, a row of zeros corresponds to a polynomial that is zero at all nodes, making existence and uniqueness impossible. Operations can stop sooner by using block matrix operations in Gaussian elimination. (Received September 16, 2008)