1056-03-75 **John Harrison*** (johnh@ichips.intel.com), Intel Corporation, JF1-13, 2111 NE 25th Avenue, Hillsboro, OR 97124. *Decidability and Undecidability in Theories of Real Vector Spaces.*

It's natural to formulate theories of real vector spaces using a 2-sorted first-order language with a sort for the scalars and a sort for the vectors. Introduction of coordinates reduces the theory of a vector space of a specific finite dimension to the first-order theory of the real numbers, known to be decidable since Tarski. Experience in the actual formalization of mathematics motivates an investigation into decision problems for various more general 2-sorted first-order theories of vector spaces, with or without with an inner product, norm, assumption of completeness, or restriction on dimension.

Solovay, Arthan and the speaker have carried out a systematic study of decidability for such theories. The theories of real vector spaces, inner product spaces, and Hilbert spaces turn out to be decidable and to admit quantifier elimination in a slightly expanded language. However, similar theories of normed spaces, Banach spaces and metric spaces are not even arithmetical: the theory of 2-dimensional Banach spaces, for example, has the same many-one degree as the set of truths of second-order arithmetic. However, by restricting quantifier alternations, we can arrive at some decidable fragments of these theories, a fact that has proved useful in mechanizing proofs. (Received September 15, 2009)