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Leanne Robertson* (robertle@seattleu.edu), Mathematics Department, Seattle University,
901 12th Ave., Seattle, WA 98122. *Monogeneity in cyclotomic fields.*

A number field is said to be *monogenic* if its ring of integers is a simple ring extension $\mathbf{Z}[\alpha]$ of \mathbf{Z} . It is a classical and usually difficult problem to determine whether a given number field is monogenic, and if it is, to find all numbers α that generate a power integral basis $\{1, \alpha, \alpha^2, \dots, \alpha^k\}$ for the ring. We consider cyclotomic fields, which are known to be monogenic, and by studying units in the ring arrive at a conjectural solution to the problem of finding all the power integral bases for these fields. G. Ranieri recently proved that if $L = \mathbf{Q}(\zeta_n)$ is a cyclotomic field whose conductor n is relatively prime to 6, then up to integer translation all the generators lie on the unit circle or the line $\operatorname{Re}(z) = 1/2$ in the complex plane. We prove that this interesting geometric restriction extends to the cases of conductor $n = 3k$ and $n = 4k$, where k is relatively prime to 6. We use this result to find all power integral bases for $\mathbf{Q}(\zeta_n)$ for $n = 15, 20, 21, 28$, and so verify our conjecture in these cases. (Received September 22, 2009)