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Euler, Dilog, and Zeta(2).

The story of Euler's original evaluation of $\zeta(2)$ and subsequent rederivations is well known. Each derivation shows the familiar Euler genius for creative manipulation of series. In the modern context, it is tempting to attempt an evaluation of $\zeta(2)$ by more mundane means, by manipulating the series $f(z) = \sum z^k/k^2$. One progresses without difficulty to find $\zeta(2) = \int_0^1 -\frac{\ln(1-t)}{t} dt$. Evaluating this integral presents an obstacle, but success is possible if one is aware of some properties of the dilog function $\text{Li}_2(z) = \int_0^z \frac{-\ln(1-t)}{t} dt$. Thus we obtain another derivation of the value of $\zeta(2)$. And who discovered the requisite properties? Euler!

Euler's initial work with Li_2 predates the evaluation of $\zeta(2)$, appearing in the same 1730 paper where he estimates $\zeta(2)$ to 6 decimal places. The critical identity for the definite integral above appears in a paper dating to 1779. There Euler might easily have evaluated $\zeta(2)$, but instead takes the value as a known result. Did he realize that his results provided yet another path to $\zeta(2)$? Could he have failed to notice? We know what Euler knew, but when did he know it? This historical puzzle remains unsolved. (Received September 18, 2009)