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An *integer part*  $I$  of a real closed field  $R$  is a discrete ordered subring containing 1 such that for all  $r \in R$  there exists a unique  $i \in I$  with  $i \leq r < i + 1$ . Shepherdson showed that  $I$  is an integer part for a real closed field if and only if  $I$  is a model of  $I\text{Open}$ , the fragment of arithmetic with induction axioms restricted to quantifier-free formulas. Mourgues and Ressayre later showed that every real closed field  $R$  has an integer part. Let  $k$  be the residue field of  $R$ , and let  $G$  be the value group of  $R$ . Let  $k\langle\langle G \rangle\rangle$  be the set of *generalized power series* of the form  $\sum_{g \in S} a_g g$  where  $a_g \in k$  and the support of the power series  $S \subseteq G$  is well ordered. Mourgues and Ressayre produce an integer part of  $R$  by building an isomorphism between  $R$  and a truncation closed subfield of  $k\langle\langle G \rangle\rangle$ . In order to understand the complexity of integer parts, we analyze an algorithmic version of the Mourgues and Ressayre construction and provide upper bounds on the ordinal lengths of the generalized power series in the image of  $R$ . These bounds are then used to show that every computable real closed field has an integer part of complexity  $\Delta_{\omega\omega}^0$ . (Received September 15, 2009)