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John D Pesek* (pesek@udel.edu), Food & Resource Economics, 213 Townsend Hall, University of Delaware, Newark, DE 19713. *Generalizations of Varignon's and Steiner's Theorems to Simplexes Using Set Partitions.*

Set partitions of the vertex set of a simplex can be regarded as a natural generalization of the concept of opposite side for triangles. Steiner's theorem for a tetrahedron says that sliding a pair of opposite edges along the lines containing them does not change the volume of the tetrahedron. A variant of Varignon's theorem says that if a plane is parallel to each of two opposite sides of a tetrahedron, it cuts the tetrahedron in a parallelogram. In each case the pair of opposite sides is determined by a partition of the vertices of the tetrahedron into two two-element sets. If a simplex and a vertex partition are considered and we 'slide' the subsimplexes defined by the points in each block of the partition within the flats determined by the subsimplexes, volume is preserved. If a simplex and a vertex partition with k blocks are considered and the simplex is intersected by a flat of dimension $k - 1$ less than that of the simplex which is parallel to each of the flats determined by the points in the blocks of the partition, then the flat intersects the simplex in a poly-simplex (a product of simplexes) with parallel 'sides' (A parallelogram is the product of two one-simplexes). A connection between these two results is also discussed. (Received September 16, 2008)