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Lobb's Generalization of Catalan's Parenthesization Problem.

Recently, A. Lobb investigated the following generalization of Catalan's well-known parenthesization problem: Find the number $L_{n,m}$ of arrangements of $n + m$ positive ones and $n - m$ negative ones such that every partial sum is nonnegative, where $0 \leq m \leq n$. Using induction, Lobb showed that

$$L_{n,m} = \frac{2m + 1}{n + m + 1} \binom{2n}{n + m}$$

So $L_{n,0} = C_n$, the n th Catalan number. Interestingly, Lobb numbers $L_{n,m}$ can be extracted from Pascal's triangle by taking successive differences of adjacent elements to the left of and in the middle column in row $2n$. We will show that $L_{n,m}$ is odd for every m if and only if either $n = 0$ or n is a Mersenne number. Consequently, $L_{n,m}$ is odd for every m if and only if C_n is odd. In addition, we will extract a number of interesting properties involving Catalan and Lobb numbers; this includes the fact that every Catalan number C_{2n} can be expressed as the sum of $n + 1$ squares. (Received September 09, 2008)