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**Hung-ping Tsao\*** (hptsao@hotmail.com), 1151 Highland Drive, Novato, CA 94949. *Triangular arrays induced from trigonometric functions.*

Using  $\cos 2nA = 2\cos nA \cos nA - 1$  and  $\cos(2n-1)A = 2\cos nA \cos(n-1)A - \cos A$ , we can obtain the polynomial expressions in  $\cos A$  for  $\cos nA$ . Let  $c(m, k)$  denote the triangular array formed by the coefficients in question. Then we can use  $c(2n-1, 2k) = c(2n, 2k-1) = 0$ ,  $c(4n-2, 0) = -1$ ,  $c(4n, 0) = 1$ ,  $c(1, 1) = 1$ ,  $c(2, 2) = 2$ ,  $c(2n+1, k) = 2c(2n, k-1) - c(2n-1, k)$  and  $c(2n+2, 2k) = 2c(2n+1, 2k-1) - c(2n, 2k)$  to generate the entire array. The triangular array induced from  $\sin(2n-1)A$  is the same as that induced from  $\cos(2n-1)A$  except the signs for even  $n$ . Let  $s(2n, k)$  be the triangular array induced from  $\sin 2(n+1)A / \cos A$ . Then  $s(2n, 2n-2k)$  is a linear combination of  $c(2n, 2n-2k)$  and  $c(2n-2, 2n-2)$  with the coefficients only involving powers of 4 and the binomial coefficient  $C(2n-k-1, k-2)$ . Furthermore, we have  $s(2n, 2k) = s(2n-2, 2k) - 2(n-k+1)s(2n, 2k-2)/k$ . By writing  $\tan(2n-1)A$  and  $\tan 2nA$  into rational functions of  $\tan A$  and considering only nonzero entries, we obtain arrays  $t$ ,  $u$ ,  $v$  and  $w$  from numerators and denominators so that  $t$  and  $u$  are the same for odd  $n$  and differ in signs, where  $u(n, k) = C(2n-1, 2k-2)$  for even  $n-k$  and  $u(n, k) = -C(2n-1, 2k-2)$  for odd  $n-k$ . We also have  $v(n, k) = C(2n, 2k-1)$  for even  $k$ ,  $v(n, k) = -C(2n, 2k-1)$  for odd  $k$ ,  $w(n, k) = C(2n, 2k)$  for even  $k$  and  $w(n, k) = -C(2n, 2k)$  for odd  $k$ .  $c$  and  $u$  are also related. (Received May 27, 2009)