

Flows of Nonsmooth Vector Fields

Consider a vector field v on the Euclidean space. The classical Cauchy–Lipschitz (also named the Picard–Lindelöf) theorem states that, if the vector field is Lipschitz in space, for every initial datum x there is a unique trajectory starting at x at time 0 and solving the ODE $\dot{\gamma}(t)=v(t,\gamma(t))$. The theorem loses its validity as soon as v is slightly less regular. However, if we bundle all trajectories into a global map allowing x to vary, a celebrated theory started by DiPerna and Lions in the 80s shows that there is a unique such flow under very reasonable conditions and for much less regular vector fields. This theory has a lot of repercussions to several important partial differential equations where the idea of “following the trajectories of particles” plays a fundamental role.

In my lectures I will review the state of the development and the state of the art of the subject, touching upon a variety of related topics, such as the most recent surprising outcomes of convex-integration techniques and the most interesting applications to evolutionary PDEs.



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Lecture I Wednesday, January 4, 2023, 1:00 p.m.–1:50 p.m.,
Hynes Convention Center, Ballroom AB

Lecture II Thursday, January 5, 2023, 1:00 p.m.–1:50 p.m.,
Hynes Convention Center, Ballroom AB

Lecture III Friday, January 6, 2023, 1:00 p.m.–1:50 p.m.,
Hynes Convention Center, Ballroom AB

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