1056-11-956 **Eric S. Rowland*** (erowland@tulane.edu), Mathematics Department, Tulane University, New Orleans, LA 70118. *The number of nonzero binomial coefficients modulo p^α.*

In 1947 Nathan Fine used Lucas' theorem to compute the number $a_p(n)$ of binomial coefficients $\binom{n}{m}$, $0 \leq m \leq n$, that are not divisible by a prime p: If $n_l \cdots n_1 n_0$ is the standard base-p representation of n, then $a_p(n) = \prod_{i=0}^{l} (n_i + 1)$.

One can set up (using generating functions, for example) a recursive algorithm to compute for a given n the number of integers $0 \leq m \leq n$ such that there are precisely c carries involved in adding m to $n - m$ in base b. For $b = p$, Kummer's theorem renders this recurrence as a generalization of Fine's theorem, giving a way to compute the number $a_{p^\alpha}(n)$ of nonzero binomial coefficients modulo p^α in terms of the base-p digits of n. For example, for $\alpha = 2$ we get the explicit expression

$$a_{p^2}(n) = \prod_{i=0}^{l} (n_i + 1) \cdot \left(1 + \sum_{i=0}^{l-1} \frac{p - (n_i + 1)}{n_i + 1} \cdot \frac{n_{i+1}}{n_{i+1} + 1} \right).$$

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